Intensity Limitations in Compact H^- Cyclotrons

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At TRIUMF, we have demonstrated 2.5 mA in a compact H^- cyclotron. It is worthwhile to explore the possibility of going to even higher intensity. In small cyclotrons, vertical focusing vanishes at the centre. The space charge tune shift further reduces vertical focusing, thus determining an upper limit on instantaneous current. The limit on average current is of course also dependent upon the phase acceptance, but this can be made quite large in an H^- cyclotron. Longitudinal space charge on the first turn can reduce the phase acceptance as well. For finite ion source brightness, another limit comes from bunching efficiency in the presence of space charge forces. We present methods of calculating and optimizing these limits. In particular, we show that it is possible to achieve 10 mA in a 50 MeV compact H^- cyclotron.

1 Introduction

The great virtue of H^- cyclotrons is that particles that get past the first few turns will make it all the way out and can be efficiently extracted. Separated turns are not required beyond the first turn. In some sense this makes the H^- cyclotron similar to the internal target H^+ cyclotron. However, with H^-, the beam is easily extracted, with high efficiency, and is of good quality. Unfortunately, it is not possible to simply enlarge the cyclotron to achieve any desired extraction energy, since for energies higher than around 50 MeV, electromagnetic stripping forces a choice of hill field less than 2 T[1]. Nevertheless, for energies up to somewhere in the 50 MeV to 100 MeV range, the compact H^- cyclotron is a far more economical choice for high intensity than a proton cyclotron.

At TRIUMF, we have a 1 MeV full-scale model of our 30 MeV H^- cyclotron TR30, called the CRM. This model has achieved 2.5 mA[2], with no bunching, from an ion source capable of 15 mA[3]. The reduction in phase acceptance from low intensity to high is minimal, so the intensity limit is expected to lie significantly beyond 2.5 mA. We wish to explore the intensity limit of cyclotrons of this kind.

Fundamental limits are expected to arise from space charge effects; both transverse and longitudinal. These effects will be discussed in the context of the existing TR30 cyclotron and also in the context of a possible scaled-up higher intensity design of the same machine. The ion source requirements depend upon the bunching efficiency. At mA intensities, bunching system parameters depend strongly upon longitudinal space charge. This aspect also will be investigated.

2 TR30/CRM Data

Fig. 1 shows the H^- current reaching 1 MeV (5 turns) as a function of the injected current, unbunched. Compared with linear, there is a dip in transmission below ($I_{\text{inj}} =$) 4 mA injected, probably because the ion source extraction optics are not optimized for such low intensity, and the emittance is not as good as it is at higher intensity. Above 6 mA, transmission drops off more or less linearly with I_{inj}. In the optimum range of $I_{\text{inj}} = 4$ to 6 mA, the transmission is 20%, for a phase acceptance of 72°. This agrees well with calculations made during the design stage of the TR30 cyclotron[4]. As an aside, we remark that this does not mean that the beam occupies 72° longitudinally. In fact the centre region, and in particular the first dee gap, bunches the beam by a factor of 2, making the local beam current in the cyclotron twice the injected current[4]. We shall come back to this point later.

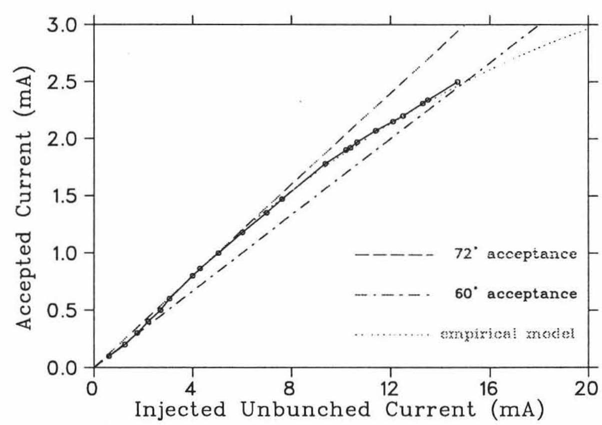

Fig. 1: Measured beam current circulating in the TR30/CRM vs. injected dc current (connected symbols). Also shown are expected curves for 72° and 60° phase acceptances and the empirical model eqn. 1.

