EQUILIBRIUM BETWEEN ELECTRON COOLING AND INTRABEAM SCATTERING

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In the static operation with electron cooling at constant energy, the resolution limits of cooled heavy ion beam are determined by the electron cooling and the heating of intrabeam scattering (IBS). According to the theory of electron cooling and IBS, simulation is made to calculate the variations of emittances and momentum spread with time for a 8.5MeV/u O^{8+} beam stored in HIRFL–CSR. Furthermore the dependence of equilibrium emittances and momentum spread on the particle numbers are presented.

1 Introduction

The phase space volume of the cooled ion beam in a cooler-storage ring is reduced by electron cooling. The ion beam cannot be cooled to infinitely small emittances and momentum spread because there exists heating mechanisms. In the case of static operation with electron cooling at constant energy (without an internal target), the final beam properties are determined by an equilibrium between the cooling power of electron cooling system and the heating source which may be residual gas scattering, intrabeam scattering and electron cooling.

Under the ultrahigh vacuum conditions ($P_{gas} = 1.0 \times 10^{-10}$ torr) of HIRFL–CSR[1], beam heating by scattering on the residual gas and the final balance between heating and cooling can be easily estimated. For example, one finds for 8.5 MeV/u O^{8+} and a cooling time $\sim 1sec$, the equilibrium emittance $\epsilon_{eq.} \leq 10^{-2}\pi mm \cdot mrad$. It will be shown that the equilibrium emittances due to intrabeam scattering will be bigger, so that the scattering on the residual gas can be neglected comparatively for the ion beam with more than 10^8 particles, and the intrabeam scattering dominates the resolution limits of the cooled ion beam.

2 Electron Cooling Rate

A practical application of the electron cooling needs knowledge of cooling time value. However, the complicated dependence of electron cooling force on parameters brings difficulties to the analytical calculation of cooling time. A simulation[2] is made to calculate the evolution of emittances and momentum spread with cooling time, taking into account the betatron oscillation and phase motion of particle in the storage ring and the influence of the cooling electron beam space charge.

In the simulation procedure, the ring is divided into two parts, the first part from the cooling section exit up to the cooling section entrance, the second part going through the cooling section. The change of the transverse coordinates x_j and θ_j (j=h,v) from the cooling section exit to its entrance is given by the following matrix expression:

$$\begin{pmatrix} x_j \\ \theta_j \end{pmatrix}_{entr.} = \begin{bmatrix} cos(\mu_j) + \alpha_j sin(\mu_j) & \beta_j sin(\mu_j) \\ -\gamma_j sin(\mu_j) & cos(\mu_j) - \alpha_j sin(\mu_j) \end{bmatrix} \cdot \begin{pmatrix} x_j \\ \theta_j \end{pmatrix}_{exit}, \qquad j = h, v$$

in which $\mu_j = 2\pi\nu_j(1-\eta)$ is the phase advance after the first part, ν_j is the betatron number of the storage ring and $\eta = \frac{l_{cooler}}{C_{ring}}$ is the fraction of cooling section length to the ring circumference.

The longitudinal coordinates at the entrance of cooling section is calculated with the use of the particle phase motion equations:

$$\begin{cases} \dfrac{d\phi}{dt} = h\omega_s\eta_\omega \dfrac{\Delta P}{P_s}, \\ \dfrac{d(\frac{\Delta P}{P_s})}{dt} = \dfrac{Q_i e V_a \omega_s}{2\pi\beta_s^2 E_s}(cos\phi - cos\phi_s) \end{cases}$$

In the cooling section, the cooling force decreases the particle velocity components and doesn't change coordinates of the particle on each certain passage through the cooling section, therefore , the new coordinates of the particle after crossing the cooling section, at its exit, are given by

$$(x_j)_{exit} = (x_j)_{entrance},$$

$$(\theta_j)_{exit} = (\theta_j)_{entrance} + \frac{F_j l_{cooler}}{P_s \beta_s c_l}$$

