# THE TRANSFER MATRICES FOR INTENSE BEAM IN SOLENOID LENS

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The transfer matrices of the solenoid lens for intense beam, in which the space charge effect and beam-divergence angle are considered, are given by solving the non-linear equation of beam envelope and using the S-Matrix. The transfer characteristics of the intense beam in the solenoid lens are also discussed.

## Ⅰ. Introduction.

The solenoid lens, as a beam focusing or transmission device, is used in the intense accelerator and the pre-injector stage of the intense accelerator. Generally, the transfer matrices calculation method is adopted for calculation of the beam optical characteristics. By using the point-by-point approximate method and the intense approach method, the transfer matrices of the intense beam accelerating tube have been given [1-3]. In order to calculate the optical characteristics of the intense beam in the solenoid lens, we need to know the transfer matrices of the intense beam in the solenoid lens. Because both the equations of motion and the beam envelop equations of charged particle are self-coupling, so the transfer matrices are not easy to be obtained directly.

In this paper, we have given the transfer matrices for the intense beam in solenoid lens by solving the non-linear equation of beam envelope and using the S-Matrix method, in the case of considering the beam space-charge effect and the beam divergence angle (or emittance). And the transfer characteristics of the intense beam in the solenoid lens are also discussed in detail. The theoretical work and the results of the calculations are accurate enough for many application purposes. The examples are given to show how the results and method would allow one to design an intense beam solenoid lens.

## Ⅱ. S-Matrix.

Let us consider axis $z$ as the beam transfer direction. In case of K-V distribution for the two-dimensional phase-plane $(x,x')$, the coordinate $x$ and $x'$ satisfy the following equation:[1][4]

$$\sigma_{22}x^2 - 2\sigma_{12}xx' + \sigma_{11}x'^2 = \varepsilon^2 \tag{1}$$

where $\varepsilon$ is the emittance in direction $x$. The beam elliptic coefficients, (which vary with the longitudinal position $z$) $\sigma_{11}>0, \sigma_{22}>0$, and $\sigma_{11}\sigma_{22} - \sigma_{12}{}^2 = \varepsilon^2$

In order to deal with equation (1) easily we take the coordinate transformation:

$$\begin{cases} \xi = \cos\varphi x + \sin\varphi x' \\ \xi' = -\sin\varphi x + \cos\varphi x' \end{cases} \tag{2}$$

Eq.(1) becomes

$$\frac{\xi^2}{a^2} + \frac{\xi'^2}{b^2} = 1 \tag{3}$$

Its parametric equation is

$$\begin{cases} \xi = a\cos\phi \\ \xi' = b\sin\phi \end{cases} \tag{4}$$

Substituting this into (2), we find

$$\begin{cases} a\cos\phi = \cos\varphi x + \sin\varphi x' \\ b\sin\phi = -\sin\varphi x + \cos\varphi x' \end{cases} \tag{5}$$

Noting that, for the initial conditions $z=0$, $x=x_0$, $x'=x'_0$, and $a=a_0$, $b=b_0$, $\varphi=\varphi_0$, we find

$$\begin{cases} \cos\phi = \dfrac{1}{a_0}(\cos\varphi_0 x_0 + \sin\varphi_0 x'_0) \\ \sin\phi = \dfrac{1}{b_0}(-\sin\varphi_0 x_0 + \cos\varphi_0 x'_0) \end{cases} \tag{6}$$

Combining this with Eq.(5), we obtain the transfer matrix

$$S = \begin{pmatrix} \dfrac{a}{a_0}\cos\varphi_0\cos\varphi + \dfrac{b}{b_0}\sin\varphi_0\sin\varphi & \dfrac{a}{a_0}\sin\varphi_0\cos\varphi - \dfrac{b}{b_0}\cos\varphi_0\sin\varphi \\ \dfrac{a}{a_0}\cos\varphi_0\sin\varphi - \dfrac{b}{b_0}\sin\varphi_0\cos\varphi & \dfrac{a}{a_0}\sin\varphi_0\sin\varphi + \dfrac{b}{b_0}\cos\varphi_0\cos\varphi \end{pmatrix} \tag{7}$$

Eq.(7) shows that if we can get $a$, $b$ and $\varphi$, we will get the transfer matrix **S**.

