SIMULATION OF RF STACKING AND MULTIPLE SINGLE-TURN INJECTION AND MULTITURN INJECTION

Y. J. Yuan J. W. Xia Y. N. Rao B. W. Wei

Institute of Modern Physics, Chinese Academy of Sciences, P.O.Box 31, Lanzhou 730000, China

RF stacking procedure was simulated using a small programming code in this paper. The evolution of longitudinal phase space and the simulation results with beam cooling effects are given. Multiple single-turn injection can also be simulated by the code. In addition, the simulation results of multiturn injection by the DIMAD program are also given.

1 Introduction

A proposal of heavy ion cooling storage ring(HIRFL—CSR) is raised in IMP. As the earlier stage of designing the machine, several methods of injection are being studied. In this paper three of the injection methods are simulated by using of computer.

2 The Programming Code

In order to study the longitudinal behaviors of the beam during the injection, the code consists of two physical procedures. The action of RF cavity on the beam and the cooling effects of an electron cooler.

The action of RF cavity on the beam is represented by solving the following differential equation[1]:

$$\begin{cases} \frac{dW}{dt} = \frac{qV}{2\pi h}[(\sin\phi_s - \sin\phi] \\ \frac{d\phi}{dt} = \frac{\omega_{rf}^2 \eta_{tr}}{\beta^2 E_s} W \end{cases} \qquad (1)$$

and for the cooling effects of an electron cooler we use the following cooling force formula[1,2]:

$$\frac{d(\delta p/p)}{dt} = \frac{2\pi r_e r_p \eta J_e}{(\delta p/p)^2 e \beta^4 \gamma^3} \frac{Z^2}{A} \ln \frac{b_{min}^2 + b_{max}^2}{b_{min}^2} \qquad (2)$$

where b_{min} and b_{max} are the minimum and maximum collision distance, respectively. This formula has the property that the cooling force rapidly reduced to zero when $\delta p/p$ approaching zero.

3 Simulation of RF Stacking

RF stacking is used to accumulate the beam in the longitudinal phase space. It consists of three steps:

(1) **Capture:** The RF voltage is switched on and the beam is captured in the stationary bucket, after 1/4 period of synchrotron oscillation the RF voltage is reduced to a lower value and still keep the beam in the bucket, then, the phase space dilution factor in longitudinal phase space is reduced by a factor of 5;

(2) **Acceleration:** The RF voltage, frequency and synchrotron phase are adiabatically changed and the beam is moved to the top of the stacking orbit;

(3) **Deposit:** The RF voltage is switched off and the beam is remained on the stacking orbit.

For every stacking cycle, the bucket passes over the stacked particles and disturbs them. The particles lose some energy and move to the stacking bottom from the top, finally, filled into the energy gap.

During the whole procedure electron cooling is applied at the stacking top. When the repetitive frequency of RF stacking is set, there is a limit in the energy spread of the stacked beam.

The momentum spread δ has

$$\frac{d\delta}{dt} = N_{RF}\delta_i \qquad (3)$$

where δ_i is the momentum spread of the beam after RF compression during step 1, N_{RF} is the repetitive frequency, normally from 20Hz to 50Hz. Assuming cooling time is τ, we have

$$\frac{d\delta}{dt} = -\frac{\delta}{\tau} \qquad (4)$$

Combining these two equations, we have the equation:

$$\delta(t) = [1 - e^{-t/\tau}]\tau N_{RF}\delta_i + \delta(0)e^{-t/\tau} \qquad (5)$$

and

$$\delta(\infty) = \tau N_{RF}\delta_i \qquad (6)$$

The parameters of RF system are changed piece by piece linearly. Parameters of RF system during RF stacking procedure of $^{40}Ar^{18+}$–25MeV/u are show in Figure 1.

During the simulation, the RF stacking procedure with cooling is repeated for 50 periods, and without cooling is repeated for 5 periods. Figure 2 illustrates the momentum phase space evolution. Figure 3 is the simulation result of RF stacking procedure with cooling(top) and without cooling(bottom), separately. In both cases the capture efficiencies are of 80% roughly.

