SPACE CHARGE EFFECTS IN CYCLOTRONS – FROM SIMULATIONS TO INSIGHTS

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In the PSI accelerator facility the beam intensity extracted from the 590 MeV ring cyclotron using the 72 MeV injector 2 cyclotron as first stage has been raised to 1.5 mA. The wish to understand the behaviour of these beams and the recently increased actuality of plans for high power accelerators have stimulated an intensive reviewing process of space charge simulation methods and their results. These activities enhance the reliability of the simulation models, make cross-checks between models of different levels of approximation and search for the intrinsic principles that govern the vast range of phenomena showing up in simulations. The quick formation of a narrow phase beam bunch and the subsequent stability of this charge distribution to be expected from simulation results are in good agreement with beam measurements in the injector 2 cyclotron.

Introduction

In isochronous cyclotrons for high intensities the effects caused by the longitudinal[1] components of space charge forces clearly dominate those from transversal[2] components. As the particles keep their longitudinal position relative to the bunch, the effects from longitudinal space charge forces pile up during acceleration and can result in a worsening of the beam quality.

At the Paul Scherrer Institute the injector 2 cyclotron is particularly conceived to produce high intensity beams [3] of 72 MeV protons to be injected into the 590 MeV ring cyclotron. The injector 2 is a separate sector cyclotron with four sector magnets. The beam injected into this cyclotron at the low energy of 870 keV comes from a Cockcroft-Walton DC accelerator. In parallel to the construction and commissioning of the injector 2 cyclotron a series of investigations on longitudinal space charge effects has been worked out [4, 5, 6, 7] at PSI.

The increase of the beam intensity extracted from injector 2 to 1.5 mA[8] has stimulated the design of a new simulation model for space charge effects [9]. While the RF power [17] of the 590 MeV ring cyclotron has been upgraded in 4 steps and the extracted intensity could correspondingly be raised to 1.5 mA [18] for the ring cyclotron as well, an intensive reviewing of the space charge simulation methods has started. This reviewing to be reported here was further motivated by new projects involving high power cyclotrons [14, 15, 16] in nuclear fission schemes and eventually in transmutation plants.

As an example of a space charge simulation including acceleration, fig. 1 shows the deformation of a beam bunch towards a narrow phase "round" charge distribution within the first few turns in the PSI injector 2 cyclotron.

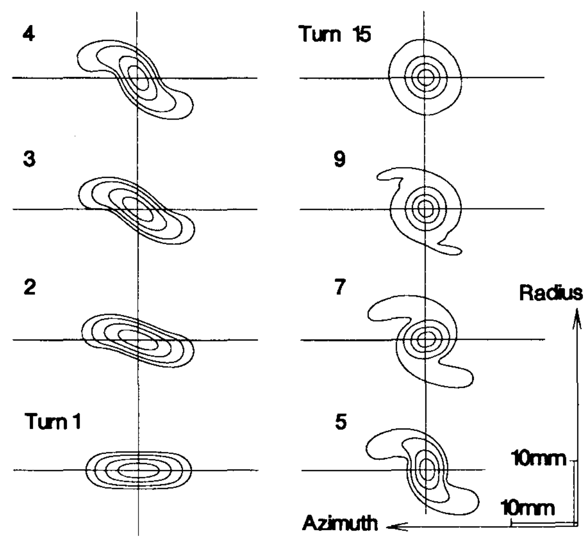

Figure 1: Simulation of the evolution on the first few turns in the PSI injector 2 cyclotron of a 1.0 mA beam bunch starting with a phase width of 15 degrees. Within 5 to 10 turns a round charge distribution is formed due to space charge forces. The contour lines represent 10%, 20%, 50% and 80% of the maximum charge density.

The Three Main Simulation Models

The **Disk Model** divides the beam bunch longitudinally into a series of uniformly charged cylinders, which become disks if the slicing is fine enough. The values of the longitudinal force acting along the axis of the cylinders then approximate the longitudinal force as a function of phase for the real beam bunch.