At $I_{\text{inj}} = 15$ mA, the transmission has fallen to 16.7%. A model which fits the data very well above $I_{\text{inj}} = 6$ mA is

$$I_{\text{circ}} = \frac{I_{\text{inj}}}{5}\left(\frac{60\,\text{mA} - I_{\text{inj}}}{54\,\text{mA}}\right). \tag{1}$$

This is a conservative approximation: one can see that it predicts zero extracted current for 60 mA injected. The formula predicts an upper limit on intensity extracted from the TR30 cyclotron of 3.3 mA. This would occur for an injected dc current of 30 mA. Another factor which makes this estimate conservative is the increase in measured source emittance at high intensity[3]. If the drop in transmission at high intensity is simply related to larger source emittance, the space charge limit could be considerably higher than 3.3 mA.

3 Vertical Space Charge Tune Shift

Space charge effects are strongest on the first turns. There are two reasons: space charge forces are strongest at low energy where the bunches are shortest, and the vertical tune is smallest at the machine centre. Because of the latter reason, flutter was kept as large as possible in the TR30 centre region design.

The calculation of incoherent tune shift is well-developed in synchrotron theory. A number of references give the following formula.[a]

$$\Delta(\nu_z^2)_{SC} = -\frac{2}{\pi}\frac{NRr_p}{\beta^2}\left[\frac{1}{b(a+b)}+\frac{\epsilon_1}{h^2}\right] \quad (2)$$

Here, a and b are horizontal and vertical beam half-sizes, h is the metal chamber half-height, r_p is the 'classical proton radius' 1.54×10^{-18} m, and N is the number of particles per unit length times $2\pi R$, R being the orbit radius. The Laslett image coefficient ϵ_1 is zero if the aperture height and width are equal, and is approximately 0.2 for a parallel plate geometry. Considering that h is small only in the dees, i.e. for 1/4 of the circumference, we see that even if the beam completely fills the aperture in the dees, the image term can be no greater than 5% of the direct space charge term. We therefore neglect images. Space charge neutralization can build up only in electric-field-free regions and on such a long time scale compared with the rf bunch structure that it can only respond to the average beam current. Neutralization can therefore not contribute more than about 5% to the tune shift and we neglect it as well. (Moreover, image effects defocus while and neutralization effects focus and so they tend to cancel.)

For a cw machine, it is more convenient to write this in terms of the local beam current $\hat{I}$; for cyclotrons, $R=\beta R_\infty$ where $R_\infty = c/\omega_{rev} = mc/e/B_0$, and B_0 is the average centre magnetic field. We find simply

$$\Delta(\nu_z^2)_{SC} = -\frac{4}{\beta}\frac{\hat{I}}{I_0}\frac{R_\infty^2}{b(a+b)}. \quad (3)$$

[a] Note that we cannot use the approximation $\Delta(\nu_z^2)\approx 2\nu_z\Delta\nu_z$ since we will be dealing with cases where $\Delta\nu_z$ is not small compared with ν_z.

$I_0 = ec/r_p = 31.3\times10^6$ A is a normalizing current as introduced by Joho[5].

This formula is to be used with some care, since as the total tune is shifted toward zero, the beam blows up, thus changing b and reducing the tune shift. The matched vertical beam size is given by

$$b^2 = \varepsilon R/\nu_z = \varepsilon_n R_\infty/\nu_z, \quad (4)$$

where ε is the emittance (phase space area $\div\pi$) and ε_n is the normalized emittance. This must be smaller than the (half-)aperture b_{max} in the dees, and this puts a lower limit on the total tune $\nu_z = \nu_{z0}+\Delta\nu_z$. In other words, we must have

$$\nu_z^2 = \nu_{z0}^2 + \Delta(\nu_z^2) > \left(\frac{\varepsilon_n R_\infty}{b_{max}^2}\right)^2. \quad (5)$$