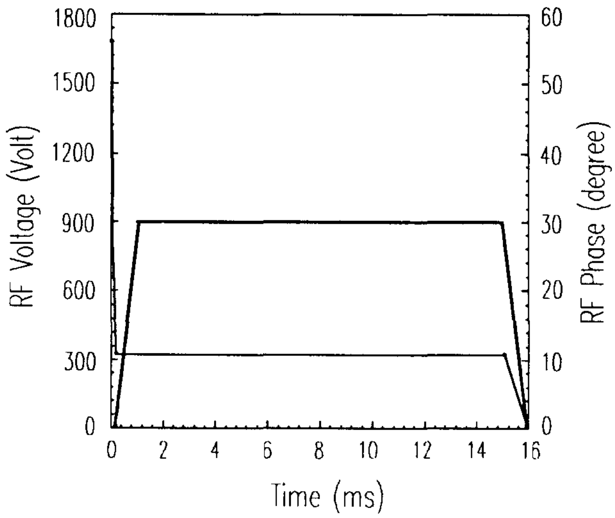

Figure 1: The parameters of RF system. (thin line: RF voltage, thick line: RF phase)

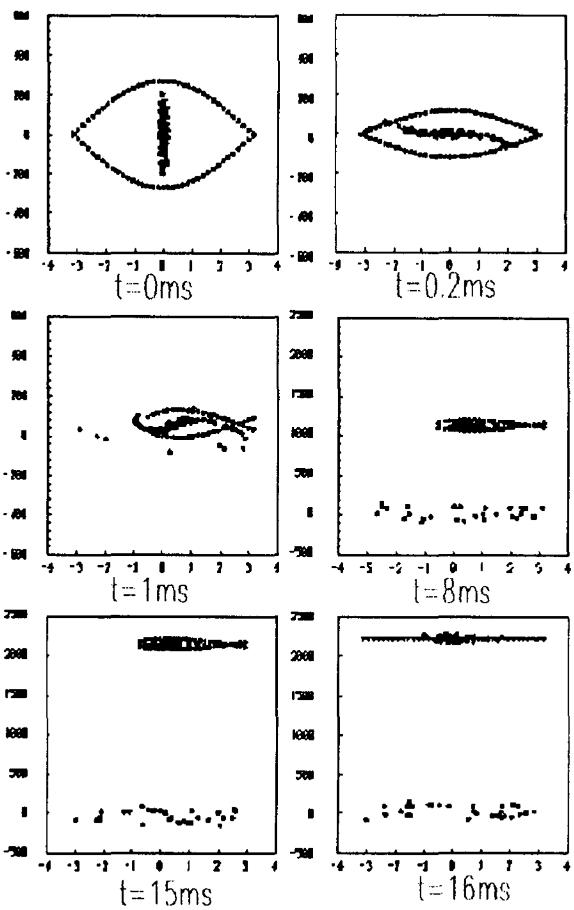

Figure 2: The simulation of momentum phase space evolution. (horizontal: RF phase, vertical: δPC)