On the two-dimensional phase-plane $(x,x')$, the beam radius $X$ and maximum slope $X'$ satisfy[1]

$$\begin{cases} \sigma_{11} = X^2, \quad \sigma_{12} = XX' \\ \sigma_{22} = \dfrac{\varepsilon^2 + \sigma_{12}^2}{\sigma_{11}} = \dfrac{\varepsilon^2 + (XX')^2}{X^2} \end{cases} \tag{8}$$

and

$$tg2\varphi = \frac{\sigma_{12}}{\sigma_{22} - \sigma_{11}} = tg\left(\frac{XX'}{A - 2X^2}\right)$$

$$a = \sqrt{2}\varepsilon\left[\sigma_{22} + \sigma_{11} + \left[(\sigma_{22} - \sigma_{11})^2 + 4\sigma_{12}^2\right]^{\frac{1}{2}}\right]^{-\frac{1}{2}} = \sqrt{2}\varepsilon\left[A + \left(A - 4\varepsilon^2\right)^{\frac{1}{2}}\right]^{-\frac{1}{2}}$$

$$b = \sqrt{2}\varepsilon\left[\sigma_{22} + \sigma_{11} - \left[(\sigma_{22} - \sigma_{11})^2 + 4\sigma_{12}^2\right]^{\frac{1}{2}}\right]^{-\frac{1}{2}} = \sqrt{2}\varepsilon\left[A - \left(A - 4\varepsilon^2\right)^{\frac{1}{2}}\right]^{-\frac{1}{2}} \tag{9}$$

In which we define the characteristic radius $A$:

$$A = \sigma_{22} + \sigma_{11} = \frac{\varepsilon^2 + (XX')^2}{X^2} + X^2 \tag{10}$$

The (9) and (10) show that, if $X^2$ and $XX'$ are got, the transfer matrix **S** will be got. Now we will show how to get $X^2$ and $XX'$ in the following paragraphs.

## III. The Equations of Motion and Envelope.

For the solenoid lens, we suppose that the uniform magnetic field with effective length $L$, which have a sudden change from 0 to $B_0$ at entrance port, on the contrary, from $B_0$ to 0 at exit port, replace the actual magnetic field. The effective length $L$ of solenoid lens is defined by[1][5]

$$\int_{-\infty}^{+\infty} B(z)dz = B_0 L \tag{11}$$

We define the magnetic action strength $Q_0$:

$$Q_0 = \left(\frac{q}{8mV}\right)^{\frac{1}{2}} B_0 \tag{12}$$

where $m$ and $q$ are the mass and electric quantity of particle, respectively. $V$ is the gauge electric potential, which is given by $V = \frac{mv^2}{2q}$, here $v$ is the velocity of particle. Eq.(12) shows that for the constant $m$, $V$ and $q$, $Q_0$ is also a constant. For the K-V distribution of axisymmetric intense beam , the equations of motion for single particle in solenoid lens are[1][6]

$$\begin{cases} x'' - 2Q_0 y' - \dfrac{\Pi}{R^2} x = 0 \\ y'' + 2Q_0 x' - \dfrac{\Pi}{R^2} y = 0 \end{cases} \tag{13}$$

In which $R$ is the maximum beam radius of axis $x$ and $y$. The generalized perveance $\Pi$ is given by

$$\Pi = \frac{\eta I}{2\pi\varepsilon_0 v^3}$$

where $\eta$ is the charge-mass ratio of the particle, $I$ is beam intensity, $\varepsilon_0$ is vacuum static dielectric constant.

