Emittance Growth in Resonance Crossing in FFAGs

K.Y. Ng

Fermilab

X. Pang, F. Wang, X. Wang, S.Y. Lee

IUCF, Indiana University

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Introduction

- FFAG • a favorable candidate for proton drivers.
- Constant guide field • rep rate can be very high, in the KHzs.
- Scaling design • nonlinear fields, large magnet apertures.
- Non-scaling design • linear fields, much smaller magnet apertures.
- Disadvantage: tunes change by many units in a ramp cycle.
- Beam quality can deteriorate when crossing resonances.
- Tune-ramp rate

$$\frac{\Delta \nu_{x,z}}{\Delta n} \sim -\left(1-\frac{D}{R}\right)\frac{\nu_{x,z}}{2\beta^2 E}\frac{\Delta E}{\Delta n}$$

typically ~ -10^{-3} to -10^{-2} per turn

An example

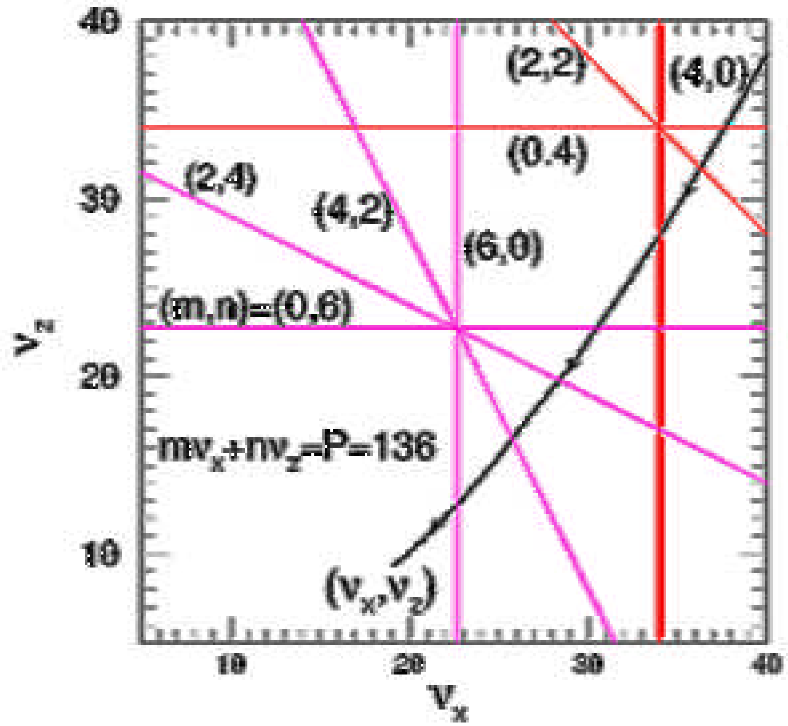
- Ruggiero suggested 3 FFAGs for BNL AGS • 10 MW proton driver.
- Tunes change from

  $(\nu_x, \nu_z) = (40,38.1)$ to $(19.1,9.3)$
- Cross systematic 4^{th} and 6^{th}

  resonances: (P=136)

  $4\bullet_x=P$, $4\bullet_z=P$, $2\bullet_x+2\bullet_z=P$

  $6\bullet_x=P$, $6\bullet_z=P$, $2\bullet_x+4\bullet_z=P$, $4\bullet_x+2\bullet_z=P$



- S.Y. Lee pointed out emittance growth can be large if crossing rate is slow, and gave a scaling relationship.
- Thus phase advance per cell cannot be near 90^0 and 60^0.
- Lattice design can become very restricted.

The Model

- Lee, et al. studied sp-ch driven 4th order systematic resonances and field-error driven linear resonances.
- We study here sp-ch driven 6th order systematic resonances and octupole driven 4th order parametric resonances.
- Our study bases on simulations.
- Lattice is similar to Fermilab Booster, with P = 24 FODO cells.
- Sp-ch kicks applied at every half cell.
- Transport matrices used from magnet to magnet.
- Kinetic energy fixed at 1 GeV; tunes allowed to ramp.
- Syn. oscillation neglected since emittance usually grows much faster.
- Assume bi-Gaussian distribution: $\rho(x,z) = \frac{Ne}{2\pi\sigma_x\sigma_z} e^{-x^2/2\sigma_x^2 - z^2/2\sigma_z^2}$