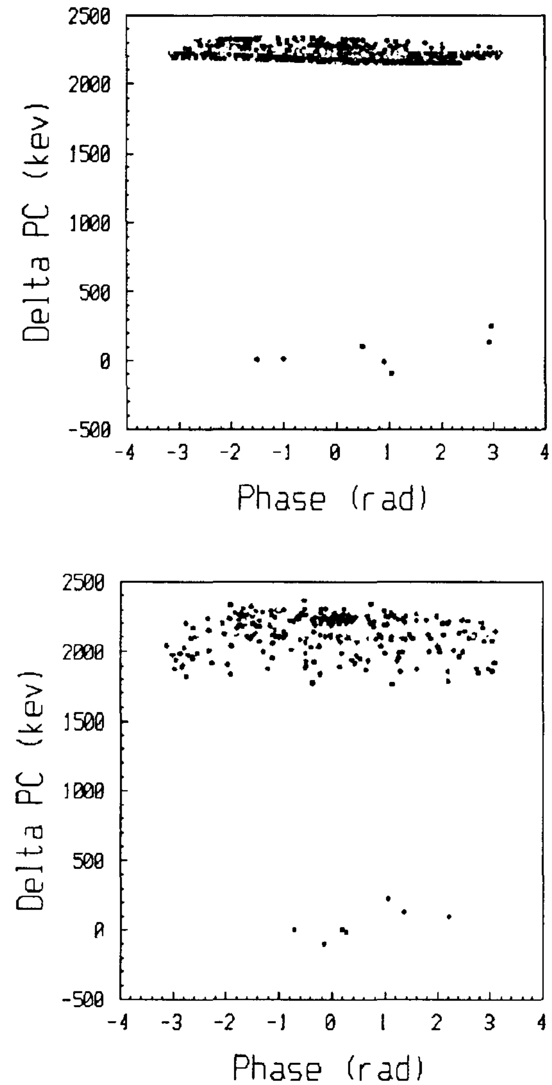

Figure 3: The simulation of RF stacking.

4 Simulation of Multiple Single-Turn Injection

Multiple Single-turn injection is preferred as the capability to retain the quality of the injected beam. In addition, by keeping the velocity of electrons equal to the mean velocity of the injected ions one can minimizes the beam transverse and longitudinal cooling time ($\tau_\perp$ and $\tau_\parallel$) of 40 to 100 ms, there by the repetition frequency of multiple single-turn injection can reach 20~50Hz.

The single turn injection is accomplished at one of the bending section of CSR main ring(CSRm). The RF system and cooling system are all working permanently during the injection period. The distortion of the closed orbit is formed by four dipole magnets, and the beam injection orbit is formed by one septum and one horizontal kicker. See appendix A.

The injection orbit of CSRm can be seen in Figure 4.

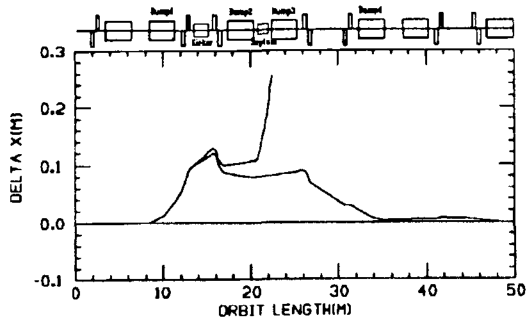

Figure 4: The injection orbit of CSRm.

The injected beam occupies 3/4 of the ring orbit. The transversal and longitudinal emittance and the beam length shrink sharply under the RF and cooling effects. When the beam length gets to 1/4 of the ring orbit(or $\pi/2$ of the stationary longitudinal phase space) the successive single turn injection can be performed.

Figure 5 is the simulation result of longitudinal phase space evolution during the first single turn injection period. The instant cooling time applied here is 40ms for $\Delta p/p =\sim 1 \times 10^{-3}$. The beam injected is considered as coasting beam for first harmonic of RF cavity. In case that the cooling force formula comes from single particle model, the final result of phase space has a hollow center.

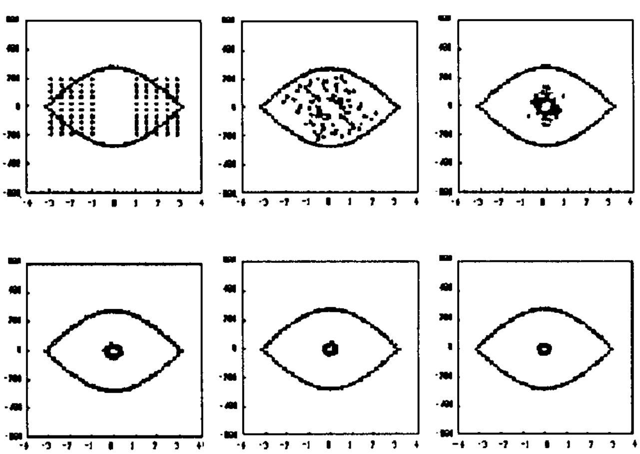

Figure 5: The longitudinal phase space evolution.