Eq.(13) shows that the particle equations of motion in axis $x$ and $y$ are self-coupling. In order to eliminate the self-coupling, we adopt the rotational coordinates $(x_r, x_r')$, which are given by

$$\begin{cases} x = x_r \cos\psi - y_r \sin\psi \\ y = x_r \sin\psi + y_r \cos\psi \end{cases} \tag{14}$$

substituting (14) into (13) ,and suppose that $\psi' = -Q_0$ and $\psi(0) = 0$ ,we find

$$\begin{cases} x_r'' + (Q_0^2 - \dfrac{\Pi}{R^2}) x_r = 0 \\ y_r'' + (Q_0^2 - \dfrac{\Pi}{R^2}) y_r = 0 \end{cases} \tag{15}$$

This shows that the $x_r$ and $y_r$ have the same equation of motion. The transformation matrix equation is

$$\begin{pmatrix} x \\ x' \\ y \\ y' \end{pmatrix} = \begin{pmatrix} \cos Q_0 z & 0 & \sin Q_0 z & 0 \\ -Q_0 \sin Q_0 z & \cos Q_0 z & Q_0 \cos Q_0 z & \sin Q_0 z \\ -\sin Q_0 z & 0 & \cos Q_0 z & 0 \\ -Q_0 \cos Q_0 z & -\sin Q_0 z & -Q_0 \sin Q_0 z & \cos Q_0 z \end{pmatrix} \begin{pmatrix} x_r \\ x_r' \\ y_r \\ y_r' \end{pmatrix} \tag{16}$$

The envelope equation of Eq.(15) is

$$X'' + Q_0^2 X - \frac{\Pi}{X} - \frac{\varepsilon^2}{X^3} = 0 \tag{17}$$

here the radius of beam envelope $X=R>0$. Multiplying both sides by $1/X$ and substituting $f(z) = X^2$ into (17), then integrating from 0 to $z$, we obtain

$$f'^2 = -4Q_0^2 f^2 + 4\Pi f \ln\frac{f}{f_0} + 4C_1 f - 4\varepsilon^2 \tag{18}$$

where

$$\begin{cases} f(z) = X^2 = \sigma_{11}(z), \quad f_0 = f(0) = X_0^2 = \sigma_{11}(0) \\ C_1 = \dfrac{f_0'^2}{4f_0} + Q_0^2 f_0 + \dfrac{\varepsilon^2}{f_0} = X_0'^2 + Q_0^2 X_0^2 + \dfrac{\varepsilon^2}{X_0^2} \\ f'(z) = 2XX' = 2\sigma_{12}(z), \quad f_0' = 2X_0 X_0' = 2\sigma_{12}(0) \end{cases} \tag{19}$$

In general, $X$ is not much bigger than $X_0$, in first-order approximation for $\ln f/f_0$ the (18) becomes

$$\int_{X_0^2}^{X^2} \left[\left(\frac{\Pi}{f_0} - Q_0^2\right) f^2 + (C_1 - \Pi) f - \varepsilon^2\right]^{-\frac{1}{2}} df = 2\int_0^z dz' \tag{20}$$

## IV. The Transfer Matrices For Intense Beam.

Eq.(20) shows that the different relations between $\frac{\Pi}{f_0}$ and $Q_0^2$ ( i.e. $\Pi$ and $Q_0^2 X_0^2$) have different result of integration .We now discuss these problems respectively.

**A**. for $\Pi = Q_0^2 X_0^2$.

Combining (20) with (19),we get

$$\begin{aligned} X^2 &= \left(X_0'^2 + \frac{\varepsilon^2}{X_0^2}\right) z^2 + 2X_0 X_0' z + X_0^2 \\ XX' &= \left(X_0'^2 + \frac{\varepsilon^2}{X_0^2}\right) z + X_0 X_0' \end{aligned} \tag{21}$$

**B**. for $\Pi < Q_0^2 X_0^2$.

Similarly, we have

$$\begin{cases} X^2 = \dfrac{1}{2} X_0^2 \left(Q_0^2 X_0^2 - \Pi\right)^{-1} \left[\sqrt{\Lambda} \sin\left(\dfrac{2z}{X_0}\sqrt{Q_0^2 X_0^2 - \Pi} + \alpha_0\right) + (C_1 - \Pi)\right] \\ XX' = \dfrac{X_0}{2}\left(\dfrac{\Lambda}{Q_0^2 X_0^2 - \Pi}\right)^{\frac{1}{2}} \cos\left(\dfrac{2z}{X_0}\sqrt{Q_0^2 X_0^2 - \Pi} + \alpha_0\right) \end{cases} \tag{22}$$

here

$$\begin{cases} \Lambda = \left(Q_0^2 X_0^2 + X_0'^2 - \Pi - \dfrac{\varepsilon^2}{X_0^2}\right)^2 + \dfrac{4X_0'^2 \varepsilon^2}{X_0^2} \\ \alpha_0 = \arcsin\left(\dfrac{Q_0^2 X_0^2 - X_0'^2 - \Pi - \dfrac{\varepsilon^2}{X_0^2}}{\sqrt{\Lambda}}\right) \end{cases} \tag{23}$$

