OPTIMISATION OF THE BEAM BUNCHER PARAMETERS IN THE INJECTION BEAMLINE OF THE NAC

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Equations which link the buncher voltage and its position in the transfer beamline, between the light ion solid-pole injector cyclotron (SPC1) and the separated-sector cyclotron (SSC), with the beam width at extraction in the SSC, have been derived. This led to the repositioning of the buncher to minimize the beam width at extraction. The resulting improvement in the beam quality relieved the constraints on the beam intensity for radiotherapy and isotope production.

1 Introduction

It is important to limit the beam pulse length in an isochronous cyclotron in order to obtain single-turn extraction with high efficiency. To limit the beam pulse length in the separated sector cyclotron (SSC) of the National Accelerator Centre (NAC), slits are used in the injector cyclotron (SPC1) and injection beamline, between SPC1 and the SSC. The injection beamline, shown in figure 1, was also equipped[1] with a klystron buncher to shorten the beam pulse length at injection in the SSC. For single-turn extraction from the SSC the beam pulse length, at injection in the SSC, should not exceed a certain maximum length which is a function of the energy spread and emittance of the injected beam and several SSC parameters. Under normal working conditions the pulse length of the beam extracted from SPC1 is longer than this maximum value, and there is a further increase in length due to the energy spread in the beam in the 25 m injection line. The effect of the buncher on the beam depends both on its voltage and position in the injection line which have therefore to be optimised to improve the quality and intensity of the beams which can be extracted from the SSC. The buncher position shown in figure 1, was chosen[1] , using as criterion that the rate of increase of the beam pulse length in front of the buncher is equal to the rate of decrease of the pulse length after the buncher, without considering the beam width at extraction in the SSC. Equations for the calculation of the buncher optimum voltage, which give the minimum beam width at extraction in the SSC have be derived. The influence of the buncher position on the beam width at the extraction radius of the SSC will also be used to find the optimum position of the buncher.

2 Presentation of the SPC1 Beam in the $\Delta\theta$-ΔE Phase Plane

To calculate the beam pulse characteristics in the $\Delta\theta-\Delta E$ phase plane (pulse length - energy spread phase plane) the sigma-matrix formalism[2] was used. This implies that in each of the three two-dimensional phase space projections, i.e. the x-x' (horizontal), y-y' (vertical) and $\Delta\theta-\Delta E$ planes, the beam is enclosed by an ellipse. For the construction of the phase ellipse in the $\Delta\theta-\Delta E$ plane at extraction in SPC1, the maximum pulse length and energy spread, as well as the orientation of the ellipse, must be known. The maximum values of ΔE and $\Delta\theta$ are functions of the slit gaps in SPC1. The energy spread in the beam pulses obtained from an isochronous cyclotron with no flat-top acceleration rf system, such as SPC1, has the distribution shown by the curve, enclosed by the ellipse, in figure 2. The main contribution to the energy spread, $\Delta E/E$, is a result of the sinusoidal dee voltage. The maximum energy spread, $(\Delta E/E)_m$, in the beam from SPC1, with pulse length θ_m, is therefore given by:

