KINETIC THEORY OF SOLITARY WAVES ON COASTING BEAMS IN SYNCHROTRONS AND STORAGE RINGS

H. Schamel, University of Bayreuth, Bayreuth, Germany
R. Fedele, INFN/University of Napoli, Napoli, Italy

Abstract

Observations in stored high-energy beams in circular accelerators show the existence of long-living coherent structures of solitary wave type. The paper focuses on a collective kinetic description of such solitary structures based on an extended Vlasov-Poisson model. Depending on the coupling impedance, on the selected dispersion branch and on the beam energy in relation to the transition energy various solutions of this system can be found. Of special interest is the one, represented by a notch in the thermal range of the distribution function, for which standard wave theory would predict strong Landau damping.

1 INTRODUCTION

Intense coasting beams in synchrotrons operating close to the linear stability limit exhibit a variety of nonlinear wave phenomena [1], the most prominent ones being coherent depletion zones in the momentum distribution excited by an external forcing. An explanation of this collective phenomenon has recently been given in terms of steady-state solitary hole solutions of the Vlasov-Poisson system in the limit of a large wall resistivity [2]. In the present paper this study is extended in a twofold manner: We allow for zero resistivity, as well, and include the whole range of phase velocities up to the hydrodynamic limit.

2 BASIC EQUATIONS

Adopting the normalization of Refs. [2, 3] we study the following set of equations [3]

$$[\partial_t + u\partial_z - \varepsilon\partial_u] f(z,u,t) = 0 \qquad (1)$$

$$(1-L)\varepsilon'' + R\varepsilon' - \mu\varepsilon = \alpha[R\lambda_1 + (g_0 - L)\lambda_1'] \qquad (2)$$

Eq. (1) is the Vlasov equation and Eq. (2) is a Poisson-like equation extended by electromagnetic correction terms which are $O(\gamma^{-2})$ where γ is the relativistic factor, $\gamma \gg 1$. In (2), α carries the sign of the slip factor η, $\mu = \left(\frac{4\pi\gamma R_0}{b}\right)^2$ and R and L are the dimensionless resistance and inductivity, respectively. Note that the capacitive space charge effect is represented by the term $g_0\lambda_1'$, with the geometry factor g_0 given by $g_0 = 1 + 2ln\, b/a$. λ_1 is the perturbed line density and dash means differentiation with respect to z.

Note also that (2) contains the two known expressions of “Poisson’s equation” in the limits of strong resistance and of purely reactive coupling impedance, respectively.

3 STANDING STRUCTURES IN CASE OF PURELY REACTIVE IMPEDANCES

To get a first impression about the possible structures we take the limit of a purely reactive impedance, $R \to 0$, in which case (2) reduces to

$$\phi'' = \bar{\mu}\phi - \bar{\alpha}\lambda_1 \qquad (2')$$

where $\bar{\mu} = \mu(1-L)^{-1}, \bar{\alpha} = \alpha(g_0 - L)(1-L)^{-1}$ and $\varepsilon \equiv -\phi'$. Assuming furthermore a standing structure $\phi(z - \Delta u t)$ with $\Delta u = 0$ and an appropriate solution $f(z,u)$ of Vlasov’s equation (1) for a positive bell-shaped potential hump $0 \leq \phi \leq \psi \ll 1$, as given in (14) of Ref. [2], we obtain solitary wave solutions of the type

$$\phi(z) = \psi \text{sech}^4\left(\frac{\sqrt{\bar{\mu} - \bar{\alpha}}}{4} z\right) \qquad (3)$$

which resembles electron hole (or hump) solutions in plasma physics [4].

Since it holds for the dimensional potential $\tilde{\phi}$, $\phi \sim \eta q \tilde{\phi}$, with q being the electric charge of a beam particle, the equivalence with the corresponding plasma situation, suggests the following general qualitative picture as shown in Fig. 1.