**C**. for $\Pi > Q_0^2 X_0^2$.

In the same way ,we obtain

$$\begin{aligned} X^2 &= \frac{X_0^2}{4}\left(\Pi - Q_0^2 X_0^2\right)^{-1} \exp l_1 - \frac{X_0^2}{2}(C_1 - \Pi)\left(\Pi - Q_0^2 X_0^2\right)^{-1} + \frac{X_0^2}{4}\left(\Pi - Q_0^2 X_0^2\right)^{-1} \\ &\quad \times \left[(C_1 - \Pi)^2 + 4\varepsilon^2\left(\frac{\Pi}{X_0^2} - Q_0^2\right)\right] \exp(-l_1) \\ XX' &= \frac{X_0}{4}\left(\Pi - Q_0^2 X_0^2\right)^{-\frac{1}{2}} \exp l_1 - \frac{X_0}{4}\left(\Pi - Q_0^2 X_0^2\right)^{-\frac{1}{2}} \\ &\quad \times \left[(C_1 - \Pi)^2 + 4\varepsilon^2\left(\frac{\Pi}{X_0^2} - Q_0^2\right)\right] \exp(-l_1) \end{aligned} \tag{24}$$

here,

$$l_1 = \ln\left[C_1 + \Pi - 2Q_0^2 X_0^2 + 2X_0'\sqrt{\Pi - Q_0^2 X_0^2}\right] + \frac{2z}{X_0}\sqrt{\Pi - Q_0^2 X_0^2} \tag{25}$$

**D**. Transfer Matrix.

Substituting $X^2$ and $XX'$ into (9) and (10), we will get three different sets of $a, b$ and $\varphi$. Then substituting $a, b$ and $\varphi$ into (7), the transfer matrix S for$(x_r, x_r')$ is obtained. Since $(x_r, x_r')$ and $(y_r, y_r')$ have similar transfer S, combining these results with (16), we will find the transfer matrix of the $(x_r, x_r', y_r, y_r')$.

According to (14), the coordinate transformation matrix

equation is
$$\begin{pmatrix} x_{r0} \\ x'_{r0} \\ y_{r0} \\ y'_{r0} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x_0 \\ x'_0 \\ y_0 \\ y'_0 \end{pmatrix} \tag{26}$$

We define matrices $M$, $N$, $K$ and $E$ as follows:

$$M = \begin{pmatrix} \cos Q_0 z & 0 \\ -Q_0 \sin Q_0 z & \cos Q_0 z \end{pmatrix} \quad N = \begin{pmatrix} \sin Q_0 z & 0 \\ Q_0 \cos Q_0 z & \sin Q_0 z \end{pmatrix}$$
$$K = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \quad E = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{27}$$

Combining (7),(16),(26) with (27), the transfer matrix $R_2$ in solenoid lens is obtained

$$\begin{pmatrix} x \\ x' \\ y \\ y' \end{pmatrix} = \begin{pmatrix} M & N \\ -N & M \end{pmatrix} \begin{pmatrix} S & 0 \\ 0 & S \end{pmatrix} \cdot \begin{pmatrix} E & -K \\ K & E \end{pmatrix} \begin{pmatrix} x_0 \\ x'_0 \\ y_0 \\ y'_0 \end{pmatrix} = R_2 (x_0 \quad x'_0 \quad y_0 \quad y'_0)^T \tag{28}$$

## V. The Transfer Matrices At Entrance Port and At Exit Port.

Since the effective magnetic field have a sudden change at entrance port and at exit port of solenoid lens, therefore, we consider that the transverse positions $x_0, y_0$ are not changed, only slope $x'_0, y'_0$ are changed at both ports.