During acceleration the integrated longitudinal forces produce energy deviations that depend on the particle phase. As it assumes an overall bunch shape similar to a straight cylinder, the disk model is restricted to cases where the energy deviations are much smaller than the beam energy. From simulations respecting this limitation the strong dependence of the energy deviations on the charge density distribution along the beam bunch became obvious[5]. The energy deviations can be divided into two parts, where the part varying linearly with phase can be compensated with changes of RF parameters, while the other, nonlinear part mainly affects the beam quality at extraction. Simulations with the disk model have shown that no realistic charge density distributions exist that can avoid the nonlinear part of the energy deviations as a function of phase.

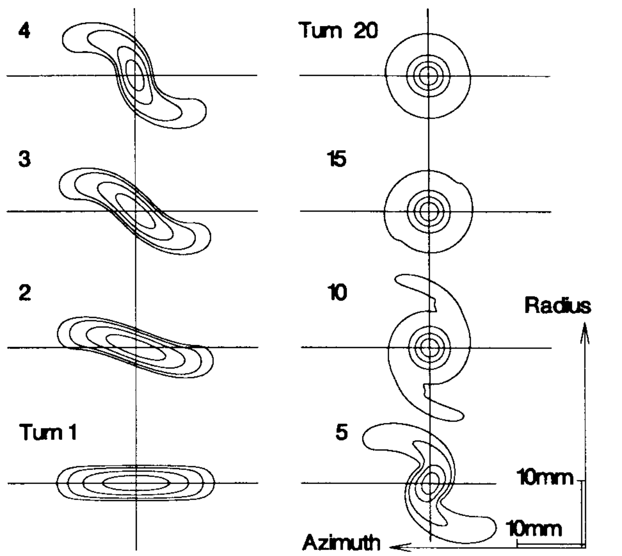

Figure 2: Simulation of a coasting beam of 1 mA with the Sphere Model. A beam bunch of initially 15 degrees phase width, remaining on 3 MeV without acceleration is deformed into a galaxy shaped charge distribution within 5 to 10 turns.

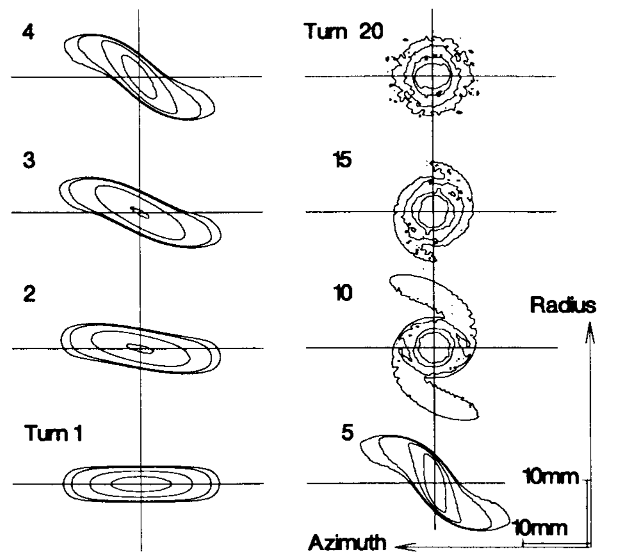

Figure 3: Simulation of a coasting beam of 1 mA with the Needle Model, remaining at 3 MeV in the injector 2 cyclotron. From the initial phase width of 15 degrees the beam bunch is deformed into a galaxy shaped charge distribution within 5 to 10 turns.

The **Sphere Model** , a two-dimensional fluid dynamical simulation method using the particle-in-cell principle, is free of restrictions related to energy deviations and bunch shape.

Separating the motion of the particles into three parts of different time scale is the key to computational simplification. The very fast part is the common motion of all particles moving along with the center of gravity of the bunch. Its calculation is avoided by choosing a reference frame moving with the bunch. A further separation occurs between the fast motion due to betatron oscillations and the slow motion caused by space charge forces. The radial betatron oscillations being tightly coupled to longitudinal oscillations let the particles move on nearly circular orbits in the plane defined by cyclotron radius and azimuth. Combining this motion with the vertical betatron oscillations, the particles perform fast motions on the surface of little vertical cylinders. For the calculation of the slow motion due to space charge forces an averaging of the values of the electrical field over the surface of the little cylinders is appropriate.