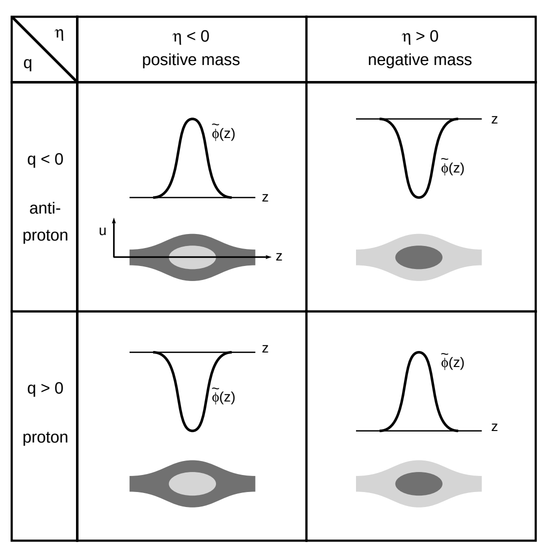

Figure 1: The dimensional electric potential $\tilde{\phi} \sim \eta q \phi$ as a function of space and the corresponding phase space pattern for four different cases (q is the particle charge, η is the slip factor). In brighter phase space regions the distribution function is less dense.

We see that for a positive mass system $(\eta < 0)$ the center of the momentum distribution is excavated whereas it is overpopulated for a negative mass system $(\eta > 0)$. This essentially is what Dory's "mass-conjugation theorem" [5] predicts.

4 GENERALIZED SOLITARY WAVE SOLUTIONS

4.1 Kinetic Regime

Next we allow for a finite propagation speed $\Delta u > 0$, in which case the perturbed line density is found to be given by the half power expansion

$$\lambda_1 = -\frac{1}{2}Z_r'\left(\frac{\Delta u}{\sqrt{2}}\right)\phi - \frac{4\bar{b}}{3}\phi^{3/2} + \frac{1}{16}Z_r'''\left(\frac{\Delta u}{\sqrt{2}}\right)\phi^2 + \ldots \quad (4)$$

which includes and extends (15) of [2]. In (4) $\bar{b}$ is given by

$$\bar{b} = \frac{1}{\sqrt{\pi}}[1 - \beta - (\Delta u)^2]\exp(-\Delta u^2/2) \equiv \bar{b}(\beta, \Delta u) \quad (5)$$

which reflects the status of particles trapped in the potential well. Note that a notch in the distribution at resonant velocity is described by β negative. Substituting (4) into (2) and concentrating on the two limits (i) $R = 0$, the purely reactive limit, and (ii) $R \gg 1$, the resistive limit, we can write (2) as

$$\phi'' = A\phi + B\phi^{3/2} + C\phi^2 \equiv -V'(\phi) \quad (6)$$

with appropriate constants A, B, C for each limit that depend on Δu.

By integration of (6) we can find

$$\frac{\phi'(z)^2}{2} + V(\phi) = 0 \quad (7)$$

with V given by

$$-V(\phi) = A\phi^2/2 + 2B\phi^{5/2}/5 + C\phi^3/3 \quad (8)$$

The condition $V(\psi) = 0$, where ψ is the amplitude of the bell-shaped potential ϕ, yields the nonlinear dispersion relation (NDR) which becomes

$$A + 4B\sqrt{\psi}/5 + 2C\psi/3 = 0 \quad (9)$$

the solution of which determines Δu the phase velocity of the solitary structure. Making use of (9), we can rewrite (8) as

$$-V(\phi) = -\frac{2B}{5}\phi^2[\sqrt{\psi} - \sqrt{\phi}] - \frac{C}{3}\phi^2(\psi - \phi) \quad (10)$$

which represents a two-parametric soliton. For $B = 0$ it holds

$$\phi(z) = \psi \text{sech}^2\left(\sqrt{\frac{-C\psi}{6}}z\right) \quad (11)$$

provided that $C < 0$ and for $C = 0$ we get

$$\phi(z) = \psi \text{sech}^4\left(\sqrt{\frac{-B\psi^{1/2}}{20}}z\right) \quad (12)$$

provided that $B < 0$. Whereas the first one is of hydrodynamic type (see later), the last one is a true result of trapping.