The equations of motion for by-axial particle in axisymmetric magnetic field are[1]

$$\begin{cases} x'' - 2Qy' - Q'y - \dfrac{\Pi}{X^2} x = 0 \\ y'' + 2Qx' + Q'x - \dfrac{\Pi}{X^2} y = 0 \end{cases} \tag{29}$$

In linear approximation, we obtain the four-dimensional transfer matrix at the entrance port

$$R_1 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & Q_0 & 0 \\ 0 & 0 & 1 & 0 \\ -Q_0 & 0 & 0 & 1 \end{pmatrix} \tag{30}$$

Similarly, the four-dimensional transfer matrix at the exit port is

$$R_3 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -Q_0 & 0 \\ 0 & 0 & 1 & 0 \\ Q_0 & 0 & 0 & 1 \end{pmatrix} \tag{31}$$

From the results of (28), (30) and(31),we obtain the four-dimensional transfer matrix $R$ for solenoid lens

$$R = R_3 R_2 R_1 \tag{32}$$

## VI. Application And Discussion On Waist And Peak of Beam.

For the solenoid lens, our interesting lies in its focusing characteristics. The ideal beam envelope is shown in Fig.1. Namely, in the case of $X'_0 > 0$, the envelope has a peak in solenoid lens, then $X'$ becomes negative after the peak, and forms a waist as beam past the exit port.

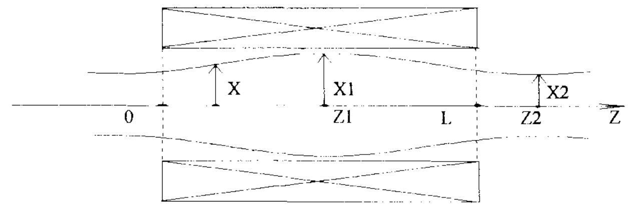


**Fig.1**. The ideal beam envelope.

Studying $X^2$ and $XX'$ in section IV ,we find that only for $\Pi < Q_0^2 X_0^2$, the ideal envelope figure maybe appear, thus we will only discuss this case in following.

We suppose that at position $z = 0$, the beam radius and maximum slope are $X_0$ and $X'_0$, respectively. The position and radius of beam peak are $z_1$ and $X_1$, and the position and radius of beam waist are $z_2$ and $X_2$,as shown in fig.1.

**A**. At the entrance port.

According to (30),the radius $X_a$ and maximum slope $X'_a$ of the intense beam are given by

$$\begin{cases} X_a^2 = X_0^2 \\ X_a X'_a = X_0 X'_0 + Q_0 X_0^2 = X_0 (X'_0 + Q_0 X_0) \end{cases} \tag{33}$$

**B**. In the solenoid lens.

According to (22), we find

$$\begin{cases} X^2 = \dfrac{1}{2} X_0^2 (Q_0^2 X_0^2 - \Pi)^{-1} \left[ \sqrt{\Lambda} \sin\left( \dfrac{2z}{X_0} \sqrt{Q_0^2 X_0^2 - \Pi} + \alpha_0 \right) + C_1 - \Pi \right] \\ XX' = \dfrac{X_0}{2} \left( \dfrac{\Lambda}{Q_0^2 X_0^2 - \Pi} \right)^{\frac{1}{2}} \cos\left( \dfrac{2z}{X_0} \sqrt{Q_0^2 X_0^2 - \Pi} + \alpha_0 \right) \end{cases} \tag{34}$$

here

$$\begin{cases} \Lambda = \left[ (X'_0 + Q_0 X_0)^2 + Q_0^2 X_0^2 - \dfrac{\varepsilon^2}{X_0^2} - \Pi \right]^2 + \dfrac{4 (X'_0 + Q_0 X_0)^2 \varepsilon^2}{X_0^2} \\ \alpha_0 = \arcsin\left( \dfrac{Q_0^2 X_0^2 - (X'_0 + Q_0 X_0)^2 - \dfrac{\varepsilon^2}{X_0^2} - \Pi}{\sqrt{\Lambda}} \right) \end{cases} \tag{35}$$

Eq.(34) shows the rule of beam radius with longitudinal coordinate $z$ ,which is drawn in Fig.2. (In this paper, we take: $X_0 = 0.005\ m$, $X'_0 = 0.04 rad$, the energy of proton is 30 *kev*, and beam intensity is 98 *mA*).