Substituting from eqn. 3, we get an upper limit on local beam current:

$$\frac{\hat{I}}{I_0} < \frac{\beta}{4}\frac{b_{max}(b_{max}+a)}{R_\infty^2}\left[\nu_{z0}^2 - \left(\frac{\varepsilon_n R_\infty}{b_{max}^2}\right)^2\right]. \quad (6)$$

If the emittance of the incoming beam is sufficiently small, we can neglect both the radial beam size and the second term in square brackets. 'Sufficiently small' means that the aperture b_{max} is generous compared with the natural beam size $\sqrt{\varepsilon_n R_\infty/\nu_{z0}}$. This is true in any well-designed cyclotron, and moreover, the two neglected effects tend to cancel. The formula for maximum local beam current is then exceedingly simple.

$$\hat{I}_{max(vert)} = \beta\left(\frac{\nu_{z0}b_{max}}{R_\infty}\right)^2 (7.8\times10^6\,\text{Amp}). \quad (7)$$

For the TR30, $\nu_{z0}\approx 0.3$, $R_\infty = 2.6$ m. The vertical aperture half height is 5 mm in the inflector and through the first dee, and 10 mm thereafter. The best aperture filling therefore occurs not for a matched beam, but a mismatched beam oscillating in size between these two extremes. Under these conditions, the effective beam size is 7 mm. For β, we take 0.02 which is the average over the first 2 turns, since 2 turns is roughly a quarter betatron oscillation of the depressed tune. This gives $\hat{I}_{max} = 100$ mA.

To find the resulting maximum extracted average current, we divide by 2 ($=\hat{I}/I_{inj}$) to account for the centre region bunching as mentioned above, and multiply by the experimentally-verified 20% phase acceptance. This gives $I_{circ} = 10$ mA; a factor of 3 better than that extrapolated from the experimental results. This factor of 3 is easily explained. The above analysis assumes the beam fills the vertical aperture entirely and uniformly. In fact, the beam density is not nearly uniform and to protect the inflector we tend to keep most of the beam inside half the inflector aperture. This is verified by directly inspecting the height of the erosion marks on the centre post after the first dee.

More important than the absolute value predicted by the formula (7) is the implication for scaling. For example, it indicates that large tune gained at the expense of magnet gap is not worthwhile. Also, higher central magnetic field (lower R_∞) is beneficial, in spite of the fact that this leads to smaller beam size. However, because of electromagnetic stripping, this limits the maximum attainable energy in H^- machines.

The centre region geometry of the TR30 is highly optimized for the case of $R_\infty = 2.6\,\mathrm{m}$, injection energy 25 keV, and dee voltage 50 kV. Without changing the geometry, we can ask what is the increase in intensity limit if this design is simply scaled up. Both β and $b_{\max}$ are proportional to the size scale, so the intensity limit is proportional to the cube of the size scale. If we wish to increase the (conservative) intensity limit by a factor of 3 from 3.3 mAto 10 mA, we need to scale up by a factor of $3^{1/3} = 1.44$. Through β, this requires increasing both the injection energy and the dee voltage by a factor $3^{2/3} = 2.08$, to 52 keV and 104 kV respectively. This scaling cannot be increased indefinitely, however, since voltages are scaling up faster than distances, and so the Kilpatrick limit will eventually be reached. No experiments have been performed on the TR30 to determine maximum achievable dee voltage, but one is encouraged by the fact that under present operating conditions there are no sparking problems.

4 Longitudinal Space Charge

As already pointed out, H^- cyclotrons do not require separated turns. However, in a compact cyclotron of the TR30 type, posts are needed between the first two turns to localize the rf electric fields, because $\beta\lambda$ is comparable with the beam gap. Longitudinal space charge causes the trailing particles in a bunch to lose energy. An intensity limit occurs when these trailing particles lose enough energy that they cannot get round the posts.

