# KINETIC TREATMENT OF THE HEAVY ION CHARGE EXCHANGE INJECTION

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***Abstract***

The paper represents the results of a kinetic treatment of the charge exchange injection of heavy ions into synchrotrons.

Analytical expressions for the particle density evolution in the transverse phase plane and for the emittance growth due to the elastic scattering and to the energy losses in the stripper have been derived.

Numerical examples for the superconducting heavy ion synchrotron Nuclotron in JINR - Dubna are given as well.

## 1. PROCESSES IN CHARGE EXCHANGE INJECTION

A comprehensive review of the processes taking place during the heavy ion charge exchange injection could be found in [1]. Here we will summarize only those results which are of importance for the following description.

As the beam travel through the stripping foil the relative content of ions in different charge states changes due to the processes of electron loss and electron capture. For thick enough foils the charge state distribution reaches an equilibrium - [2]. This equilibrium distribution is independent on the charge state distribution in the incident beam and is determined only by the relations between different charge - exchange cross sections and by the ion velocity.

The equilibrium charge state distribution is well described by a Gaussian - [3]:

$$\Phi_q = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(q-\bar{q})^2}{2\sigma^2}} \qquad (1)$$

In (1) $\Phi_q$ denotes the relative content of ions in charge state 'q' , $\bar{q}$ is the average charge state and $\sigma$ is the standard deviation of the charge state distribution.

Several semiempirical formulae have been proposed for the average charge state $\bar{q}$ , for example the Nikolaev - Dmitriev's formula - [4]:

$$\frac{\bar{q}}{Z_{pr}} = (1 + X^{-\frac{1}{0.6}})^{-0.6} \qquad (2)$$

$$X = \frac{v}{v' Z_{pr}^{0.45}} \quad , \qquad v' = 3.6.10^8 \; cm/s \qquad (3)$$

In (2) and (3) v is the ion velocity and $Z_{pr}$ the projectile atomic number.

For the standard deviation of the distribution Nikolaev and Dmitriev propose the following expression-[4]:

$$\sigma = 0.5\sqrt{\bar{q}(1-(\frac{\bar{q}}{Z_{pr}})^{1.67}} \qquad (4)$$

Two processes are of big importance for the charge exchange injection: Coulomb elastic scattering and ionization losses of ion energy.

The Coulomb elastic scattering causes a change in the ion trajectory slope. In [5] the following experimental formula for the heavy ion multiple scattering mean square angle in solid foils is given:

$$<\Delta\theta^2> = 0.250\frac{Z_t(Z_t+1)}{A_t}\frac{Z_{pr}^2}{E_{pr}^2}t \qquad (5)$$

where the scattering angle $\theta$ is in mrad, the stripper thickness t is in $\mu g/cm^2$ and the projectile energy $E_{pr}$ is in MeV.

The distribution of the multiple scattering angle could be approximated with good accuracy by a Gaussian.

The losses of the ion energy in the striping foil is due to the excitation and ionization of the foils atoms. Mean losses are described by the well-known Bethe - Bloch formula -[6].

The ionization losses straggling is distributed according to Landau's, Vavilov's or normal distributions depending on the ion velocity. In practice the straggling is small and could be neglected.

## 2. EQUATION OF BOLTZMANN

The Flocquet normalized coordinates are the most convenient for the description of the kinetics of charge exchange injection. In this paper we will use:

i.) generalized azimuth $\phi$ as an independent ('time') variable:

$$\phi = \int_0^s \frac{ds}{Q\beta(s)} \qquad (6)$$

ii.) normalized transverse displacement:

$$u = \frac{x}{\sqrt{\beta}} \qquad (7)$$

iii.) conjugate momentum:

$$p_u = \frac{du}{d\phi} = Q(\frac{\alpha x + \beta x'}{\sqrt{\beta}}) \qquad (8)$$

In the above formulae x denotes the horizontal coordinate, s - the longitudinal coordinate and $\alpha$, $\beta$, $\gamma$ are the Twiss functions. We will use " ′ " for the differentiation with respect to $\phi$.

Let us consider the evolution of the particle density in a thin slice of the beam.

In the consecutive moments $\phi$=0, 2$\pi$, 4$\pi$, ... the slice will pass through the stripper and the particles in the slice will undergo elastic Coulomb scattering. The elastic scattering results in kicks in the slope of the particle trajectories.

A bit earlier new portions of particles are injected into the accelerator.

As we will see, not all of the particles which have passed the stripper will be accepted by the accelerator; the $(1-\Phi_{<q>})\, f(u, u', \phi)$ of them will be lost.

