A NEW METHOD OF FINDING THE POLE PROFILE IN QUADRUPOLE MAGNETS FOR OBTAINING HIGH FIELD QUALITY

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The quality of field in quadrupole magnets depends mainly on the pole profile. For reducing the field errors, one often uses a circular pole of optimized radius or hyperbolic profiles modified at the pole edges. In any magnet design one first chooses a profile and then calculates the error harmonics. Here we describe a new method in which we choose the error harmonics first and then find the profile thus ensuring a good field quality. The method can be applied to other multipole magnets also.

1 Introduction

Quadrupole magnets are essential elements in any beam line used for transporting beams of charged particles. Quadrupoles of various sizes and strengths are used for this purpose. Electrostatic quadrupoles are also sometimes used for low energy beams. In all these applications especially in high resolution beam lines and in synchrotrons one requires a highly pure quadrupole field so that the beam quality remains good after the beam passes through the magnet either once or many times. The field quality in an iron magnet depends mainly on the profile of the pole. The quality of the magnetic field or the magnetic potential can be judged from the harmonic expansion of the potential due to the particular pole configuration of the magnet. The potential $U(r,\theta)$ at any point (r,θ) in a multipole element can be written as

$$U(r,\theta) = C_0 + C_1 r cos(\theta) + C_2 r^2 cos(2\theta) + ... \quad (1)$$

where C's are the coefficients of various harmonics. A pure quadrupole potential contains only the r^2-term and so the pole profile of a perfect quadrupole is given by

$$r^2 cos(2\theta) = constant \quad (2)$$

This is a hyperbola. However, the field in a practical quadrupole contains the higher harmonic terms because one has to place the energizing coils around the poles and so the pole width has to be truncated. In electrostatic quadrupoles also one cannot use electrodes of very large dimension. The higher order terms are the error harmmonics and these limit the so called 'good field region' in a magnet. The 12-pole term i.e the C_6 term is the dominant error harmonic and one tries to reduce this.

There are various ways of reducing the C_6-term. Often a circular pole profile is used and its radius is optimized to make C_6 vanish. Grivet and Septier tried various radii experimentally. The optimized radius comes out to be 1.145 times to 1.15 times the half-aperture of the magnet [1,2]. Circular profile has another advantage, its fabrication is easy whereas fabrication of a hyperbolic profile requires a sophisticated numerically controlled machine. Of course, in laminated magnets this is not a problem once a dice is made with a sophisticated machine.

For obtaining a better field quality than that of a circular pole people have used various other pole profiles [3]. Hinterberger and others [4] used a pure hyperbolic profile terminated by shims at the coil windows. Some others use hyperbolic poles truncated with straight lines at the outer edges [5,6]. More Complicated profiles have also been used [7,8]. Danby and others designed and made poles consisting entirely of plane surfaces [9,10].

In this work we have described a new and simple method of determining the pole profile by which one can reduce not only the C_6-term to zero but also the

C_{10} and some other terms to zero. One can obtain a very good field quality and at the same time the pole width also can be controlled with the help of this method.

2 The method

In the design of the pole profile of a magnet what is generally done is to choose a profile first and then find the harmonic errors due to the chosen profile. The geometric parameters involved in the given profile are then varied to reduce the field errors. The calculations are generally done with the help of standard codes like POISSON, SUPERFISH etc. which solve the Poisson's equation by the finite difference method or the finite element method. Another method is a semi-analytical method [11,12] in which the harmonic expansion equation of the potential is solved by the matrix inversion technique. In these methods the accuracy depends on the number of points cnsidered in the region where the field or potential is calculated and one gets large errors wherever sharp boundaries are involved. We do not have to face this problem in the method described below.

In our method we choose the error harmonics first and then find the profile which generates the chosen errors. Thus the quality of the magnet is chosen a priori in our method. The magnetic potential for a four-fold symmetric system such as a quadrupole involves only the odd harmonics of the basic quadrupole potential and is given by

$$U(r,\theta) = C_2 r^2 cos(2\theta) + C_6 r^6 cos(6\theta)$$
$$+ C_{10} r^{10} cos(10\theta) + C_{14} r^{14} cos(14\theta) + ... \qquad (3)$$

By optimizing the radius of a circular pole one obtains $C_6 = 0$. Since we want a better quality than this we take $C_6 = C_{10} = 0$. We assume that the pole is an equipotential surface and so the pole profile is determined by

$$C_2 r^2 cos(2\theta) + C_{14} r^{14} cos(14\theta)$$
$$+ C_{18} r^{18} cos(18\theta) + ... = constant \qquad (4)$$

The pole profile can now be found out numerically by putting values for r in the above equation and solving for θ. The deviation of the profile from the ideal one will obviously depend mainly on the value of C_{14}. In the numerical procedure one can take a small value for C_{14} and then fiddle with the higher harmonics to get a profile which gives a finite and constant width for the pole. In practice the root-finding procedure is very easy, only one has to be careful to choose the right roots when multiple roots are involved. Fig. 1 shows how by varying the value of C_{18} one gets a geometrically good profile which has a finite and constant width.