$$(\Delta E/E)_m = 1-\cos(\theta_m/2) \quad (1)$$

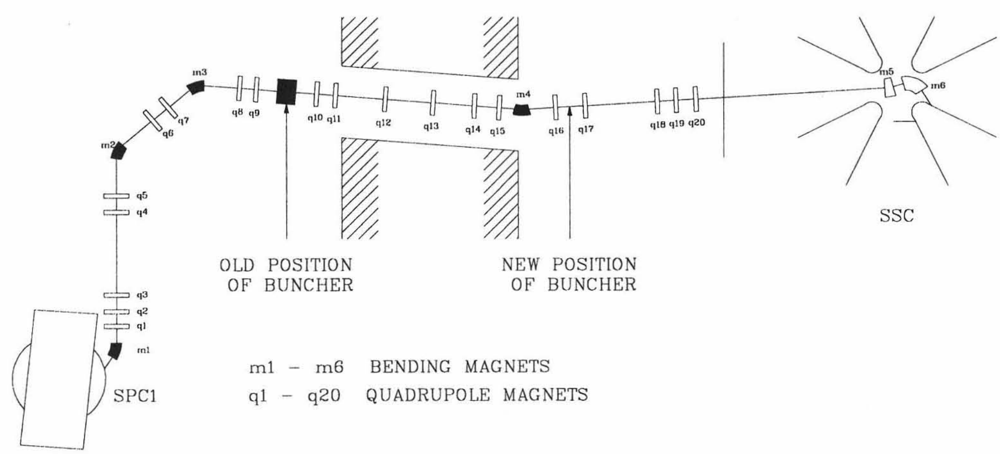

Fig 1 Layout of the injection beamline between SPC1 and the SSC with the old and new buncher positions indicated.

$$\approx \tfrac{1}{2}\Delta\theta_m^2 \tag{2}$$

with

$$\Delta\theta_m = \theta_m/2 \tag{3}$$

To specify the beam in the sigma-matrix formalism, an ellipse which encloses the particles has to be determined. The simplest approach is to use an ellipse with half-axes $\Delta\theta_m$ and $(\Delta E/E)_m$. This ellipse does not enclose all the particles in the pulse. Equations for the half-axes of an ellipse of minimum area which includes all the particles, are given below. Figure 2 shows such an ellipse which includes all the particles in a beam pulse with a length of $2\Delta\theta_m$. The equation for the curve included by the ellipse is:

$$\frac{\Delta E}{E} = -\frac{\Delta\theta^2}{2} + k \qquad \text{for } -\Delta\theta_m \le \Delta\theta \le \Delta\theta_m \tag{4}$$

and the equation for the ellipse is:

$$\frac{\Delta\theta^2}{a^2} + \frac{(\Delta E/E)^2}{k^2} = 1 \tag{4}$$

with

$$k = \frac{\left(\Delta\theta_m^2/2\right)}{1-\sqrt{1-\frac{\Delta\theta_m^2}{a^2}}} \tag{5}$$

The area, A, of the ellipse is given by:

$$A = \pi a k = \pi a\left[\frac{\left(\Delta\theta_m^2/2\right)}{1-\sqrt{1-\frac{\Delta\theta_m^2}{a^2}}}\right] \tag{6}$$

and is a minimum when its first derivative with respect to a, is equal to zero and the second derivative is positive. The major half-axes a and k for an ellipse with minimum area can be written as a function of $\Delta\theta_m$ by using the equation formed by requiring that the first derivative of the area is equal to zero, and is given by:

$$a = \frac{2}{\sqrt{3}}\Delta\theta_m \tag{7}$$

$$k = \Delta\theta_m^2/3 \tag{8}$$

The major and minor half-axes, a and k, of the ellipse are, respectively, 1.1547 and 1.333 times larger than those of an ellipse with one half-axis equal to half the beam pulse length and the other half-axis equal to half the energy spread given by equation 2.

3 The Minimum Pulse Length at Injection in the SSC

The transfer matrix of the injection beamline for the calculation of the beam pulse length at injection of the SSC, is formed by the product of three separate matrices, A, F and B, where:

- A = the transfer matrix from SPC1 to the buncher
- F = the buncher transfer matrix, and
- B = the transfer matrix from the buncher to the injection in the SSC

There is no coupling between the y-y' phase plane and the $\Delta\theta - \Delta E$ phase plane in the injection line. The y-y' phase plane is therefore omitted from the transfer matrices. If we regard the buncher as a thin lens of focal length f acting in the longitudinal phase space, then the transfer matrix, between the buncher entrance and the injection in the SSC, can be written as:

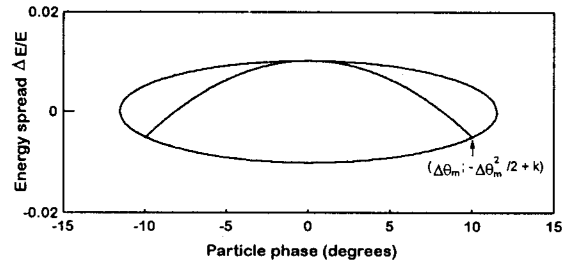

Fig 2 An ellipse which includes all the particles in a beam pulse with pulse length of 20°. The half-axes of the ellipse are $\Delta\theta = \frac{2}{\sqrt{3}}\Delta\theta_m$ and $\Delta E/E = \Delta\theta^2_m/3$.

$$R = BF$$

$$= \begin{bmatrix} B_{11} & B_{12} & 0 & B_{16} \\ B_{21} & B_{22} & 0 & B_{26} \\ B_{51} & B_{52} & 1 & B_{56} \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -\frac{1}{f} & 1 \end{bmatrix}$$

$$= \begin{bmatrix} B_{11} & B_{12} & -\frac{1}{f}B_{16} & B_{16} \\ B_{21} & B_{22} & -\frac{1}{f}B_{26} & B_{26} \\ B_{51} & B_{52} & 1-\frac{1}{f}B_{56} & B_{56} \\ 0 & 0 & -\frac{1}{f} & 1 \end{bmatrix} \tag{9}$$

The sigma-matrix, σ_f, at the SSC, is related to the sigma-matrix, σ_i, at the beginning of the system[2] :

$$\sigma_f = R\sigma_i R^T \tag{10}$$

Substituting equation 9 in 10 gives the σ-matrix at injection in the SSC. The σ_{55}- and σ_{66}-elements in equation 10, i.e. the square of the maximum projections onto the $\Delta\theta$- axis and $\Delta E/E$-axis respectively, are given by:

$$\begin{aligned}\sigma_{55}(SSC) = \; & B_{51}(\sigma_{11}B_{51} + \sigma_{12}B_{52} + \sigma_{15}(1-\tfrac{1}{f}B_{56}) + \sigma_{16}B_{56}) + \\ & B_{52}(\sigma_{12}B_{51} + \sigma_{22}B_{52} + \sigma_{25}(1-\tfrac{1}{f}B_{56}) + \sigma_{26}B_{56}) + \\ & (1-\tfrac{1}{f}B_{56})(\sigma_{15}B_{51} + \sigma_{25}B_{52} + \sigma_{55}(1-\tfrac{1}{f}B_{56}) + \sigma_{56}B_{56}) + \\ & B_{56}(\sigma_{16}B_{51} + \sigma_{26}B_{52} + \sigma_{56}(1-\tfrac{1}{f}B_{56}) + \sigma_{66}B_{56})\end{aligned} \tag{11}$$

and

$$\sigma_{66}(SSC) = \tfrac{1}{f}\sigma_{55} - \tfrac{2}{f}\sigma_{56} + \sigma_{66} \tag{12}$$

The minimum beam pulse length at injection in the SSC can be obtained by differentiating equation 11 with respect to the buncher strength, 1/f, and by putting the result equal to zero:

$$(1-\tfrac{1}{f}B_{56}) = -\frac{B_{51}\sigma_{15}+B_{52}\sigma_{25}+B_{56}\sigma_{56}}{\sigma_{55}} = K \tag{13}$$

The result obtained from the differentiation substituted in equation 11 gives the minimum beam pulse length, θ_{min} at injection in the SSC:

$$\theta_{min} = 2\sqrt{\sigma_{55}(SSC)}$$
$$= 2[B_{51}(\sigma_{11}B_{51} + 2\sigma_{12}B_{52} + 2\sigma_{15}BK + 2\sigma_{16}B_{56}) + B_{52}(\sigma_{22}B_{52} + 2\sigma_{26}B_{56} + 2\sigma_{25}K) + 2\sigma_{56}B_{56}K + B_{56}^2\sigma_{66} + K^2\sigma_{55}]^{1/2} \quad (14)$$