The sphere model further assumes that the results of the averaging are similar for all the cylinders having the same center, independent of their differences in height and radius. These cylinders then form a group that should move in common. Taking into account that very few particles simultaneously have a high radial and vertical betatron amplitude (narrow cylinders are higher than wide ones), each group of cylinders can be replaced by a charged sphere represented by its centerpoint.

In a careful analysis the radial space charge force has to be separated into an incoherent part acting like a transversal space charge force and a coherent part that has to be included in the context of the longitudinal space charge force. With the separation of motions based on different time scales and the principle of averaging, the sphere model intrinsically includes this separation and establishes the equivalence between longitudinal and coherent radial space charge forces.

Summarizing the layout of the sphere model, the motion of points in the median plane representing centers of charged spheres that can overlap each other, models the behaviour of a 3-dimensional charge distribution due to longitudinal and coherent radial space charge forces in the cyclotron. The law describing the force between two uniformly charged spheres can be calculated analytically. In the simulation the fact that the sphere model describes the motion in an isochronous cyclotron is taken into account by applying the rule that the motion always takes place in a direction perpendicular to the force.

Due to the long range the calculation of the force in radius and phase requires a two dimensional convolution of the charge density distribution with the function of the force between two spheres. Modern computational mathematics allow to perform such a convolution using fast fourier transforms before and after a simple Hadamard-product (multiplying corresponding components of the two large vectors). In the particle-in-cell method the information is transferred back and forth between the coordinates of moving particles and a regular grid used to calculate the force functions. Each of the particles can be assigned an individual value for the charge density of the sphere it represents.

Figure 2 shows how a distribution of sphere centers and the corresponding charge density distribution deform due to space charge forces. At low energies and high intensities, within a few turns, an elongated beam bunch is transformed into a galaxy shaped distribution with a vertical extension that remains essentially unchanged.

The **Needle Model** is based on similar ideas as the sphere model, but without requiring some of the important simplifying assumptions. The principles, to use a reference frame moving with the center of gravity of the bunch and, to separate the vertical motion from the horizontal one, are kept, but the motion in the radial-azimuthal plane is freely following the combined action of radial-azimuthal focusing and space charge forces. In the needle model the beam is treated as an ensemble of charged vertical needles of the same height as the beam. The charge density distribution in the radial-azimuthal plane defined by the locations of thousands of needles with individual charge values again provides a two dimensional model of the three dimensional reality. The particle-in-cell method and the convolution using fast fourier transforms could be derived from the code for the sphere model with moderate modifications.

Due to limited computer resources, simulations often have to use models with more simplifying assumptions than would be desirable. When Shane R. Koscielniak [9] came to PSI for a two month visit, the large increase of computing power that had become available allowed us to replace the old sphere model by the more realistic needle model.

The high similarity of the results comparing the sphere model to the needle model becomes apparent looking at the figures 2 and 3. One has to compare the charge density distributions. For the sphere model these are shown on the left half of each of the pictures of the beam bunches.

Increasing the Reliability

The results from the sphere model were quite unexpected in spite of the knowledge of the principle of the vortex motion and of similar results[10] from A. Chabert. In order to increase the credibility of the results a series of cross-checks and improvements of the code have been carried out: For small deformations of the bunch the sphere model had to agree with the cylinder model; two charged spheres, at a distance such that they are clearly separated, had to perform a circular motion around their symmetry point, where the rotation speed could be checked by analytical calculations, and finally, the results of the sphere model proved to be similar to those of the newly developed needle model as well as comparable to results from other publications[11, 12, 13]. In order to check the numerical precision of the simulation, variations of the integration step size, of the number of bytes reserved for floating point numbers and of the mesh size of the grid used in the particle-in-cell method were investigated, as well a comparison of several integration algorithms, but none of these modifications produced substantial changes of the simulation results.