The cooling force in the moving frame with the average electron velocity at finite magnetic field strength can be represented as follows[3]:

$$\vec{F}_\perp = -C \cdot \vec{\theta}_\perp \cdot \begin{cases} \frac{1}{\theta^3}(2L_{FH} + k_\perp \cdot L_{MH}), & \theta > \theta_{e\perp} \\ \frac{2(L_{FL}+N_L L_{AL})}{\theta_{e\perp}^3} + \frac{k_\perp \cdot L_{ML}}{\theta^3}, & \theta_{e\parallel} < \theta < \theta_{e\perp} \\ \frac{2}{\theta_{e\perp}^3}(L_{FS} + N_S L_{AS}) + \frac{L_{MS}}{\theta_\parallel^3}, & \theta < \theta_{e\parallel} \end{cases}$$

$$\vec{F}_{\parallel} = -C\cdot\vec{\theta}_{\parallel}\cdot\begin{cases} \frac{1}{\theta^3}(2L_{FH}+k_{\parallel}\cdot L_{MH}+2), & \theta > \theta_{e\perp} \\ \frac{2(L_{FL}+N_L L_{AL})}{\theta^2_{e\perp}\theta_{e\parallel}} + \frac{k_{\parallel}\cdot L_{ML}+2}{\theta^3}, & \theta_{e\parallel} < \theta < \theta_{e\perp} \\ \frac{2}{\theta^2_{e\perp}\theta_{e\parallel}}(L_{FS}+N_S L_{AS}) + \frac{L_{MS}}{\theta^3_{e\parallel}}, & \theta < \theta_{e\parallel} \end{cases}$$

in which $C = 2\pi n_e\cdot Q_i^2 r_e^2\cdot m_e c^2/(\beta_s^2\gamma_s^2)$, $k_\perp = 1-3\left(\frac{\theta_l}{\theta}\right)^3$, $k_\parallel = 2+k_\perp$, and $\theta = \sqrt{\theta_h^2+\theta_v^2+\theta_l^2}$ is the full velocity of particle in the moving frame in units of $\gamma\beta c_l$, and 'L' are the Coulomb logarithms[3].

The evolution of transverse emittances and longitudinal momentum spread with time resulting from the above computations are shown in Fig.1, supposing 8.5MeV/u O^{8+} beam with initial emittance of $25\pi mm\cdot mrad$ and momentum spread of $\pm 1.5!$ respectively. The emittances and momentum spread are decreased by one order in a cooling time of less than 160ms and 80ms respectively. Additionally, the phase motion leads to the oscillatory decrease of momentum spread.

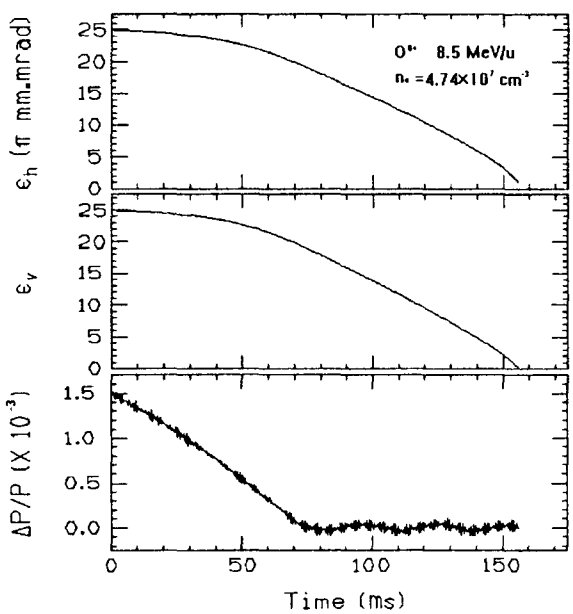

Figure: 1 Electron cooling with neutralized electron beam

3 Intrabeam Scattering Growth Rate

The growth rates of intrabeam scattering depend on the number of ions as well as the beam dimensions which are given by the focusing properties of the storage ring and the beam emittances. Following the intrabeam scattering(IBS) theory of Bjorken and Mtingwa[4], including the variations of β–functions and dispersion around the accelerator, more general formulae for the growth rates of horizontal, vertical emittances and longitudinal momentum spread are derived in a simpler form of elliptic integrals, which is expressed as below:

$$\frac{1}{\tau_j} = \frac{1}{\epsilon_j}\frac{d\epsilon_j}{dt} = \Gamma\cdot\langle\ (F_{1j}A_1+F_{2j}A_2)\,E(\xi,k)+ (F_{1j}B_1+F_{2j}B_2)\,F(\xi,k)+ (F_{1j}C_1+F_{2j}C_2)\ \rangle,\quad j=h,v,l$$

The brackets $\langle\cdots\rangle$ indicate this must be averaged around the ring. For a bunched beam,

$$\Gamma = \frac{N_i\cdot r_i^2\cdot c_l\cdot L_c}{8\pi\beta_s^3\gamma_s^4\cdot\epsilon_h\epsilon_v\sigma_\eta\sigma_s}$$

in which N_i is the number of particles/bunch, $r_i = (Q_i^2/A_i)\cdot r_p$ is the ion radius given by the ion charge state Q_i , the mass number A_i and the classical proton radius r_p, c_l is the speed of light, $L_c \simeq 20$ is the Coulomb logarithms, β_s,γ_s are the relativistic factors of ion, ϵ_h,ϵ_v are the emittances, σ_η is the rms relative momentum $\frac{\Delta P}{P}$ and σ_s is the rms bunch length. F_{1j} and F_{2j} are given by

$$\begin{aligned} F_{1j} &= 2(S_{1j}+S_{3j}+S_{4j})(T_1+T_3+T_4)- \\ &\quad 3(S_{1j}T_3+S_{1j}T_4-2S_{2j}T_2+S_{3j}T_1+ \\ &\quad S_{3j}T_4+S_{4j}T_1+S_{4j}T_3), \\ F_{2j} &= (S_{1j}+S_{3j}+S_{4j})(T_3T_4+T_1T_4+T_1T_3-T_2^2)- \\ &\quad 3(S_{1j}T_3T_4-2S_{2j}T_2T_4+S_{3j}T_1T_4+ \\ &\quad S_{4j}T_1T_3-S_{4j}T_2^2). \end{aligned}$$

and S_{1j}, S_{2j}, S_{3j}, S_{4j} are elements of the symmetric matrix $L^{(j)}$ which is expressed as follows for horizontal, vertical and longitudinal components respectively

$$L^{(j)} = \begin{pmatrix} S_{1j} & S_{2j} & 0 \\ S_{2j} & S_{3j} & 0 \\ 0 & 0 & S_{4j} \end{pmatrix},$$

$$L^{(h)} = \frac{\beta_h}{\epsilon_h}\begin{pmatrix} 1 & -\gamma\phi & 0 \\ -\gamma\phi & \frac{\gamma^2 D_h^2}{\beta_h^2}+\gamma^2\phi^2 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$L^{(v)} = \frac{\beta_v}{\epsilon_v}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

$$L^{(l)} = \frac{2\gamma^2}{\sigma_\eta^2}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

in which $\phi = D_h' - \beta_h' D_h/(2\beta_h)$, $\beta_h' = d(\beta_h)/ds$ and $D_h' = d(D_h)/ds$ are the variations of β_h and the dispersion D_h around the ring. T_1, T_2, T_3 and T_4 are the elements of the sum matrix $L^{(m)}$:

$$L^{(m)} = L^{(h)}+L^{(v)}+L^{(l)} = \begin{pmatrix} T_1 & T_2 & 0 \\ T_2 & T_3 & 0 \\ 0 & 0 & T_4 \end{pmatrix}$$

with the eigenvalues λ_1, λ_2 and λ_3 given by

$$\begin{aligned}\lambda_1 &= \frac{1}{2}\left[(T_1+T_3)+\sqrt{(T_1-T_3)^2+4T_2^2}\right],\\ \lambda_2 &= \frac{1}{2}\left[(T_1+T_3)-\sqrt{(T_1-T_3)^2+4T_2^2}\right],\\ \lambda_3 &= T_4.\end{aligned}$$