Summarizing this three effects we can write the following kinetic equation :

$$\frac{\partial f}{\partial \phi} + \frac{\partial f}{\partial u} u' + \frac{\partial f}{\partial u'}(-Q^2 u + \sum_{0\le k\le \phi/2\pi} Q\sqrt{\beta_t}\,\frac{\Delta\theta_k}{\Delta\phi} h_k(\phi)) = q_{source} - q_{drain} \tag{9}$$

where $h_k(\phi)$ is a unity pulse from $2\pi k$ to $(2\pi k+\Delta\phi)$, $\Delta\phi$ being the phase thickness of the striping foil ( $\Delta\phi$ << $2\pi$ ).

In equation (9) $q_{source}$ denotes the power of the sources of particles. If $f_0$ (u, u′ ) is the distribution function in the injected beam we could write for the power of the sources:

$$q_{source} = \sum_{0\le k\le \phi/2\pi} f_0(u,u')\,\delta(\phi - 2\pi k) \tag{10}$$

In equation (9) $q_{drain}$ denotes the power of drains of particles. In order to obtain an expression for $q_{drain}$ we will use the following assumption. Let a beam of ions in equilibrium charge state <q> crosses the stripper ( <q> is an integer closest to the parameter $\bar{q}$ in (1) ). As a result of a chain of electron losses and captures behind the stripper the beam will embrace a whole spectrum of charge states. This spectrum of charge states is given by the experimental formula (1). If the stripping foil has equilibrium thickness the distribution of charge states behind the foil will not depend on the charge state distribution in the incident beam.

We will consider that the machine is able to accelerate only ions in equilibrium charge state <q> . Ions in charge states different from the equilibrium will finally be lost on the vacuum chamber walls and hence the circulating beam will consists only of ions in <q> charge state. Under this assumption we can write for the power of the particle drains:

$$q_{drain} = (1-\Phi_{<q>}) \sum_{0\le k\le \phi/2\pi} f(u,u', 2\pi k + \Delta\phi - 0)\cdot\delta(\phi - (2\pi k + \Delta\phi))$$

The Boltzmann equation (9) contains the random quantities $\Delta\theta_k$ and is in essence a stochastic PDE. Our first step will be to cope with this stochasticity. In order to do this we will try to find an equation for the averaged over the realizations of $\Delta\theta_k$ particle density <f>. If we neglect all the fast oscillating members and retain only the main, slowly varying members we reach to the following equation for <f>:

$$\frac{\partial <f>}{\partial\phi} + u'\frac{\partial <f>}{\partial u} - Q^2 u\frac{\partial <f>}{\partial u'} - \sum_{0<k<\phi/2\pi}\frac{1}{2}Q^2\beta_t\,\frac{<\Delta\theta^2>}{\Delta\phi}\frac{\partial^2 <f>}{\partial u'^2} h_k(\phi) = q_{sourse} - <q_{drain}> \tag{12}$$

This PDE is already free of any stochasticity. From this point further we will omit the ugly brackets, writing f instead of <f>.

We must solve the equation (12 ) under zero initial and boundary conditions.

## 3. SOLUTION OF THE KINETIC EQUATION

We will consider that the particles in the injected beam have normal distribution in the transverse phase plane (u, $p_u$ ):

$$f_0(u,u') = \frac{\Delta N}{2\pi\, Q\sigma_0^2} e^{-\frac{Q^2u^2 + u'^2}{2Q^2\sigma_0^2}} \tag{13}$$

where $\Delta$N is the number of injected into the slice particles.

In our treatment we will work with accuracy $O(\frac{\beta <\Delta\theta^2>}{2\sigma_0^2})$ . As we will see later this means that we consider the emittance growth $\Delta\varepsilon$ in single stripper crossing much smaller than the initial emittance $\Delta\varepsilon/\varepsilon$ << 1.

The charge exchange injection coves the following processes: injection of new particles, diffusion along u′ coordinate in the stripper, ion losses and betatron oscillations between two successive crossings the stripper.

It can be shown that the solution of the kinetic equation (12) consists of main stationary part which represents a sum of Gaussians and a small ( of order $\beta_0<\Delta\theta^2>/2\sigma_0^2$ ) and depending on $\phi$ part with more complicated structure.