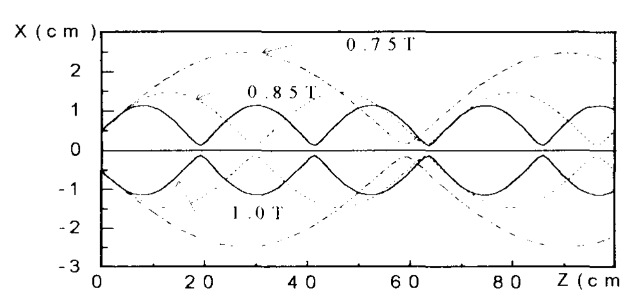


**Fig.2**. Curves of beam radius $X$ of proton beam with longitudinal coordinate $z$ for various magnetic field $B_0$.

At the position $z_1$ of beam peak, $X'_{z=z_1} = 0$, namely, $XX'_{z=z_1} = 0$. From (34) we obtain the position of beam peak

$$z_1 = \frac{X_0}{2}\left(Q_0^2X_0^2 - \Pi\right)^{-\frac{1}{2}} \arccos\left(\frac{Q_0^2X_0^2 - (X_0' + Q_0X_0)^2 - \frac{\varepsilon^2}{X_0^2} - \Pi}{\sqrt{\Delta}}\right) \quad (36)$$

and the radius $X_2$ of beam peak is

$$X_1^2 = \frac{X_0^2}{2}\left(Q_0^2X_0^2 - \Pi\right)^{-1}\left[\sqrt{\Delta} + C_1 - \Pi\right] \quad (37)$$

Eq.(36) and (37) show how the radius $X_1$ and position $z_1$ of beam peak change with $\Pi$and $Q_0$, which are shown in Fig.3.

The radius $X_b$ and maximum slope $X_b'$ of the beam at the end of solenoid lens are given by

$$\begin{cases} X_b^2 = \frac{1}{2}X_0^2\left(Q_0^2X_0^2 - \Pi\right)^{-1}\left[\sqrt{\Delta}\sin\left(\frac{2L}{X_0}\sqrt{Q_0^2X_0^2 - \Pi} + \alpha_0\right) + C_1 - \Pi\right] \\ X_bX_b' = \frac{X_0}{2}\left(\frac{\Delta}{Q_0^2X_0^2 - \Pi}\right)^{\frac{1}{2}}\cos\left(\frac{2L}{X_0}\sqrt{Q_0^2X_0^2 - \Pi} + \alpha_0\right) \end{cases} \quad (38)$$

here $L$ is the effective length of the solenoid lens.

**C**. At the exit port.

According to (31), the radius $X_c$ and maximum slope $X_c'$ are

$$\begin{cases} X_c^2 = X_b^2 \\ X_cX_c' = X_bX_b' - Q_0X_b^2 = X_b(X_b' - Q_0X_b) \end{cases} \quad (39)$$

**D**. In the free-drift space (after the lens).

The beam envelope equation is

$$X'' - \frac{\Pi}{X} - \frac{\varepsilon^2}{X^3} = 0 \quad (40)$$

The solution is

$$\begin{cases} X^2 = \frac{X_b^2}{4\Pi}\exp l_2 - \frac{X_b^2}{2\Pi}(C_1 - \Pi) + \frac{X_b^2}{4\Pi}\left[(C_1 - \Pi)^2 + \frac{4\Pi\varepsilon^2}{X_b^2}\right]\exp(-l_2) \\ XX' = \frac{1}{4}\sqrt{\frac{X_b^2}{\Pi}}\exp l_2 - \frac{1}{4}\sqrt{\frac{X_b^2}{\Pi}}\left[(C_1 - \Pi)^2 + \frac{4\Pi\varepsilon^2}{X_b^2}\right]\exp(-l_2) \end{cases} \quad (41)$$

here
$$\begin{cases} C_1 = (X_b' - Q_0X_b)^2 + \frac{\varepsilon^2}{X_b^2} \\ l_2 = 2\sqrt{\frac{\Pi}{X_b^2}}(z - L) + \ln\left(C_1 + \Pi + 2\sqrt{\Pi}(X_b' - Q_0X_b)\right) \end{cases} \quad (42)$$