The energy spread, $(\Delta E/E)_f$, for a beam which is bunched to a minimum length θ_{min}, at injection in the SSC, is obtained from equation 12. For no coupling between the x-x' and $\Delta\theta - \Delta E$ phase planes, i.e. with B_{16}, B_{26}, B_{51} and B_{52} equal to zero, equation 14 is reduced to:

$$\theta_{min} = 2\left[\frac{B_{56}^2(\sigma_{55}\sigma_{66} - \sigma_{56}^2)}{\sigma_{55}}\right] \quad (15)$$

4 Derivation of the Equation for the Optimum Buncher Voltage

To derive an equation for the optimum buncher voltage for a minimum beam width at extraction of the SSC the following equation for the beam width, W, in a cyclotron is used[3] :

$$W = \frac{R_e(E_e - E_i)(1 - k - \cos\Delta\theta_f + k\cos 3\Delta\theta_f)}{E_e\gamma_e(\gamma_e + 1)(1 - k)} + \frac{1}{2}R_i\left(\frac{\Delta E}{E}\right)_i\frac{\gamma_i}{\gamma_e} + 2\frac{\gamma_i}{\gamma_e}\sqrt{\frac{\varepsilon_{ri}R_i}{\nu_{ri}\pi}} \quad (16)$$

with:

E_e = extraction energy of the cyclotron
E_i = injection energy of the cyclotron
R_e = extraction radius of the cyclotron
R_i = injection radius of the cyclotron
ν_{ri} = radial betatron frequency at injection
ε_{ri} = emittance in the x-x' phase plane
k = ratio of a flat-top voltage to the main voltage $(\Delta E/E)_i$
$(\Delta E/E)_i$ = energy spread at injection in the cyclotron $= 2\sqrt{\sigma_{66}(SSC)}$
$\Delta\theta_f$ = the maximum phase deviation with respect to the central particle $= \theta_m/2$ (with θ_m the beam pulse length) $= \sqrt{\sigma_{55}(SSC)}$

Equation 16 can be written in terms of the σ-notation :

$$W = \frac{R_e(E_e - E_i)(1 - k - \cos\sqrt{\sigma_{55}(SSC)} + k\cos 3\sqrt{\sigma_{55}(SSC)})}{E_e\gamma_e(\gamma_e + 1)(1 - k)} + R_i\frac{\gamma_i}{\gamma_e}\sqrt{\sigma_{66}(SSC)} + 2\frac{\gamma_i}{\gamma_e}\sqrt{\frac{\varepsilon_{ri}R_i}{\nu_{ri}\pi}} \quad (17)$$

The optimum buncher voltage is determined by requiring that the first derivative of the beam width, W, with respect to the buncher strength, 1/f, is equal to zero:

$$\frac{R_e(E_e - E_i)}{E_e\gamma_e(\gamma_e + 1)(1 - k)}(-A + B) + \frac{R_i\gamma_i}{\gamma_e}\left[\frac{\frac{\sigma_{55}}{f} - \sigma_{56}}{\sqrt{\frac{1}{f^2}\sigma_{55} - \frac{2\sigma_{56}}{f} + \sigma_{66}}}\right] = 0 \quad (19)$$

With

$$A = -\frac{\sin(\sqrt{\sigma_{55}(SSC)})}{2\sqrt{\sigma_{55}(SSC)}} \times \frac{d}{d\frac{1}{f}}(\sigma_{55}(SSC)) \quad (20)$$

$$B = -k\left[\frac{3\sin\left(3\sqrt{\sigma_{55}(SSC)}\right)}{2\sqrt{\sigma_{55}(SSC)}} \times \frac{d}{d\frac{1}{f}}(\sigma_{55}(SSC))\right] \quad (21)$$

$$\frac{d}{d\frac{1}{f}}(\sigma_{55}(SSC)) = -2B_{56}[(B_{15}\sigma_{15}(V) + B_{25}\sigma_{25}(V) + B_{56}\sigma_{56}(V) + \sigma_{55}(V)(1 - \tfrac{1}{f}B_{56})] \quad (22)$$