A better approximation to the reality of accelerated beams in the cyclotron than just simulating coasting beams could be achieved with two extensions to the sphere model. First, a simplified scheme to simulate the acceleration in the cyclotron was added. In contrast to our expectations, an elongation of the round charge distribution at higher energies did not show up in the simulations. A bunch once being deformed to a round charge distribution in radius and phase seemed to keep this shape a long way through the acceleration process. The second extension introduced the effects from neighbouring orbits into the simulation. It exploited the fact that the method based on the fast fourier transform actually performs a circular convolution. Restricting the radial extent of the grid (used for the calculation of the forces) to the turn separation, an infinite array of equally spaced bunches could be simulated. An example of the simulation of an accelerated beam with the sphere model is anticipated in figure 1.

For the simulations including acceleration the number of turns where the simulation remained stable was initially limited to about 20. It could be extended to more than 100 adapting the grid size more often, but by a smaller factor, to cope for the growing extent of the beam with increasing energy. Another trick to increase the long term stability of the integration was the "grid-shuffling", where an offset randomly chosen between zero and the mesh size defines the position of the particles relative to the grid used for the calculation of the force function.

Insights: Interpretation of the results

Trying to extract the common features from a large variety of simulation results, the trend that the charge density distribution is deformed towards a galaxy shaped cloud clearly sticks out. It can be understood as the vortex motion principle[1] applied to the case of fully separated turns. In the discussions, the question of the proper naming of this effect came up: with careful judging it is neither a resonance, nor an instability, it is a mismatch. This hypothesis has been tested successfully with a beam bunch having a round charge distribution from the start showing no change of shape in the simulation.

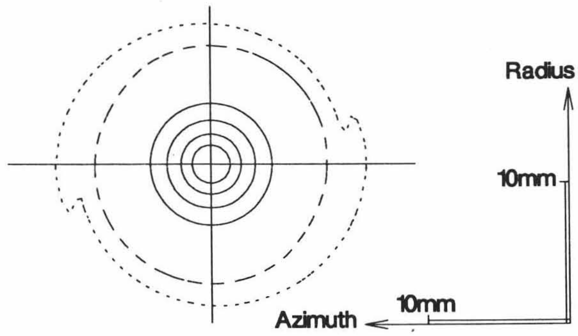

Figure 4: The formation of a "round" charge density distribution due to space charge forces is accompanied by the production of wide haloes. For turn # 40 of a 3 MeV coasting beam with 1 mA and initially 15 degrees phase width the 1% (dotted) and 2% (dashed) contours are shown in addition to the full lines for 10%, 20%, 50% and 80% of the maximum charge density.

Apart from this general trend to deform the beam towards a round charge distribution as the limiting case, the longitudinal and coherent radial space charge forces can produce really strange effects like two or three rotation centers, or beams with a narrow core having distinct satellites. A tendency that is common to most of the simulation results is the formation of substantial, long range haloes in the radial and azimuthal directions around the central beam. Figure 4 shows the wide haloes predicted by the simulation of a coasting beam. From the experience with the cyclotron setup it is well-known that haloes can increase extraction losses and that it is important to cut haloes away at the lowest possible energy in order to avoid machine activation.

An important information gained from these simulations is the knowledge that, due to the low injection energy in the PSI injector 2 cyclotron, for beams above 1 mA the formation of a round distribution happens very fast, within the first 4 to 8 turns and that this charge distribution with extremely narrow phase width remains then essentially unchanged until extraction. The fact, found by the operators, that the best settings for high intensity beams in the injector 2 need no flattop voltage, reflects this theoretical result in the reality of the beam.

Conclusions

A careful design of the models, intensive work to improve the algorithms and thorough cross-checking of results have brought the simulation of space charge dominated beams in cyclotrons from the status of an academic game towards a tool that helps to understand the complex effects occurring with real beams. Before it will be possible however, to quantitatively predict extraction losses in the order of 10^{-3} or 10^{-4} of the beam intensity there is still a long way to go.

Acknowledgements

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