Beam lifetime in the JINR Phasotron

L. Onischenko, N. Shakun, P. Shishlyannikov
Laboratory of Nuclear Problems
Joint Institute for Nuclear Research
141980 Dubna, Moscow reg.Russia

The circulating beam lifetime in the JINR Phasotron limited by the multiple scattering on the residual gas is calculated and measured. The beam stretching system was used as a tool for this measuring.

In the JINR Phasotron [1] the C–electrode [2] is used to stretch the extracted beam. The beam particles, after switching off the dee voltage before extraction, are circulating for some time until switching on the C–voltage. At this time the betatron oscillation amplitudes grow because of proton scattering on the residual gas. Then they can reach the maximum value A_{max} limited by the chamber (or extraction channel) apperture and will be lost.

We shall consider the influence of multiple scattering on particle dynamics following the reference [3]. It is convenient to take value $y = A^2/A^2_{max}$ as an independent variable because it is the integral of motion. The instant proton scattering is equivalent to the proton velocity direction change by the angle χ. Then δy is the increase of the betatron motion integral y; it is equal in average to

$$\overline{\delta y} = \frac{\overline{\chi^2} R^2}{A^2_{max} Q^2} \tag{1}$$

where R – circulating beam radius,
Q – betatron oscillation frequency.

According to Rutherford's formula [4] the scattering angle mean square is equal to

$$\overline{\chi^2} = \overline{\chi^2}_{min} lg(183 Z^{-1/3}) \tag{2}$$

and the elastic scattering cross-section is

$$\sigma = \frac{4\pi Z^2 r_p^2}{\beta^4 \gamma^2 \bar{\chi}^2_{min}} \tag{3}$$

where Z – scattering atom number
$r_p = 1.53 \cdot 10^{-18} m$ – classical proton radius.

Then the mean increase of y in time will be equal to

$$\overline{\Delta y} = \overline{\delta y} \cdot \beta \cdot c \cdot \sigma \cdot N_0 = \frac{4\pi Z^2 lg(183 Z^{-1/3})}{\gamma^2 \beta^3} \frac{r_p^2 R^2 c}{Q^2 A^2_{max}} N_0 \tag{4}$$

where $N_0[m^{-3}] = 0.7 \cdot 10^{23} p$ – number of scattering atoms in a residual gas volume unit
p[mm Hg] – residual gas pressure.

If the residual gas is the air (Z=8) formula (4) is simplified to

$$\overline{\Delta y}[s^{-1}] = \frac{0.08}{\gamma^2 \beta^3 Q^2} \frac{R^2}{A^2_{max}} p \tag{5}$$

The particle distribution function $Y(y,t)$ under influence of the statistically independent small perturbations is described by the Einstein–Fokker equation

$$\frac{\partial Y(y,t)}{\partial t} = -\frac{\partial}{\partial y}(\overline{\Delta y} \cdot Y(y,t)) + \frac{\partial^2}{\partial y^2}(\frac{\overline{\Delta y^2}}{2} \cdot Y(y,t)) \tag{6}$$

where $\overline{\Delta y}$ and $\overline{\Delta y^2}$ are the mean increase and the mean square increase of y during a time unit. It is known that

$$\overline{\Delta y^2} \simeq 2y \cdot \overline{\Delta y} \tag{7}$$

We shall use the dimensionless time

$$\tau = \int_0^t \overline{\Delta y}(t) dt \tag{8}$$

It is numerically equal to the relative mean square amplitude of the oscillations excited by the particle scattering process during the time t.

If $\overline{\Delta y}$ does not depend on y and (7) and (8) are taken into account equation (6) is transformed to

$$\frac{\partial Y(y,t)}{\partial t} = \frac{\partial}{\partial y}(y \cdot \frac{\partial Y(y,t)}{\partial y}) \tag{9}$$

with boundary conditions

$$Y(1,\tau) = 0 \tag{10}$$

$$Y(y,0) = Y(y) \tag{11}$$

The first condition means that particles with the oscillation amplitude bigger than A_{max} will be lost. The second one describes the particle amplitude distribution at the initial moment of time. With these boundary conditions and with standardising

$$\int_0^1 Y(y,0)dy = 1 \tag{12}$$

the solution of (9) looks like

$$Y(y,t) = \sum_{i=1}^{\infty} C_i e^{-\mu_i \tau} J_0(2\sqrt{\mu_i y}) \tag{13}$$

where μ_i are fundamental meanings of

$$J_0(2\sqrt{\mu_i}) = 0 \tag{14}$$

and constants C_i are determined as

$$C_i = J_1^{-2}(2\sqrt{\mu_i}) \int_0^1 Y(y) J_0(2\sqrt{\mu_i y})dy \tag{15}$$

Equation (15) can be found by multiplying (13) by $J_0(2\sqrt{\mu_i y})$ and integrating along y from 0 to 1 when $\tau = 0$ [5].