The limitation on the number of ions comes from two ways. First is the space charge instability which manifests itself in the increase of the transverse beam emittance. Second is the gradual increase of the fraction of the orbit length occupied by the stored beam bunch. The transversal effect is small enough in this case ($\Delta Q \sim 10^{-3}$). The effect caused by the longitudinal repulsion will be evaluated in Appendix B[3]. The orbit length occupied by the beam bunch is proportional to $N_i^{1/3}$, where N_i is the number of ions in the bunch.

5 Simulation of Multiturn Injection

The multiturn injection system consists of the following components:

(1) focusing elements to form the injecting beam into an extra small horizontal size at the injection point;

(2) septum-magnets to inflect the beam into the ring;

(3) instant offsets of the close orbit are formed at the injection region without change of the closed orbit beyond.

The simulation of the multiturn injection procedure has been done using DIMAD program. The offset of closed orbit is reduced from 3.5cm to 0 during 30 revolution periods. For each period, one hundred particles are injected. Since we have a small beam size at the exit of the injector (2.23mm for 5πmm·mrad) and the septum is near to the beam, a good result has been obtained. One thousand particles are injected with a dilution factor[4,5] of 2.0 . Figure 6 is the simulation result of the horizontal transverse phase space. For each multiturn injection, roughly 10 times particles of single turn injection are injected into the ring. Figure 7 shows the injection efficiency for each turn.

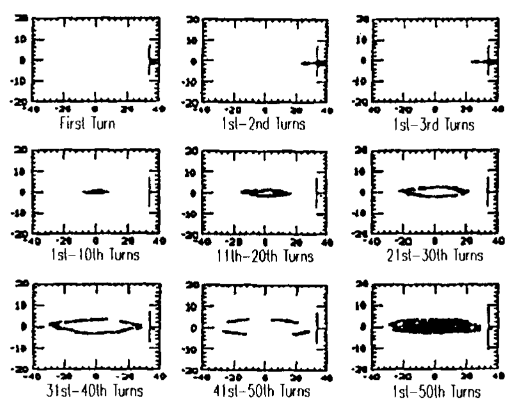

Figure 6: The simulation result of multiturn injection.

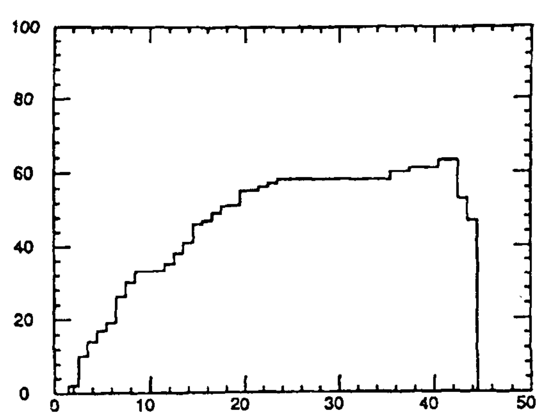

Figure 7: The injection efficiency for each turn.

References

1. M. Conte et. al., *An Introduction of the Physics of Particle Accelerators*,(World Scientific, Singapore, 1991).
2. J. D. Jackson, *Classical Electrodynamics*, (John Wiley & Sons, American, 1975).
3. *HEAVY ION STORAGE RING COMPLEX K4-K10*, A Technical Proposal, Dubna 1992.
4. S. Fenster et. al., Multiturn Injection, *IEEE Trans. On Nuc. Sci., Vol. NS-28, No. 3*, June 1981.
5. G. H. Rees, INJECTION, *CAS 84*, CERN 85-19.