In equation (4) we have taken the coefficients upto C_{26} only as the higher harmonics have very little contribution to the profile for values of r smaller than about 3 (in terms of the half-aperture).

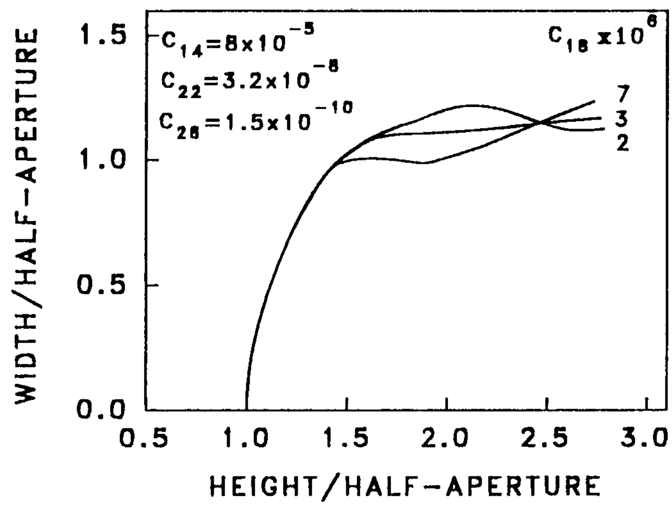

Fig.1. *Change of pole profile as C_{18} changes*

3 Results and discussions

We have calculated the quadrupole pole profile for various fixed values of C_{14}. The results are shown in Figs. 2 and 3. Fig. 2 shows the profile for $C_{14} = 0.00004$. The width of the new pole is 1.120 times the half-aperture and is nearly equal to that of the circular pole. The pole width can be adjusted by changing the value of C_{14}. As can be seen from Fig. 3 the width increases when we choose smaller values of C_{14}. For larger values of C_{14} the width decreases and becomes lower than the width of the circular pole. A smaller pole width obviously allows a larger space for the coil.

Fig. 4 compares the field quality of the new pole with that of the circular pole as a function of the radial distance for various poles. It is clear that the new poles have very good field quality. Fabrication of the pole profile obtained by us is more difficult than that of the circular pole but so is the fabrication of the hyperbolic profiles mentioned earlier.

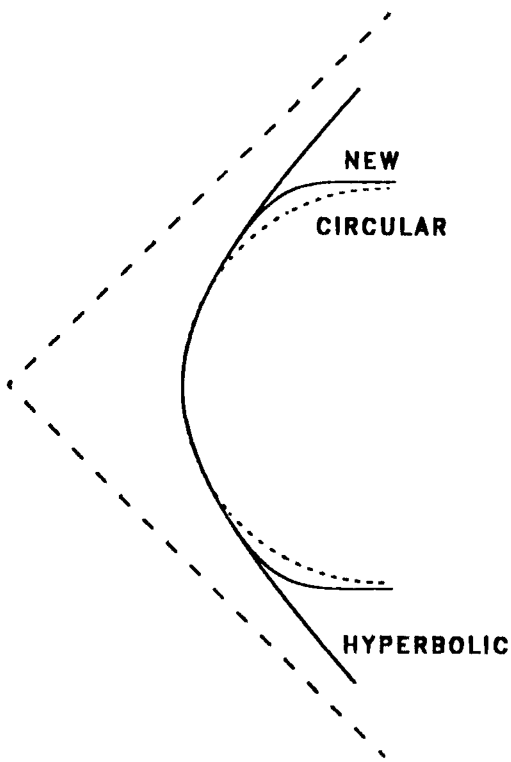

Fig.2. *Pole profile of a quadrupole obtained with the new method for C_{14}=0.00004. An optimized circular profile and an ideal hyperbolic profile are also shown.*