Source of Systematic Resonances

- Sp Ch potential: $V_{sc}(x,z) = \frac{K_{sc}}{2}\int_0^{\infty} \frac{\exp\left[-\frac{x^2}{2\sigma_x^2+t} - \frac{z^2}{2\sigma_z^2+t}\right] - 1}{\sqrt{(2\sigma_x^2+t)(2\sigma_z^2+t)}} dt, \qquad K_{sc} = \frac{2Nr_0}{\beta^2\gamma^3}$
- Expansion: $(r = \sigma_z/\sigma_x)$

linear

4th order

$$V_{sc}(x,z) = -\frac{K_{sc}}{2}\left\{\left[\frac{x^2}{\sigma_x(\sigma_x+\sigma_z)} + \frac{z^2}{\sigma_z(\sigma_x+\sigma_z)}\right] - \frac{1}{\sigma_x^2(\sigma_x+\sigma_z)^2}\left[\frac{2+r}{3}x^4 + \frac{2}{r}x^2z^2 + \frac{1+2r}{3r^3}z^4\right]\right.$$
$$\left. + \frac{1}{72\sigma_x^3(\sigma_x+\sigma_z)^3}\left[\frac{8+9r+3r^2}{5}x^6 + \frac{3(3+r)}{r}x^4z^2 + \frac{3(3r+1)}{r^3}x^2z^4 + \frac{8r^2+9r+3}{5r^5}z^6\right] + \cdots\right\}$$

6th order

- Sp-ch force: $F_{x,sc} = -\frac{\partial V_{sc}}{\partial x} \approx \frac{K_{sc}x}{\sigma_x(\sigma_x+\sigma_z)}\exp\left[-\frac{x^2+z^2}{(\sigma_x+\sigma_z)^2}\right],$

$$F_{z,sc} = -\frac{\partial V_{sc}}{\partial z} \approx \frac{K_{sc}z}{\sigma_z(\sigma_x+\sigma_z)}\exp\left[-\frac{x^2+z^2}{(\sigma_x+\sigma_z)^2}\right].$$

- Effective force is easier to use than the exact one.
- Exact analytic expression has an apparent singularity when $\sigma_x = \sigma_z$

6^{th} order Systematic Resonances

§ In action-angle variables,

$$V_{sc,6}(J_x, J_z, \psi_x, \psi_z, \theta) \approx -\frac{1}{R}\sum_{\ell} |G_{60\ell}| J_x^3 \cos(6\psi_x - \ell\theta + \chi_{60\ell})$$ ← $6\nu_x = P$

$$-\frac{1}{R}\sum_{\ell} |G_{06\ell}| J_z^3 \cos(6\psi_z - \ell\theta + \chi_{06\ell}) - \cdots$$ ← $6\nu_z = P$

$$G_{60\ell} = \frac{1}{5760\pi}\oint \frac{K_{sc}\beta_x^3(8\sigma_x^2 + 9\sigma_x\sigma_z + 3\sigma_z^2)}{\sigma_x^5(\sigma_x+\sigma_z)^3} e^{i(6\phi_x - 6\nu_x\theta + \ell\theta)} ds$$

$$G_{06\ell} = \frac{1}{5760\pi}\oint \frac{K_{sc}\beta_z^3(8\sigma_z^2 + 9\sigma_x\sigma_z + 3\sigma_x^2)}{\sigma_z^5(\sigma_x+\sigma_z)^3} e^{i(6\phi_z - 6\nu_z\theta + \ell\theta)} ds$$

§ Can factor out sp-ch dependent part of resonance strength, giving dimensionless reduced strength g_{mnl}:

$$G_{mn\ell} = g_{mn\ell}\frac{K_{sc}R}{4\epsilon_{\rm rms}^3}$$

lattice dependent → $g_{mn\ell}$

sp-ch dependent → $\frac{K_{sc}R}{4\epsilon_{\rm rms}^3}$

Sample Simulation

§ Crossing systematic resonances:

$6\nu_x = P$, P=24

$6\nu_z = P$, P=24

§ Resonance strengths:

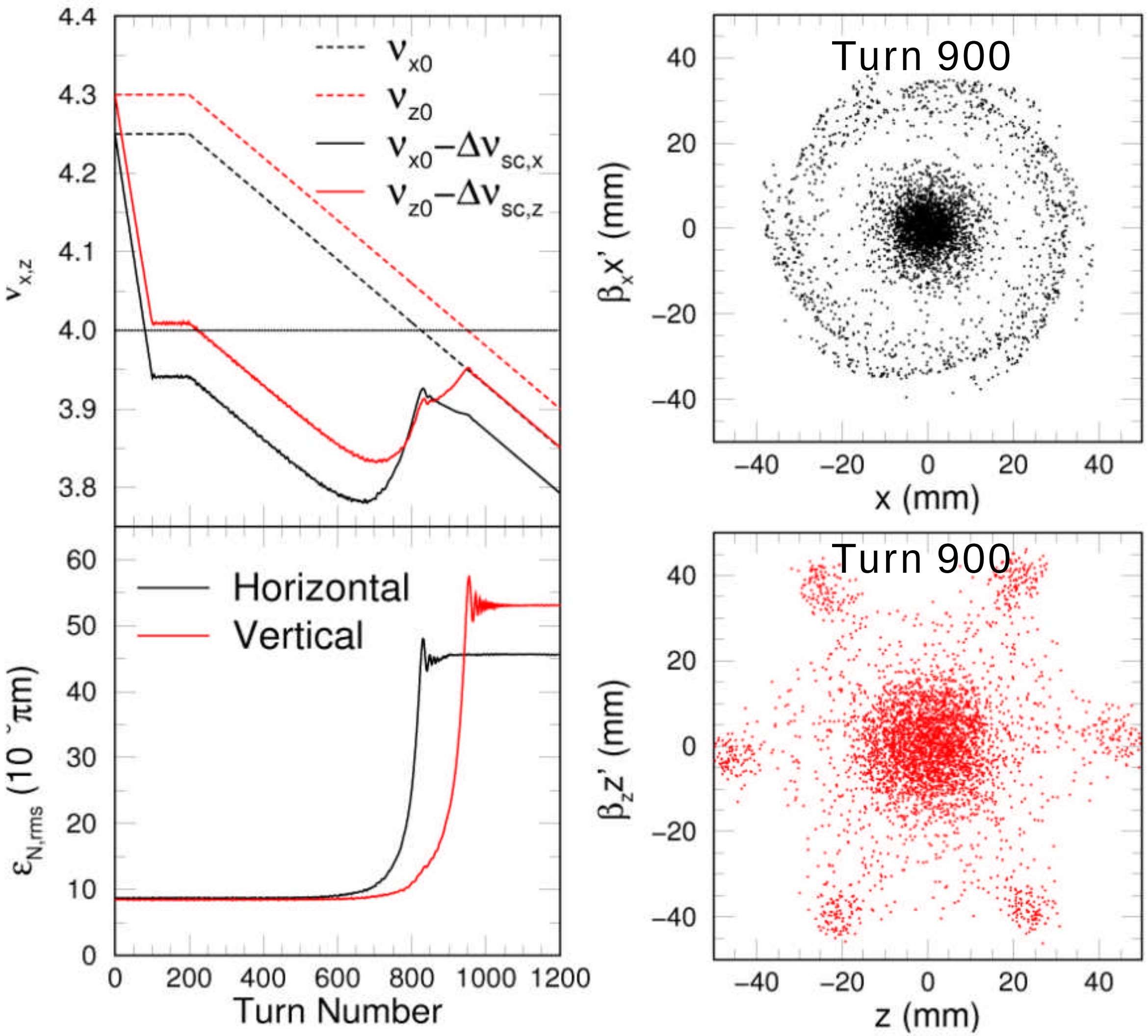
$|g_{60P}| = 0.0062$

$|g_{06P}| = 0.0046$

$d\nu_{x,z}/dn = -0.004$

§ Emittance growth factor (EGF):

Final emit./initial emit.