For these well-separated first two turns, a 'cigar' model is appropriate for the beam bunches. Joho[5] finds that the energy gain per turn due to longitudinal space charge is $377\,\Omega \times \hat{I}R_\infty/a$. This is however for a round beam. A good approximation for radial and vertical beam half sizes a and b is to replace the a by $\sqrt{ab}$:

$$\left.\frac{dE}{dn}\right|_{\mathrm{SC}} = 377\,\Omega \times \hat{I}\,\frac{R_\infty}{\sqrt{ab}}. \tag{8}$$

One can now calculate the accumulated energy loss over the first two turns, find the increased radial width of the bunch using $R = \beta R_\infty$, and compare with the radial aperture. However, the phase band accepted by the cyclotron is determined by the same constraint, so it is more straightforward to compare the space charge energy loss with the loss of the extreme phase particle with respect to the synchronous particle:

$$\left.\frac{dE}{dn}\right|_{\mathrm{RF}} = \hat{V}(1-\cos\delta\phi) \approx \hat{V}\,\frac{(\delta\phi)^2}{2} \tag{9}$$

Here $\hat{V}$ is the rf voltage per turn and $\delta\phi$ is half the rf phase width of the circulating bunch. Insisting that the space charge energy loss be smaller than the off-phase energy loss, we find the following maximum local beam intensity.

$$\hat{I}_{\max(\mathrm{long})} = \frac{\hat{V}}{377\,\Omega}\,\frac{(\delta\phi)^2}{2}\,\frac{\sqrt{ab}}{R_\infty}. \tag{10}$$

As before, we apply this to the TR30 by assuming the vertical aperture is filled. The radial size is $\approx \sqrt{\nu_z}b$, $\hat{V}$ is 200 kV, and $\delta\phi$ is 18°. We find $\hat{I}_{\max} = 50\,\mathrm{mA}$, a factor of two smaller than the limiting intensity found for the vertical space charge case. However, we have neglected a very important effect. For over 80% of the first two turns up to the point where there is no longer a radial constraint, longitudinal space charge is shielded by the presence of an inner wall. Imagine that a trailing particle is just skimming this wall. Since it sees an image bunch of opposite charge, longitudinal space charge is cancelled (but radial force is augmented).

Another factor which has been neglected is the fact that longitudinal space charge can be compensated to some extent by shifting the bunch to earlier phase. This gives the trailing particles extra energy gain from the rf, and the lower energy gain of leading particles is compensated by space charge as well. One cannot carry this too far, though, because the leading particles are electrically defocused at the dee gaps. The well-known formula is:

$$\Delta(\nu_z^2)_e = \frac{gh}{4\pi n}\sin\phi, \tag{11}$$

where n is the turn number and $g = 1/2$ on the first turn where the gaps are symmetric, and $g = 1$ on subsequent turns where the gap is planar. So for $\nu_{z0} = 0.3$ and $h = 4$, phases $\phi > 34°$ have imaginary vertical tune, even without the additional defocusing due to vertical space charge.

So vertical and longitudinal space charge limits for the TR30 are similar, and without additional data, it is not possible to say which of the two is responsible for the drop in beam transmission observed in the CRM between 6 mA and 15 mA injected.

In scaling the machine size, we note that $\hat{V}$ is proportional to the length scale squared. Therefore, remarkably, the longitudinal limit scales with size in the same way as the vertical limit, namely, as the cube of length. So in spite of the fact that we cannot determine the origin of the intensity effects observed for the TR30/CRM, we can say with some confidence that scaling up by a factor 1.44 can increase the intensity limit by a factor of 3, to 10 mA.

5 Bunching

When the TR30 was designed, there already existed H^- ion sources of sufficient brightness that bunching was not necessary to achieve the 500 μA design goal. A dc beam is desirable, since it allows near 100% space charge neutralization, and no intensity-dependent space charge detuning in the injection line. However, a double-gap buncher was tested in the CRM and is now installed in the TR30, since even though it complicates the tuning and gives less than a factor of 2 gain, any lengthening in ion source filament lifetime is very desirable.

The performance achieved by this buncher is shown in Fig. 2. The buncher is located 1 metre from the inflector. Remarkably, there is no gain from the buncher for ion source currents in excess of 15 mA. Why is this so? Also, why does the gain factor reach no higher than 2? The former question is related to space charge effects, and the latter to transit time effects in the buncher.