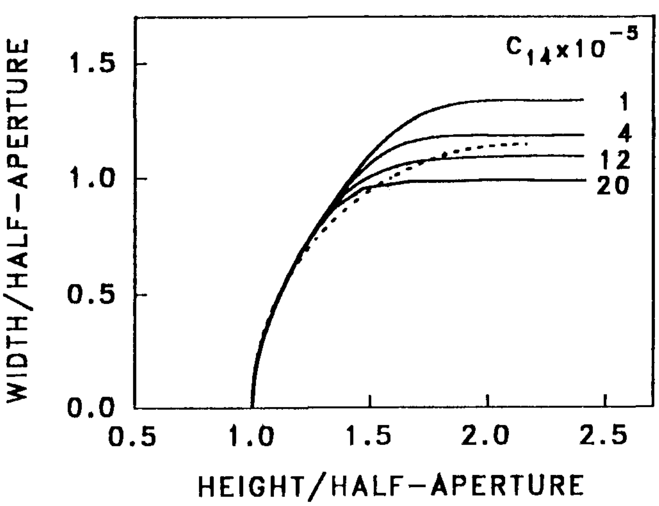

Fig.3. *Variation of the width of the new pole profile for various values of C_{14}.*

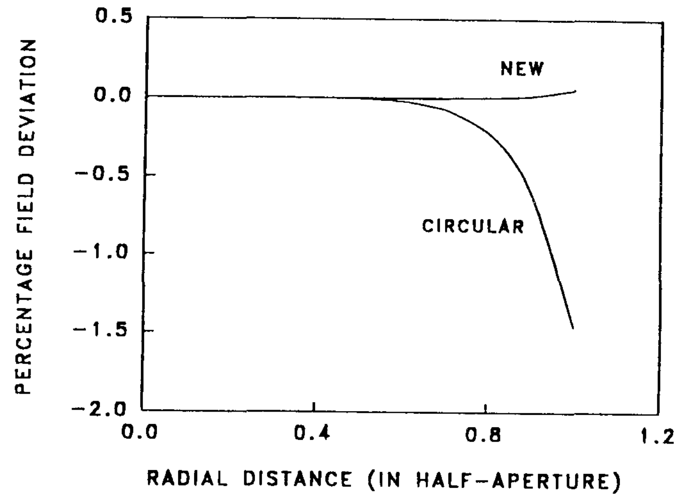

Fig. 4. *Comparison of field error of the new profile with that of the circular profile*

One interesting feature is observed when we calculate the pole shape for various values of C_{14}. The field deviation mainly depends on the magnitude of C_{14} and not on its sign. Fig.5 shows the pole shapes for two values of C_{14} which have the same magnitude but opposite in sign. We see that for negative values of C_{14} the pole width is much higher than that for the positive value. The pole shapes adopted in references [7-8] are examples of such designs. It is obvious that the pole widths could have been reduced in those cases by a large amount.

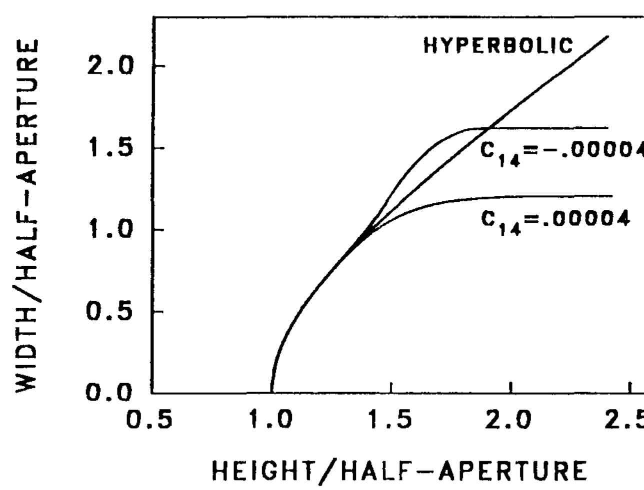

Fig.5. *Comparison of two pole shapes giving the same field deviation in amplitude but opposite in sign. The pole shape for positive C_{14} has a smaller pole width.*

The method discussed above is very simple and can be applied to other multipoles like sextupoles, octupoles etc. This method is simpler than choosing an arbitrary profile and then try to optimize the parametrs involved for reducing the field errors. The accuracy of the calculaton is not limited by the number of points considered. For achieving even better field quality one can start with $C_6 = C_{10} = C_{14} = 0$ but then other factors like the location of the coils etc. become important and one has to consider their contributions also to the field. The end effects also become very important. We note here that the procedure is valid only when the pole faces are equipotentials. This is strictly valid in electrostatic quadrupoles and also in low field magnets.

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