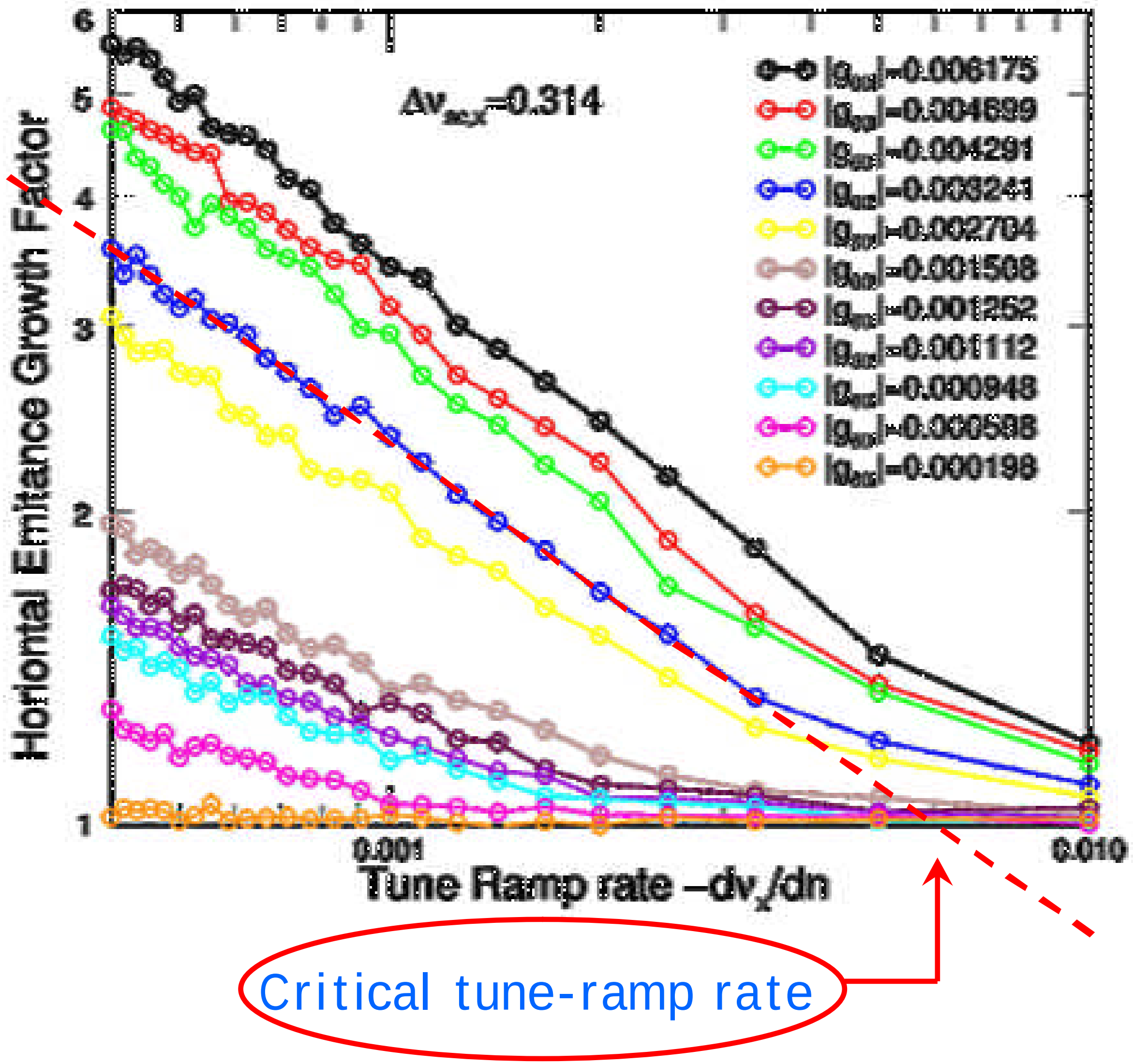


Scaling Power Laws

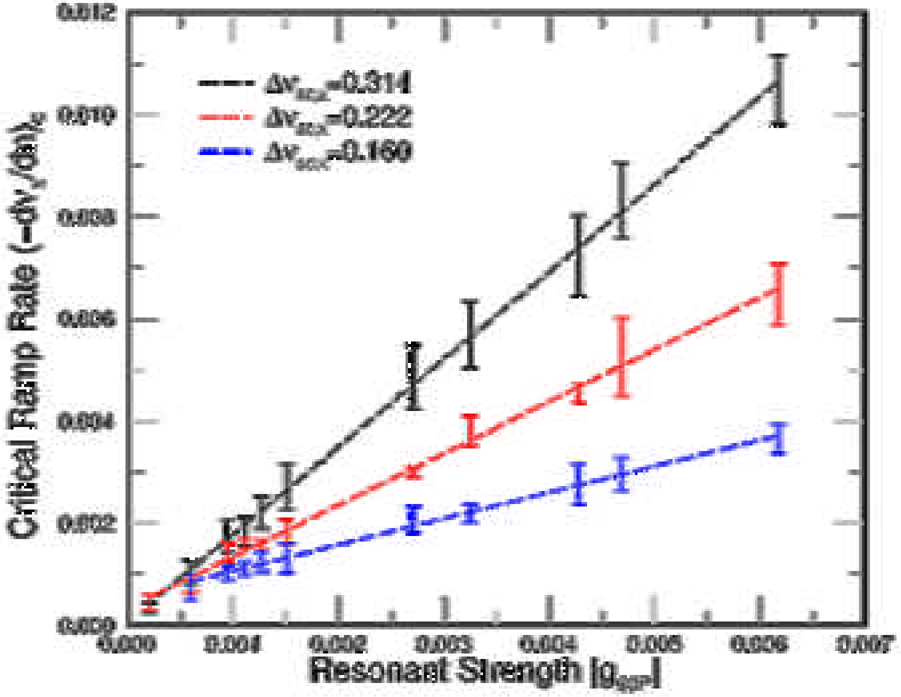
- § Log-log plots
- § Scaling law

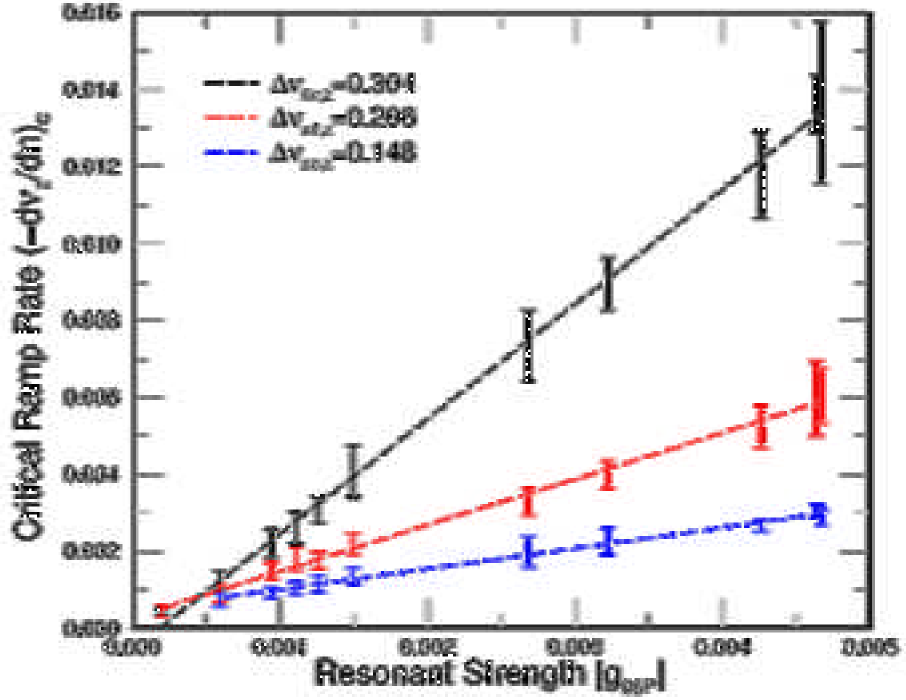
$$\mathrm{EGF} = (-d\nu/dn)^{-a}$$

a ~ 0.23 to 0.53



Critical Tune Ramp Rate





§ Given $\Delta\nu_{sc,x}$ and $|g_{60P}|$ or $\Delta\nu_{sc,z}$ and $|g_{06P}|$,

plots give min. tune ramp rate so that EGF remains tolerable.

§ Can serve as a guideline for FFAG design.

4^{th} Order Parametric Resonance

- One octupole is added at D-magnet in last cell to mimic random 4^{th} order parametric resonance.
- Potential: $V_4(x,z) = -\frac{1}{4!}\frac{B'''}{B\rho}(x^4 - 6x^2z^2)$
- In action-angle variables:

$$V_4(J_x,J_z,\psi_x,\psi_z,\theta) \approx -\frac{1}{R}\sum_\ell |G_{40\ell}|J_x^2\cos(4\psi_x - \ell\theta + \chi_{40\ell}) - \frac{1}{R}\sum_\ell |G_{04\ell}|J_z^2\cos(4\psi_z - \ell\theta + \chi_{04\ell}) - \cdots$$

$4\nu_x = l$

$4\nu_z = l$

$$G_{40\ell} = \frac{1}{96\pi}\oint \frac{B'''\beta_x^2}{B\rho} e^{j(4\psi_x - 4\nu_x\theta + \ell\theta)} ds$$

$$G_{04\ell} = \frac{1}{96\pi}\oint \frac{B'''\beta_z^2}{B\rho} e^{j(4\psi_z - 4\nu_z\theta + \ell\theta)} ds$$

- Octupole kick: $\begin{cases} \Delta x' = \frac{1}{6}S_4(x^3 - 3xz^2), \\ \Delta z' = \frac{1}{6}S_4(z^3 - 3x^2z), \end{cases}$ $S_4 = \frac{B'''\ell}{B\rho}$

- Dimensionless reduced resonant strength: $g_{mn\ell} = G_{mn\ell}\epsilon_{rms}$

Sample Simulation

§ Octupole strength:

S_4=20 m^{-3}

§ Resonance strength:

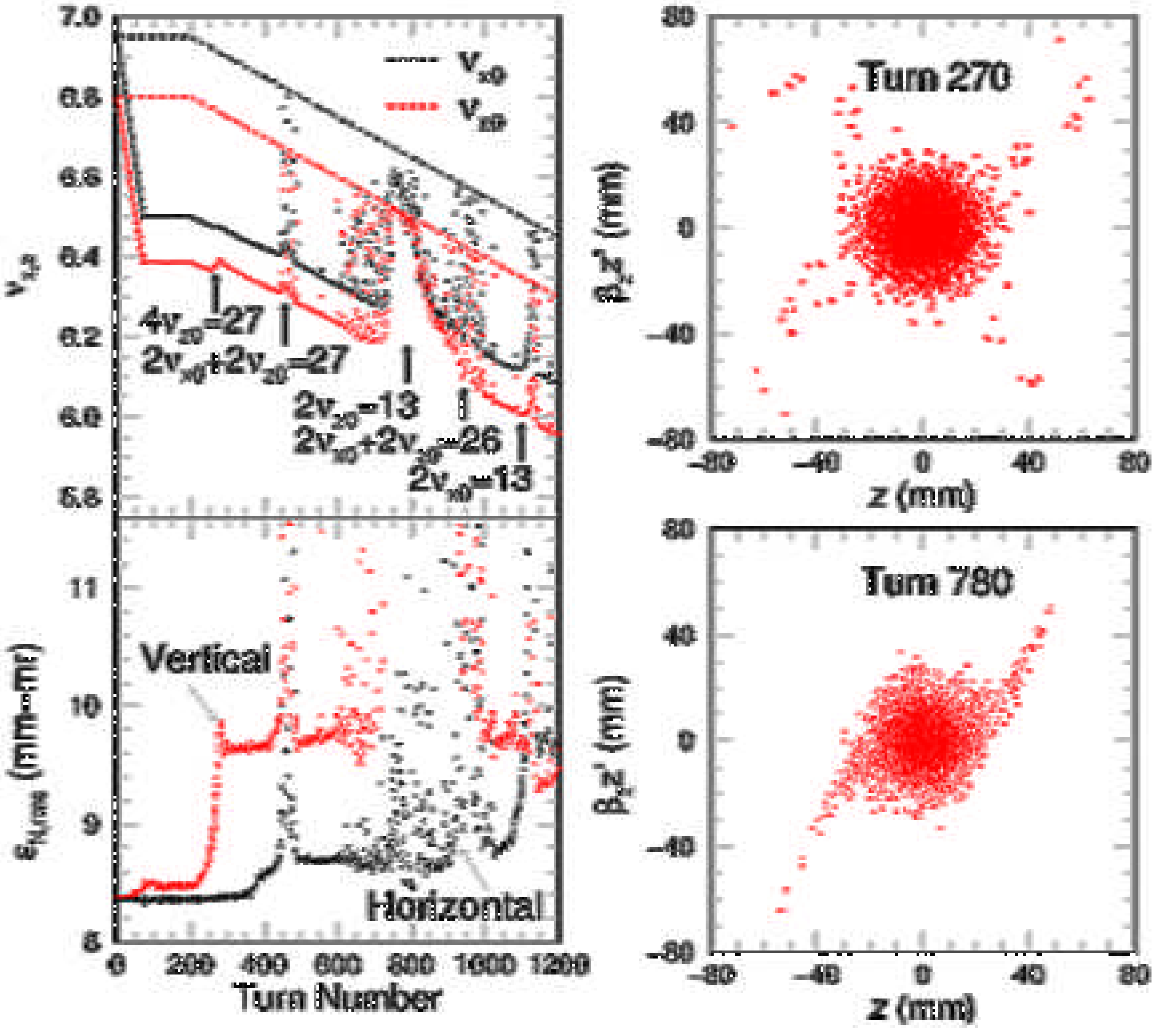
$|g_{04l}|$ = 0.0038

$d\nu_{x,z}/dn$ = -0.0005

$\Delta\nu_z$ = 0.21

§ Crossing many parametric resonances

§ Unlike previous simulations, there is big beam loss.