The coefficients A, B, C are given by

$$A_1 = \frac{2V}{\lambda_1^2\cdot U^{\frac{5}{2}}\cdot(1-V)^{\frac{7}{2}}}\cdot\frac{(1+V)k^4+(V-3)k^2+2}{k^4(1-k^2)^2},$$

$$B_1 = \frac{2V}{\lambda_1^2\cdot U^{\frac{5}{2}}\cdot(1-V)^{\frac{7}{2}}}\cdot\frac{(2-V)k^2-2}{k^4(1-k^2)},$$

$$C_1 = \frac{2V}{\lambda_1^2\cdot U^{\frac{5}{2}}\cdot(1-V)^{\frac{7}{2}}}\cdot\frac{(-V)(k^2+1)}{k^2(1-k^2)^2}\cdot\sqrt{\frac{1-V}{U}},$$

$$A_2 = \frac{2}{\lambda_1^3\cdot U^{\frac{5}{2}}\cdot(1-V)^{\frac{7}{2}}}\cdot \frac{(-2)[(V^2-V+1)k^4-(2-V)k^2+1]}{k^4(1-k^2)^2},$$

$$B_2 = \frac{2}{\lambda_1^3\cdot U^{\frac{5}{2}}\cdot(1-V)^{\frac{7}{2}}}\cdot \frac{(V^2-2V+1)k^4+(2V-3)k^2+2}{k^4(1-k^2)},$$

$$C_2 = \frac{2}{\lambda_1^3\cdot U^{\frac{5}{2}}\cdot(1-V)^{\frac{7}{2}}}\cdot\frac{V[(2V-1)k^2+1]}{k^2(1-k^2)^2}\cdot\sqrt{\frac{1-V}{U}},$$

$$U=\lambda_2/\lambda_1, \qquad V=\lambda_3/\lambda_1.$$

$E(\xi,k)$,$F(\xi,k)$ are elliptic integrals of the first and second kinds:

$$E(\xi,k)=\int_0^{\xi}\sqrt{1-k^2sin^2\theta}\ d\theta,$$

$$F(\xi,k)=\int_0^{\xi}\frac{d\theta}{\sqrt{1-k^2sin^2\theta}}$$

with

$$k^2=\frac{\lambda_1(\lambda_2-\lambda_3)}{\lambda_2(\lambda_1-\lambda_3)}, \qquad \xi=arcsin\sqrt{1-\frac{\lambda_3}{\lambda_1}}$$

We can apply these expressions to the HIRFL–CSR with the varying lattice parameters. Taking the horizontal and vertical emittances of $0.3\pi mm\cdot mrad$ and momentum spread of 7.0×10^{-5} for 8.5MeV/u O^{8+} beam of 1.0×10^9 particles after electron cooling, the calculated results of beam blow-up are shown in Fig.2. The emittances and momentum spread grow by a factor of 5 in a time of $\sim 30s$ and $\sim 1s$ respectively. In contrast to the electron cooling rate with the electron density of $4.74\times10^7/cm^3$, the growth rate of intrabeam scattering is much lower. It also can be seen from the figure that the beam doesn't tend to an equilibrium, but show an overall heating, which is caused by the coupling of the horizontal and longitudinal degree of freedom because of the dispersion in the storage ring.