The $\sigma_{55}(SSC)$ terms in equations 20, 21, and 22 are to be obtained from equation 11. The second derivative of equation 19, with respect to the buncher strength, is positive, which implies that the buncher strength obtained from equation 19 gives a minimum beam width. Equation 19 is simplified considerably if no coupling between the x-x' and $\Delta\theta - \Delta E$ phase planes exists, and the cyclotron into which the beam is injected does not have a flat-top acceleration system. With the approximation $\sin\theta \approx \theta$, equation 19 reduces to:

$$\frac{R_e(E_e - E_i)B_{56}}{E_e\gamma_e(\gamma_e + 1)}\left[-\sigma_{55}(V)(1 - \frac{B_{56}}{f}) - \sigma_{56}(V)B_{56}\right] + \frac{R_i\gamma_i}{\gamma_e}\left[\frac{\frac{\sigma_{55}(V)}{f} - \sigma_{56}(V)}{\frac{\sigma_{55}(V)}{f^2} - \frac{2\sigma_{56}(V)}{f} + \sigma_{66}(V)}\right] = 0 \quad (23)$$

All the $\sigma(V)$ elements refer to the elements at the buncher entrance. The optimum strength of the buncher can be calculated numerically from equation 19 for given beam characteristics at the beginning of the beamline. With the optimum buncher voltage known it is possible to calculate $\sigma_{55}(SSC)$ and $\sigma_{66}(SSC)$ by using equations 11 and 12. The minimum width of the beam at extraction in the SSC can then be calculated from equation 17.

The operating buncher voltage for the best extraction efficiency in the SSC varies between 36 kV and 40 kV. With the σ-elements and the transfer matrix elements substituted in equation 19, an optimum buncher voltage of 38.14 kV is calculated. That calculated optimum buncher voltage lies in the centre of the experimentally determined range.

5 Derivation of the Equations for the Optimum Buncher Position

Although for a given buncher position the optimum buncher voltage can be determined by minimising the beam width at extraction, the beam width can further be optimised by determining the optimum buncher position. That position corresponds to an upright ellipse in the $\Delta\theta - \Delta E$ phase plane with a specific beam pulse length at injection in the SSC.

The optimum buncher position can be determined by applying Liouville's theorem: When an upright ellipse, with half-axes of $\Delta\theta_i$ and $(\Delta E/E)_i/2$ at the beginning of the beamline, is transferred to an upright ellipse, with half-axes $\Delta\theta_f$ and $(\Delta E/E)_f/2$ at injection in the SSC, the following relation exists:

$$(\Delta E/E)_f = \frac{\Delta\theta_i(\Delta E/E)_i}{\Delta\theta_f} \quad (24)$$

The radial beam width, W, at extraction in the SSC is obtained by substituting equation 24 in equation 16. For a minimum beam width at extraction, the first derivative of W with respect to $\Delta\theta_f$, the beam pulse length at injection in the SSC, should be zero and gives[4]:

$$\frac{R_e(E_e-E_i)}{E_e\gamma_e(\gamma_e+1)(1-k)}\left[\sin\Delta\theta_f - 2k\sin 3\Delta\theta_f\right] - \frac{\gamma_i R_i \Delta\theta_i^3}{4\gamma_e\theta_f^2} = 0 \qquad (25)$$

Equation 25 can be solved numerically. With the approximation $\sin\Delta\theta_f \approx \Delta\theta_f$ for a cyclotron without a flat-top system, equation 25 reduces to:

$$\Delta\theta_f = \Delta\theta_i\left(\frac{\gamma_i R_i E_e(\gamma_e+1)}{4R_e(E_e-E_i)}\right)^{1/3} \qquad (26)$$