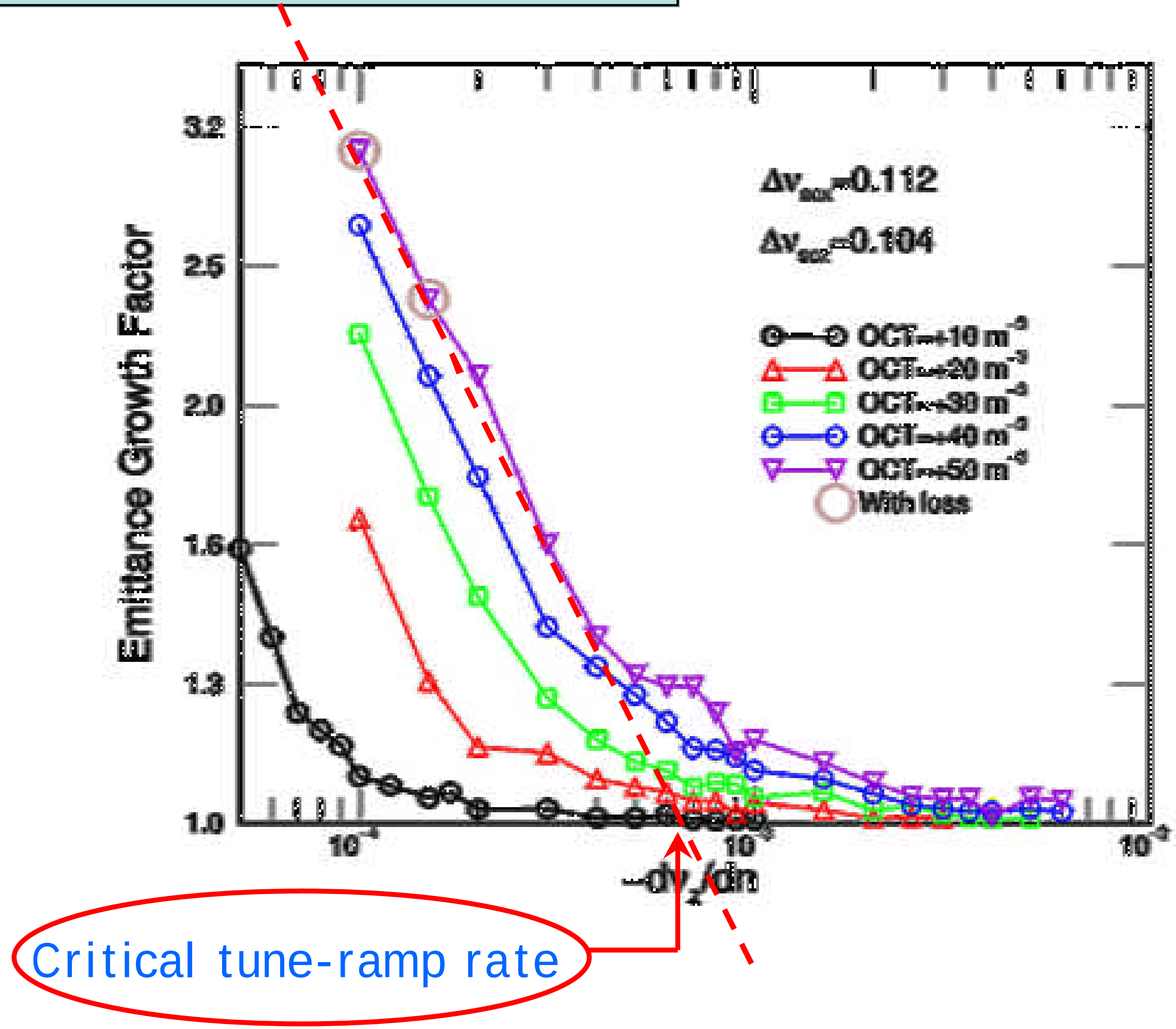


Scaling Power Laws

- Log-log plots
- Scaling law

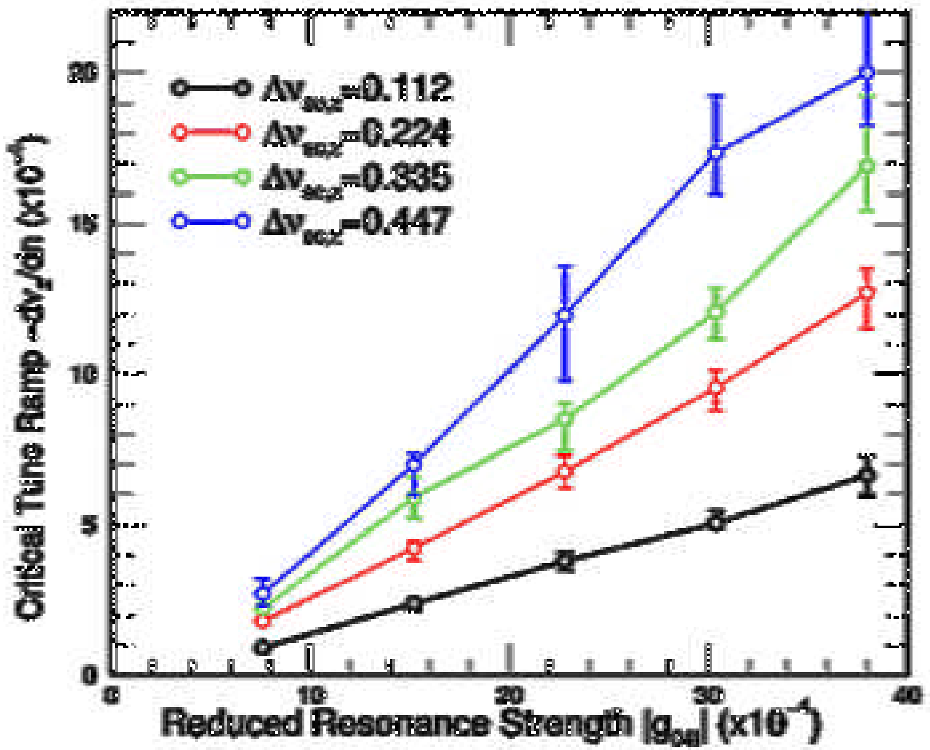
$$\mathrm{EGF} = (-d\nu/dn)^{-a}$$

a ~ 0.23 to 0.53



Critical Tune-Ramp Rate

- § Given $\Delta\nu_{sc,z}$ and $|g_{04l}|$, this gives min. tune ramp rate so that EGF remains tolerable.
- § It is clear that sp-ch contribution is very significant.



Conclusion

§ Power scaling laws obtained between EGF and $d\nu_z/dn$ for crossing sp-ch driven systematic 6th order resonances and octupole-driven 4th order parametric resonance.