For the stationary part of the solution we can obtain:

$$\bar{f}(u,u') = \Phi_{<q>}\,\bar{f}_1(u,u') + \Phi^2_{<q>}\,\bar{f}_2(u,u') + \ldots + \Phi^n_{<q>}\,\bar{f}_n(u,u') \tag{14}$$

with:

$$\bar{f}_k(u,u') = \frac{\Delta N}{2\pi\ Q\sigma_k^2} e^{-\frac{Q^2u^2 + u'^2}{2Q^2\sigma_k^2}} \quad (15)$$

$$\sigma_k^2 = \sigma_0^2 + \frac{1}{2} k \beta_t < \Delta\theta^2 > \quad (16)$$

For the variable with ϕ part of the solution we can obtain:

$$\tilde{f}(u,u',\phi) = \sum_{k=0}^{n} \sum_{i=0}^{n-k} \Phi_{<q>}^{n+1-k} \frac{a}{2Q^2\sigma_i^4} \bar{f}_i(u,u')$$

$$[(\frac{u'^2 - Q^2u^2}{2}) \cos 2Q(\phi - (i+k)2\pi)$$

$$+ \ Quu' \sin\ 2Q\ (\phi\ - (i+k)2\pi\ )\ ] \quad (17)$$

We are interested mainly in the stationary ( averaged over betatron oscillations) distribution function.

In equation (14) 'n' denotes the number of realized injection turns, in (17) a= $1/2\beta<\Delta\theta^2>$.

Expressed in words the meaning of this formula is as follows: a portion of ΔN particles with normal distribution $f_0(u, p_u) \sim N(0, \sigma_0)$ is injected in the accelerator ; passing through the stripper the distribution gets wider in u′ direction; the following betatron oscillations spread this widening also on the coordinate u ; a part $(1 - \Phi_{<q>}) f$ of this particles goes to charge states different from the equilibrium charge state <q> after the striping foil and is cut later by the accelerator, so only $\Phi_{<q>} f$ of the slice survives. This process repeats 'n' times.

## 4. EMITTANCE GROWTH

Integrating (14) over u and u′ from (-∞) to (+∞) and over the azimuth from 0 to 2π we receive the number of particles successfully injected in the accelerator:

$$N = I_0\ T\,(\Phi_{A,1}^X \Phi_{A,1}^Z \Phi_{<q>} + \Phi_{A,2}^X \Phi_{A,2}^Z \Phi_{<q>}^2 + \dots + \Phi_{A,n}^X \Phi_{A,n}^Z \Phi_{<q>}^n\ ) \quad (18)$$

where Io is the injection current, T - the period of the synchronous particle and the 'aperture' factors $\Phi_{A,k}$ are given by:

$$\Phi_{A,k} = \begin{cases} 1, \ if \ \ 4\sigma_k^2 < A \\ \dfrac{1 - e^{-\frac{2A}{\varepsilon_{RMS}}}}{1 - e^{-2}} = 1.16\,(1 - e^{-\frac{A}{2\sigma_k^2}}) & ,4\sigma_k^2 \geq A \end{cases} \quad (19)$$

A being the acceptance.

The diffusion in the stripper due to the Coulomb scattering and the jumps in the off-momentum orbit after the foil crossings due to the ionization losses of energy lead to transverse emittance growth.

On the base of (14) the following formula for the emitance growth can be deduced:

$$\sqrt{\beta_t \varepsilon} = \sqrt{\beta_t(\varepsilon_0 + 2n\beta_t < \Delta\theta^2 >)} + + \sqrt{D_t^2 + \beta_t^2\ D_t'^2}\ \ n\Delta\delta \quad (20)$$

## 5. CHARGE EXCHANGE INJECTION IN NUCLOTRON

We have applied the above derived formulae for the charge exchange injection in the JINR - Dubna superconducting heavy ion synchrotron Nuclotron - [7].

A natural development of the JINR LHE spin physics programme will be the acceleration of polarized beams of deuterons in Nuclotron. The scheme of acceleration covers : a cryogenic source of polarized deuterons Polaris, a 5 MeV/u linac, charge exchange $D^-\uparrow \rightarrow D^+\uparrow$ injection into Nuclotron and acceleration in it up to 6 GeV/u - [8].

Fig. 1 shows the process of ion storage.

An intensity gain of about 40 could be achieved for a 100 turn stripping injection.

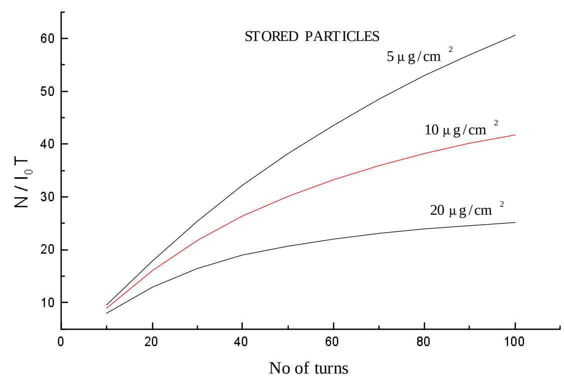


**Fig. 1.** Ion storage during $D^-$ injection into Nuclotron.

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