Appendix A: Orbit Distortion by Four Bumping Magnets

The distortion must be formed only to affect the closed orbit among the injection region without any change of the closed orbit beyond. In order to produce a horizontal offset of closed orbit x_i at x'_i at the exit of the septum, the strength of the four bumping dipoles should be given by:

$$\psi_1 = -\frac{(\sin\phi_{2-i}\alpha_i - \cos\phi_{2-i})x_i + \sin\phi_{2-i}\beta_i x'_i}{\sqrt{\beta_i\beta_1}\sin\phi_1}$$

$$\psi_2 = -\frac{\sqrt{\beta_1}(\sin(\phi_1+\phi_{2-i})\alpha_i - \cos(\phi_1+\phi_{2-i}))\psi_1 + \sqrt{\beta_i}x'_i}{\sqrt{\beta_2}(\sin\phi_{2-i}\alpha_i - \cos\phi_{2-i})}$$

$$\psi_3 = -\frac{\sqrt{\beta_1}(\sin(\phi_a) - \sin(\phi_b) + \cos(\phi_a)\alpha_i - \cos(\phi_b)\alpha_i)\psi_1 + 2.\sqrt{\beta_i}\sin(\phi_2+\phi_3)x'_i}{\sqrt{\beta_3}(\sin(\phi_3-\phi_{2-i}) + \sin(\phi_3+\phi_{2-i}) - \cos(\phi_3-\phi_{2-i})\alpha_i + \cos(\phi_3+\phi_{2-i})\alpha_i)}$$

$$\psi_4 = \frac{\sqrt{\beta_1}(\sin(\phi_c) + \sin(\phi_d) + \cos(\phi_c)\alpha_i - \cos(\phi_d)\alpha_i)\psi_1 + 2.\sqrt{\beta_i}\sin\phi_2 x'_i}{\sqrt{\beta_4}(\sin(\phi_3-\phi_{2-i}) + \sin(\phi_3+\phi_{2-i}) - \cos(\phi_3-\phi_{2-i})\alpha_i + \cos(\phi_3+\phi_{2-i})\alpha_i)} \quad (A.1)$$

where $\alpha s, \beta s$ are the horizontal betatron oscillation parameters at the exit of the septum and the four bumping dipoles, $\phi_1, \phi_2, \phi_3, \phi_{2-i}$ are the betatron oscillation phase shift between the four bumping dipoles and between the second bumping dipole and the exit of the septum. $\phi_a = \phi_1 - \phi_2 - \phi_3 + \phi_{2-i}$, $\phi_b = \phi_1 + \phi_2 + \phi_3 + \phi_{2-i}$, $\phi_c = \phi_1 - \phi_2 + \phi_{2-i}$, $\phi_d = \phi_1 + \phi_2 - \phi_{2-i}$.

Appendix B: Longitudinal Repulsion of Ions

Let us consider the ion bunch circulating on a ring orbit and being captured in a stationary RF bucket. The ions are cooled permanently. The longitudinal dimension of the bunch will be determined by the equality of the potentials of the longitudinal repulsive field of the bunch and of RF field. The RF potential ΔU over the length $l_b/2$ from the bunch center can be written as:

$$\Delta U = \frac{\pi^2}{2}\frac{h^2 l_b^2}{P^2}U_{RF} \quad (B.1)$$

and potential U_a between the center of the beam bunch and the wall of the vacuum chamber can be written as:

$$U_a = \frac{eq_i N_i}{2\pi\epsilon_0 l_b h}[0.5 + \ln(R/a)] \quad (B.2)$$

where U_{RF}, the RF amplitude; h, the harmonic number; P, the perimeter of the ring; e, the electron charge; q_i and N_i, the charge state and number of ions; R, the radius of the vacuum chamber; a, the beam radius.

Assuming $\Delta U = U_a$, then the longitudinal dimension of the bunch could be determined:

$$l_b/P = \left\{\frac{eq_i N_i}{PU_{RF}\pi^3\epsilon_0 h^2}(0.5 + lnR/a)\right\}^{1/3} \quad (B.3)$$

The following table gives the length l_b and the ratio l_b/P for different numbers of $^{40}Ar^{18+}$ ions in CSRm, where h=1, P=141m, q_i=18 and (0.5+ln R/a)=10.

Table: l_b and l_b/P

N_i	U_{RF}=237V		U_{RF}=500V	
	l_b (m)	l_b/P	l_b (m)	l_b/P
1×10^7	4.45	0.03	3.47	0.02
1×10^8	9.59	0.07	7.47	0.05
1×10^9	20.6	0.15	16.1	0.11
4×10^9	32.8	0.23	25.6	0.18