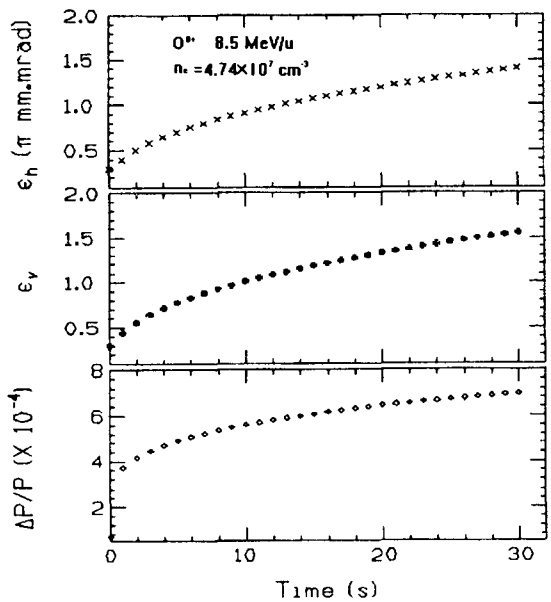

Figure: 2 Beam blow-up due to intrabeam scattering

4 Equilibrium Calculation

Now that the electron cooling rate and the intrabeam scattering growth rate are obtained, the evolutions of emittances and momentum spread can be calculated by integration of the differential equations in the presence of electron cooling and intrabeam scattering as below:

$$\frac{d\epsilon}{dt}=\frac{d\epsilon_{ec}}{dt}+\frac{d\epsilon_{ibs}}{dt},$$

$$\frac{d(\frac{\Delta P}{P_s})}{dt}=\frac{d(\frac{\Delta P}{P_s})_{ec}}{dt}+\frac{d(\frac{\Delta P}{P_s})_{ibs}}{dt}$$

The equilibrium properties are resulted from the conditions:

$$\frac{d\epsilon}{dt}=0, \qquad \frac{d(\frac{\Delta P}{P_s})}{dt}=0$$

Taking the HIRFL–CSR lattice parameters, the equilibrium calculation was performed for 8.5MeV/u O^{8+} beam of 1.0×10^9 particles, and the resulted equilibrium emittances and momentum spread are $\epsilon_h\sim0.5\pi mmmrad$,

$\epsilon_v \sim 0.9\pi mmmrad$ and $\frac{\Delta}{P_s} \sim 7.0 \times 10^{-5}$ respectively. The equilibrium emittances and momentum spread as a function of number of stored particles are shown in Fig.3. An increase of the phase space volume with the number of stored ions is evident, and the calculations show a power scaling of $\epsilon \propto N_i^{0.42}$ for the emittances and $\Delta P/P_s \propto N_i^{0.2}$ for the momentum spread.

In Conclusion, the intrabeam scattering is an dominant effect which present a limit to the available ion beam resolutions for experiments.

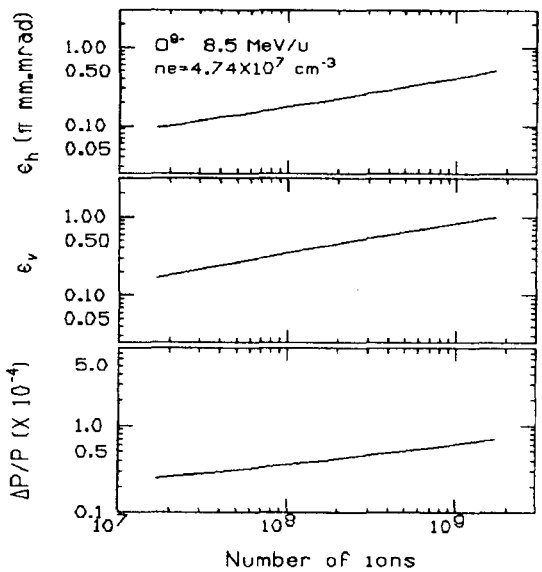

Figure: 3 The dependence of equilibrium emittances and momentum spread on number of particles/bunch

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