§ For a ring like Fermilab Booster,

with $|g_{60P}|$~0.0062, $\Delta\nu_{sc,x}$=0.31, $(d\nu_x/dn)_{crit}$~ -0.0014/turn

§ For octupole driven resonance,

with $|g_{04I}|$~0.0038, $\Delta\nu_{sc,z}$=0.45, $(d\nu_z/dn)_{crit}$~ -0.0020/turn

Conclusion

- Effective sp-ch force

$$F_{x,sc} = -\frac{\partial V_{sc}}{\partial x} \approx \frac{K_{sc}x}{\sigma_x(\sigma_x + \sigma_z)} e^{-\frac{x^2+z^2}{(\sigma_x+\sigma_z)^2}},$$

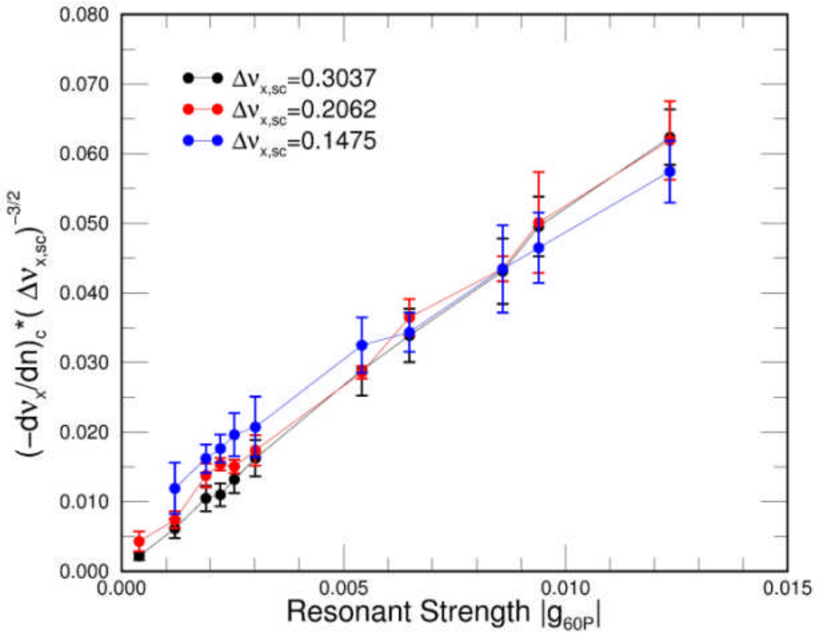
$$F_{z,sc} = -\frac{\partial V_{sc}}{\partial z} \approx \frac{K_{sc}z}{\sigma_z(\sigma_x + \sigma_z)} e^{-\frac{x^2+z^2}{(\sigma_x+\sigma_z)^2}}.$$

  is easy to use, but not derivable from a potential.

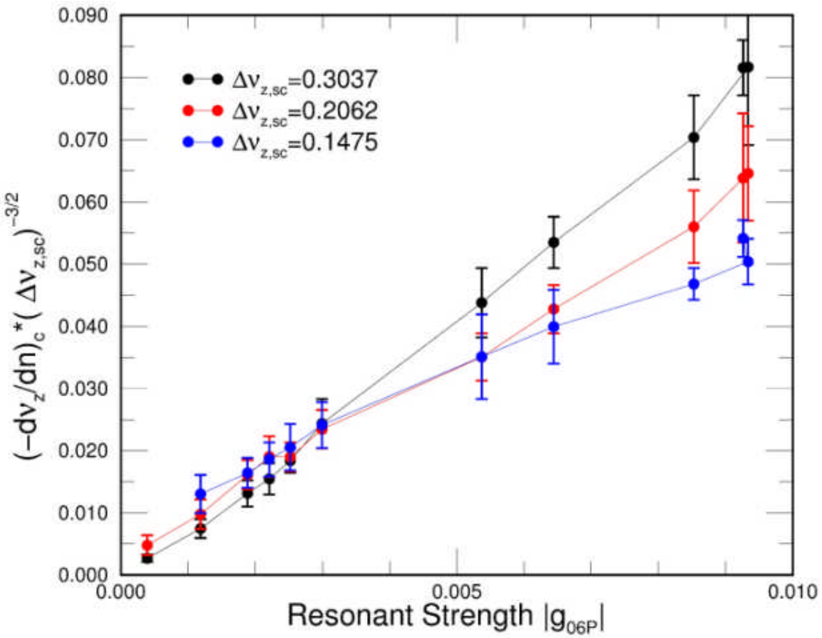
- Nevertheless, Cauchy-Riemann theorem is approx. satisfied; thus the potential is approximately correct.

- There may be problems when 2 transverse spaces are mixed together like the systematic sum resonances.

- We are currently working on a better approximation for the sp-ch force.



$\Delta\nu_{x,sc}$=0.3037
$\Delta\nu_{x,sc}$=0.2062
$\Delta\nu_{x,sc}$=0.1475
$(-d\nu_x/dn)_c{}^*(\Delta\nu_{x,sc})^{-3/2}$
Resonant Strength $|g_{60P}|$



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