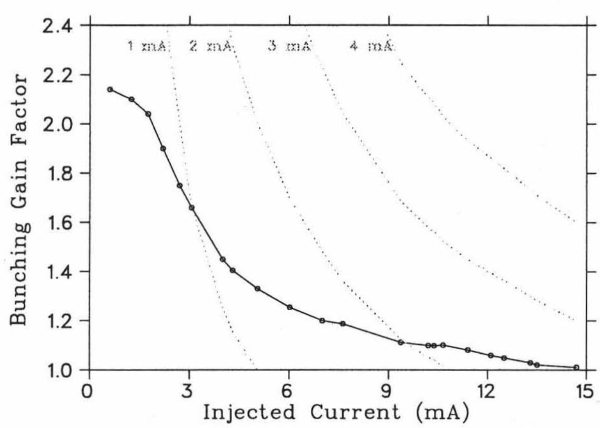

Fig. 2: Measured bunching gain factor as a function of injected current. Also shown are contours of constant accepted current.

5.1 Space Charge

To develop some feeling for the effect of longitudinal space charge on bunching, we use a simple spherical-bunch model. This is a surprisingly good approximation especially at the injection gap where the bunch half sizes are 2.5 mm vertically by 1.5 mm horizontally by 3.0 mm longitudinally. The injected sphere is allowed to expand freely as it is tracked backward from the injection gap to determine the point at which it has debunched to some approximation of the initial dc beam.

As is well known, the electric field in a sphere with uniform charge density is proportional to radial position. Therefore, if the sphere is allowed to expand freely from a stationary start, it will remain uniform. The electric field at the edge of the sphere (radius r, charge Q) is

$$\mathcal{E} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}. \tag{12}$$

We equate this to $(m/e)\ddot{r} = 2V_0(d^2r/ds^2)$, where s is the longitudinal coordinate and V_0 is the ion source potential. Q is the charge per bunch, i.e. just the intended circulating beam intensity $I_{\rm circ}$ divided by the rf frequency. We can therefore write the equation of motion of the bunch radius as

$$\frac{d^2r}{ds^2} = \left[\frac{377\,\Omega \times I_{\rm circ}\, R_\infty}{2hV_0}\right]\frac{1}{2r^2}, \tag{13}$$

and solve with the initial conditions $r(0) = r_0, r'(0) = 0$. The solution can be written as $s(r)$:

$$s = L_0\left(\sqrt{\alpha^2-\alpha} + \cosh^{-1}\alpha\right), \tag{14}$$

where $\alpha \equiv r/r_0$ and we have introduced the length scale

$$L_0 \equiv \sqrt{\frac{2hV_0}{377\,\Omega \times \hat{I}}\frac{r_0^3}{R_\infty}}. \tag{15}$$

For the TR30/CRM at the highest intensity reached ($I_{\rm circ} = 2.5$ mA, $V_0 = 25$ kV, $r_0 = 3.0$ mm, $h = 4$, $R_\infty = 2.6$ m), L_0 is 47 mm. For this bunch to expand from $\pm 36°$ to unbunched size $\pm 180°$, $\alpha = 5$ and the drift required is $s = 6.8 \times L_0 = 0.32$ m. This would be the optimum distance of a buncher from the inflector. However, in the TR30 this area is too congested (see Fig. 3). At the location of 1 m from the inflector, bunches can be created of the necessary length, but these have expanded back to a dc beam by the time they reach the injection gap. Lowering the buncher voltage helps somewhat at low intensity, but as intensity is raised, a point is reached where there is no longer any gain from the buncher and the only way in which to recover buncher efficiency is to move it downstream. For the TR30 case with the buncher 1 m from the inflector, the intensity at which this happens is 15 mA.