To see under what circumstances such a solutions exists, we have to analyze the NDR (9). Inserting A, B and C in to (8) we find

$$-\frac{1}{2}Z_r'\left(\frac{\Delta u}{\sqrt{2}}\right) = D \quad (13)$$

Fig. 2 shows the permissible solutions of (13). If D is negative and small, $-0.285 < D \leq 0$, we obtain two branches, the kinetic branch with $\Delta u \approx 0(1)$ and the hydrodynamic branch with $\Delta u \gg 1$.

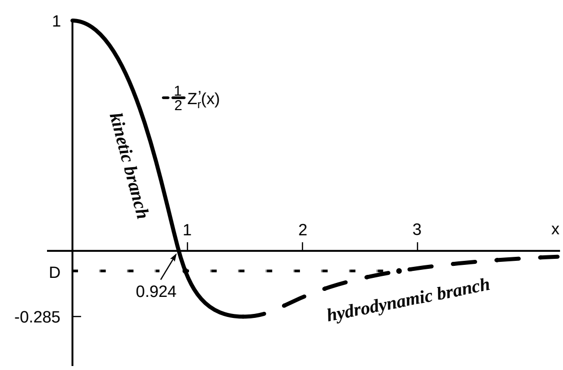

Figure 2: The real part of $-\frac{1}{2}Z'(x)$, $Z(z)$ being the plasma dispersion function, as a function of $x = Rez$.

In the kinetic thermal regime, the solution is given by

$$\Delta u = 1.307(1 - D) \quad (14)$$

where $\mid D \mid \ll 1$. The existence condition $B < 0$ becomes in the strongly resistive case $(R \gg 1)$, $\alpha\bar{b} < 0$, and in the purely reactive case $(R = 0)$, $\bar{\alpha}\bar{b} < 0$. Hence below transition energy $(\alpha < 0)$ $\bar{b}$ must be positive which, together with (14), implies

$$-\beta > 0.71 \quad (15)$$

The trapping parameter β must be sufficiently negative corresponding to a depletion zone in the momentum distribution.

Above transition energy, $\bar{b}$ must be negative and hence $\beta > -0.71$, implying that a hump-like resonant distribution is an admissible solution.

Hence, the general picture of Fig. 1 can be transferred to propagating structures as well, valid for both limits of impedances. (Note, however, that standing resistive structures of O-type separatrix exist only for beams below transition energy [3].)

4.2 Hydrodynamic Regime

Finally, for large Δu we find that $(\Delta u)^{-2} = -D \ll 1$ and that B is negligible (no contribution from trapping). We, hence, have $C < 0$, i.e. the situation of eq. (11). An evaluation of these inequalities shows that KdV-solitons of type (11) **cannot** exist in the strongly resistive case. In the purely reactive case, however, they can exist both below and above transition energy provided that L is appropriately chosen. For a beam above transition energy, the existence condition requires a negative imaginary coupling impedance i.e. the dominance of inductivity over space charge.

We conclude that a negative mass instability can saturate in a nonlinear soliton only for zero resistivity and for a sufficiently large inductivity.

5 CONCLUSION

The collective kinetic treatment of coasting beam dynamics exhibits a rich world of solitary waves and associated structures being more intricate than conjectured by DORY's "mass conjugation theorem". They typically rest on a new type of kinetic acoustic modes propagating with thermal phase velocities where Landau (resp. Keil-Schnell theory) theory would predict strong damping. The resolution of this seeming discrepancy is that the latter is not applicable due to the involved wave-particle resonance valid even in the infinitesimal wave limit. These structures remain nonlinear no matter how small their amplitudes are [6].

REFERENCES

[1] P.L. Colestock and L.K. Spentzouris, in "The Tamura Symposium Proceedings", Austin, TX, 1994.

[2] H. Schamel, *Phys. Rev. Lett.* **79**, 2811, 1997.

[3] H. Schamel and R. Fedele, *Phys. Plasmas* **7**, No. 8, 2000.

[4] H. Schamel, *Phys. Reports* **140**, 161, 1986.

[5] R.A. Dory, *Plasma Phys.* **6**, 511, 1964.

[6] H. Schamel (submitted to *Phys. Plasmas*, 2000).