At position $z_2$ of beam waist, $\left.\frac{dX^2}{dz}\right|_{z=z_2} = 2XX'|_{z=z_2} = 0$, we find

$$z_2 = \frac{1}{4}\sqrt{\frac{X_b^2}{\Pi}}\ln\left[\frac{(C_1 - \Pi)^2 + \frac{4\Pi\varepsilon^2}{X_b^2}}{\left[C_1 + \Pi + 2\sqrt{\Pi}(X_b' - Q_0X_b)\right]^2}\right] + L \quad (43)$$

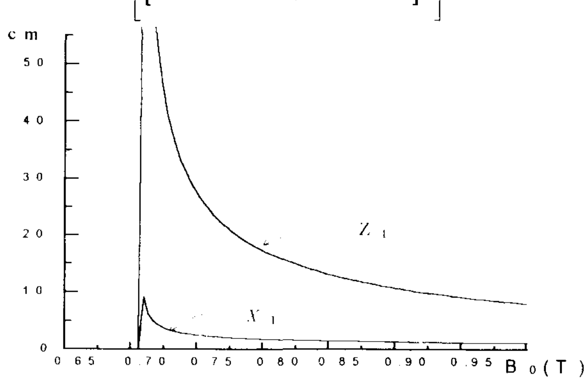


**Fig.3**. Curves of the position $z_1$ and radius $X_1$of beam peak of proton beam as functions of $B_0$.

This shows how the position $z_2$ of beam waist changes with $Q_0$ , $L$ and $\Pi$. Fig.4.shows the relation of beam waist position $z_2$ with $B_0$.

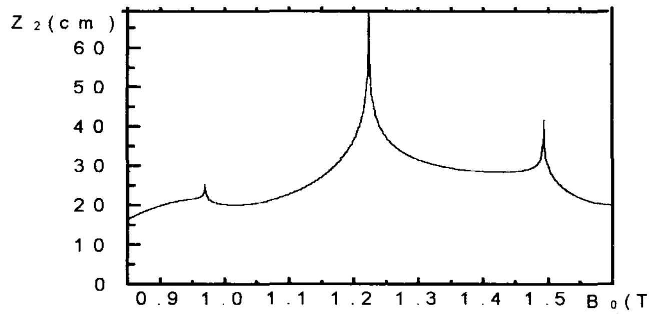


**Fig.4**. Curves of the position $z_2$ of beam waist of proton beam with the magnetic field $B_0$.(here, take $L$=20 *cm*)

The beam radius$X_2$ of waist is given by

$$X_2^2 = \frac{X_b^2}{2\Pi}\left\{\left[(C_1 - \Pi)^2 + \frac{4\Pi\varepsilon^2}{X_b^2}\right]^{\frac{1}{2}} - (C_1 - \Pi)\right\} \quad (44)$$

This shows the change relation of beam-waist radius $X_2$ with $B_0$, $L$ and $\Pi$. Fig.5.shows the relation of beam waist radius $X_2$ with $B_0$.

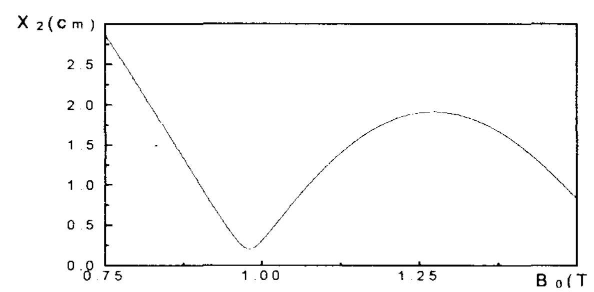


**Fig.5.** Curves of beam waist radius $X_2$ of proton beam with the magnetic field $B_0$.(here, take $L$=20*cm*)

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