This is illustrated in Fig. 4, where the results of **SPUNCH** calculations have been plotted. **SPUNCH**[6] is a computer code which calculates longitudinal space charge forces by dividing the initial beam into discs. It also includes the effect of images in the surrounding vacuum chamber. An interesting effect shown in this calculation

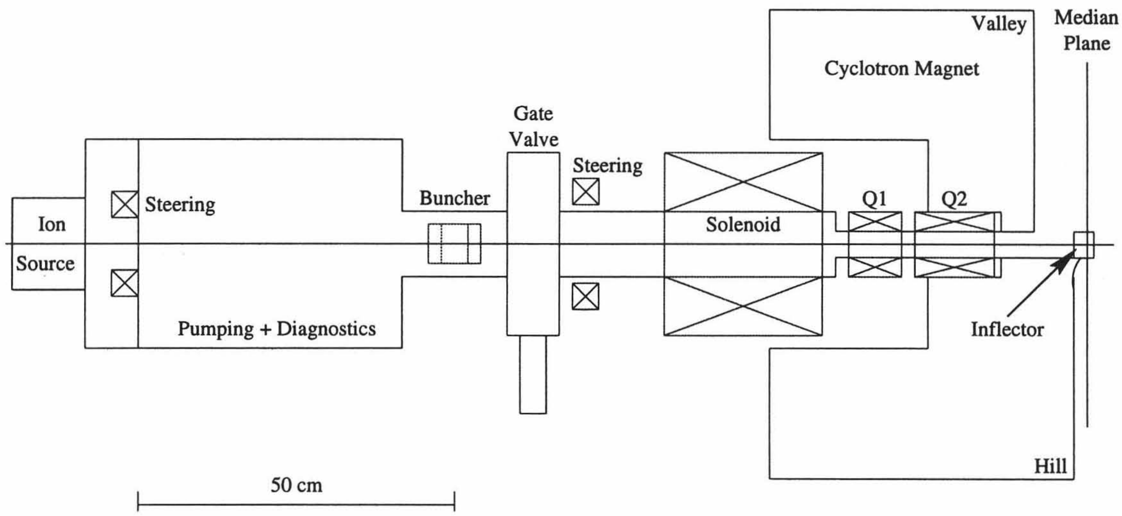

Fig. 3: Layout of the TR30/CRM injection line. (Note that the CRM is 'on its side' with a vertical median plane.)

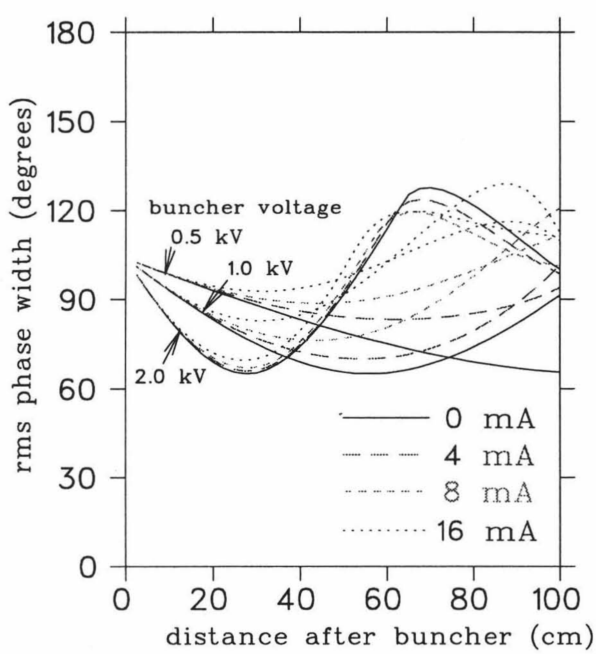

Fig. 4: Bunching as a function of distance downstream, intensity, and buncher voltage, as calculated by SPUNCH. Beam energy is 25 keV, rf frequency is 74 MHz.

is that bunches can debunch due to space charge, rebound off neighbouring bunches, and rebunch. So far, this effect has not been seen in the TR30/CRM. This negative result may be due to the disruptively strong transverse and longitudinally space charge effects at the minimum bunch length.