It is possible to find analitically the solution of (15) for two special cases:

a) all particles at $\tau = 0$ have zero amlitudes of oscillations or $Y(y) = \delta(0)$. Then

$$C_i = J_1^{-2}(2\sqrt{\mu_i}) \tag{16}$$

b) homogeneous distribution $Y(y) =$ const. Then

$$C_i = (\sqrt{\mu_i} J_1(2\sqrt{\mu_i}))^{-1} \tag{17}$$

The number of particles which have the oscillation amplitudes inside the interval $0 < y < 1$ in the time τ is

$$N(\tau) = \int_0^1 Y(y,\tau)dy = \sum_{i=1}^{\infty} \frac{C_i}{\sqrt{\mu_i}} J_1(2\sqrt{\mu_i}) exp(-\mu_i \tau) \tag{18}$$

For case a)

$$N(\tau) = \sum_{i=1}^{\infty} \frac{exp(-\mu_i \tau)}{\sqrt{\mu_i} J_1(2\sqrt{\mu_i})} \tag{19}$$

For case b)

$$N(\tau) = \sum_{i=1}^{\infty} \frac{exp(-\mu_i \tau)}{\mu_i} \tag{20}$$

These dependences are plotted in fig.1

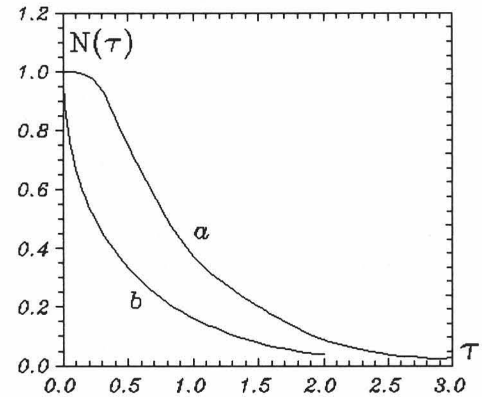

Fig.1. Calculated dependences $N(\tau)$ for a) δ–function oscillation amplitude distribution and b) for homogeneous one.

To measure the beam lifetime, as was mentioned above, we used the following procedure. Protons accelerated to the radius of 265 cm are stopped by switching off the dee voltage at the moment t_1. After a pause the C–voltage is switched on at the time t_2 and the circulating beam is accelerated to the extraction channel and extracted. The dependence of the extracted beam intensity is measured when changing the pause between t_1 and t_2.

The results of this measurement are plotted in fig.2 by vertical lines, the line length demonstrating the accuracy of the intensity measurement.

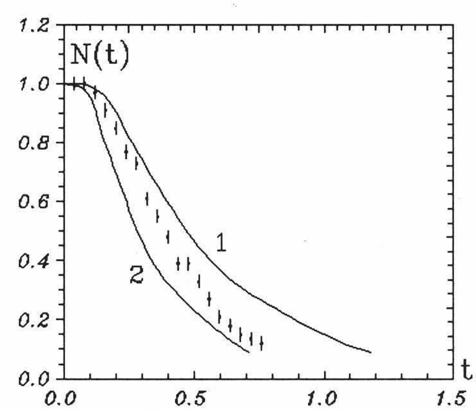

Fig.2. Measured dependence (points) N(t) and calculated ones for δ–function distribution with 1– A_{max}=1.5 cm and 2– A_{max}=1.1 cm.

The calculated dependences N(t) for the δ–function distribution are plotted by the solid lines in the same fiqure for the phasotron parameters: $\gamma = 1.7$; $\beta = 0.8$; $R = 265\ cm$; $Q_z = 0.14$; $P = 2 \cdot 10^{-5}\ mmHg$. The vertical aperture of the beam extraction channel is 3 cm ($A_{max} = 1.5\ cm$) for curve 1 and 2.2 cm ($A_{max} = 1.1\ cm$) for curve 2. The corresponding values of $\overline{\Delta y}$ are 1.68 s^{-1} and 3.11 s^{-1}. The geometrical aperture of the extraction channel is 35 mm. The form of the experimental curve is as in the case when the initial distribution of the betatron amplitudes is a δ–function, i.e. betatron amplitudes are significantlly smaller, than the extraction channel aperture wich is certainly less than the geometrical one.

The experimental data agree with the calculation if we assume the dynamic aperture of the extraction channel to be equal to 80% of the geometrical one, or 27 mm.

Acknowledgments

Authors are grateful to profs. A.A. Glazov, V.P. Dmitrievsky,N.A. Lebedev for the discussions and remarks.

References

1. L.M. Onischenko. Proc of PAC-87, Washington, v.2,p.878.
2. L.M. Onischenko,N.G. Shakun,P.T. Shishlyannikov. Proc.of EPAC–92, Berlin, v.1, p.1484.
3. A.A. Kolomensky and A.N.Lebedev. "Theorija cyclicheskich uscoriteley", Moscow, 1962.
4. B. Rossi. "High energy particles", New York,1952.
5. A. Gray and G.B. Mathews. "Bessel functions". Second edition, 1931.