The optimum pulse length at injection (upright ellipse) in the SSC for a specific pulse length at the beginning of the beamline can be calculated from equation 25. By using the transfer matrix of the beamline the equations for the shape of the final ellipse and the buncher strength that yield an upright ellipse at the end of a beamline consisting of a drift space, a buncher and a further drift space is given by:

$$\frac{1}{f} = \frac{(d_1^2+\beta_i^2)+2d_1d_2 \pm \sqrt{(d_1^2+\beta_i^2)^2-4d_2^2\beta_i^2}}{2d_2(d_1^2+\beta_i^2)} \qquad (27)$$

$$\frac{\beta_f}{\beta_i} = \left[1-\frac{2d_1}{f}+\frac{1}{f^2}\left(d_1^2+\beta_i^2\right)\right]^{-1} \qquad (28)$$

where

$$\beta_i = \frac{\Delta\theta_i}{\frac{1}{2}(\Delta E/E)_i},\ \beta_f = \frac{\Delta\theta_f}{\frac{1}{2}(\Delta E/E)_f},\ d_1 = \frac{L_1 h}{R\gamma_i(\gamma_i+1)} \text{ and } d_2 = \frac{L_2 h}{R\gamma_i(\gamma_i+1)}$$

with L_1 and L_2 ($L_2 = L_t - L_1$ with L_t the distance between the two cyclotrons) the distances from the preceding cyclotron to the buncher and from the buncher to the end of the line, h is the harmonic number and R the extraction radius of the preceding cyclotron. To calculate the distance from the beginning of the line to the buncher, equation 28 is substituted in equation 27. The distance obtained is the optimum position for a specific ratio of β_f/β_i. The optimum position of the buncher as a function of the initial beam pulse length at the beginning of a 25 m beamline consisting of a drift space, a buncher and a drift space for injection in the SSC are shown in figure 4.

For a more accurate calculation of the optimum buncher position in the injection beamline, equation 25 was used to determine the optimum pulse length $\Delta\theta_f$, at injection in the SSC for a specific pulse length from SPC1. The program TRANSPORT[5] was then used to fit the position and the strength of the buncher for a upright ellipse with the length of one of the half-axes equal to $\Delta\theta_f$. The upper curve in figure 4 shows the optimum position of the buncher between the SPC1 and the SSC for a 66 MeV proton beam.

6 Repositioning of the Buncher and its Effect on the SSC Beam Quality and Intensity.

After the buncher was moved to the new position as shown in figure 1, the beam characteristics were measured and compared with the previous values. The following improvements resulted:

1 The horizontal width of a 65 μA, 66 MeV proton beam, extracted from the SSC, decreased by approximately 8 mm. the reduction in beam width corresponds to a decrease in the measured horizontal emittance of the beam from 27 π mm mrad to 13 π mm mrad.
2. For typical beam currents the beam loss on the extraction elements of the SSC decreased by a factor of between 5 and 10, for proton beams at energies below 120 MeV.

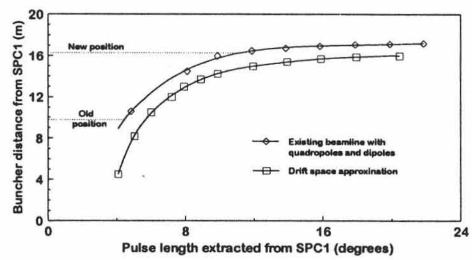

Fig 4 The optimum buncher position, for minimum radial beam width at extraction in the the SSC, as a function of the beam pulse length at the start of the injection beamline.

The improvement in the beam quality occurred at energies below 150 MeV. Because of the smaller turn separation at 200 MeV the maximum beam pulse length from SPC1 which can be injected into the SSC is only 8 degrees.

For the new position the buncher voltage had to be increased by 35% to 70 kV. Minor changes had to be made to the buncher to accomplish this.

Space-charge effects have been neglected in the calculations above. Such effects at high beam intensities, may play an important role and could cause deviations from the predicted results

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