Moving the buncher downstream is also beneficial for another reason. Since the space charge neutralization time constant is of the order of 100 μsec or more[7], the neutralization cannot respond at the bunching frequency. Therefore, the local instantaneous charge density is no longer completely neutralized in a bunched beam. There will be a dependence of the injection line tune on intensity, and there may be nonlinear space charge effects which increase the beam halo content. These effects are the smaller, the closer the buncher is to the inflector.

The possibility of moving the buncher in the TR30/CRM to the point 0.4 m from the inflector, inside the solenoid just upstream of the quadrupoles, is presently being investigated. (See Fig. 3.)

In a system scaled up to achieve 10 mA, we have already noted that the scale factor is $3^{1/3}$, and the potentials scale as the square of this factor. Perhaps not surprisingly, we therefore find from eqn. 15 that L_0 scales by $3^{1/3}$ as well. If the scaled-up design uses a scaled-up injection line as well, we have the same problem that the best location for the buncher is also the location of the quadrupoles. However, the inflector-to-quadrupole distance need not scale in this way, and besides, in a new design, one can open the quadrupole apertures to allow the possibility of a buncher residing inside them.

5.2 Transit Time Effects

Simple calculations show that if there is no dependence of energy gain on radial position in the buncher, then with a sinusoidal buncher, 70% of the dc beam can be bunched into the 72° phase acceptance. This represents a bunching gain factor of 3.5. The measured gain factor for intensities where space charge does not play a role is 2.15.

A problem for bunching in compact cyclotrons is that

$\beta\lambda$ is comparable with the beam size in the injection line. At high intensity, it is not feasible to use a gridded buncher. In this case, the net energy gain through a buncher gap varies across the aperture because on-axis particles ($r = 0$) see a varying electric field as they go through the gap, while particles travelling at the aperture limit ($r = R_b$) see a varying field for a much shorter time. This can be summarized in a 'transit time factor' which in the limit of a zero physical gap length can be written as[8]:

$$T(r) = 2\sum_{n=1}^{\infty} \frac{x_n J_0(x_n r/R_b)}{(x_n^2 + [2\pi R_b/(\beta\lambda)]^2)J_1(x_n)}, \qquad (16)$$

where x_n is the n^{th} zero of the Bessel function $J_0(x)$.

In the TR30/CRM, R_b was reduced to 1 cm using a partial grid. It is not possible to reduce R_b further because the outer edge of the beam would melt the grid. With $\beta\lambda = 3.0$ cm, we find $T(0) = 0.4$. In other words, there is a factor of 2.5 difference in energy gain between large-radius particles and on-axis particles. This can easily explain the discrepancy between the observed bunching gain factor of 2.15 and the expected factor of 3.5.

In the scaled-up version, $\beta\lambda$ is larger and beam size is similar or slightly smaller. Therefore, one can expect higher bunching efficiency in the 10 mA design than in the present smaller design.

Taken together, the relocation of the buncher to a point nearer injection, and increased bunching efficiency due to a more favourable ratio of $\beta\lambda/R_b$, raises the possibility of achieving a bunching gain factor of 2 to 3.

6 Conclusions

The observed drop in transmission in the TR30/CRM as intensity is raised, can be explained by space charge effects. Extrapolating yields an upper intensity limit of 3.3 mA. There is good reason to believe that a design scaled up by a factor 1.44 in size and 2.1 in injection energy and rf voltage can achieve 10 mA. With an improved buncher system, the required ion source intensity could be as low as 30 mA.

Imposing an upper limit of 1% on beam spill due to electromagnetic stripping yields a maximum energy of 50 MeV for such a 10 mA 'compact' H^- cyclotron[1]. To reach 70 MeV, for example, requires that the hill field be reduced from 1.9 T to 1.4 T[9]. According to eqns. 7 and 10, this would require proportionately larger vertical aperture (in addition to the scale factor of 1.44) to achieve the same 10 mA intensity limit. Beyond energies of this order, H^- cyclotrons of this design lose their compactness and therefore their advantages over separated-turn proton cyclotrons. Nevertheless, if higher energy is required, the compact H^- design can be used to economic advantage as an injector to a 10 mA proton machine.

Acknowledgements

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