# FUNDAMENTAL DESIGN PRINCIPLES OF LINEAR COLLIDER DAMPING RINGS, WITH AN APPLICATION TO CLIC

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## Abstract

Damping Rings for Linear Colliders have to produce very small normalised emittances at a high repetition rate. A previous paper presented analytical expressions for the equilibrium emittance of an arc cell as a function of the deflection angle per dipole. In addition, an expression for the lattice parameters providing the minimum emittance, and a strategy to stay close to this, were proposed. This analytical approach is extended to the detailed design of Damping Rings, taking into account the straight sections and the damping wigglers. Complete rings, including wiggler and injection insertions, were modelled with the MAD [1] program, and their performance was found to be in good agreement with the analytical calculation. With such an approach it is shown that a Damping Ring corresponding to the Compact Linear Collider (CLIC) parameters at 0.5 and 1 TeV centre-of-mass energy, and tunable for two different sets of emittance and injection repetition rate, can be designed using the same ring layout.

## 1 INTRODUCTION

Damping Rings are used to reduce beam emittances at a speed compatible with the high repetition rate of Linear Colliders. The present paper describes a sequential approach to achieve both the damping speed and the equilibrium emittance. It is based on an analytical calculation tailored to Linear Collider Damping Ring design [2], with particular emphasis on the optical parameters, disregarding momentum issues which are more related to collective effects like Intra-Beam Scattering (IBS).

Starting from the length of the bunch train to be damped and the repetition rate of the Linear Collider, a quantity called *reduced damping time* is introduced in order to quantify the damping performances of rings of different sizes.

Then, for a constant reduced damping time and a constant wiggler length, different combinations of arc field and wiggler field are calculated, allowing large variations of $R_{ra}$, the ratio of the total radiation loss *around the ring* to the radiation loss *in the arcs only.*

The choice of a combination of wiggler field and arc field for a given reduced damping time determines the transverse equilibrium emittance reduction, as well as the parasitic emittance created by the wiggler itself.

In most low equilibrium emittance Damping Rings, high horizontal phase advances, small dispersion and $\beta_x$ values in the bending magnets are used. This results in a very small momentum compaction, and consequently a low turbulence impedance threshold. Using a strategy to detune a *theoretical minimum emittance* (TME) lattice close to its optimum [2], these effects can be alleviated. The choice of the emittance detuning ratio fixes the arc cell lattice and thus finalises all Damping Ring parameters.

## 2 WIGGLER DAMPING AND ENERGY SPREAD

In a Damping Ring of circumference $C$, with a bunch train length $l_s$ (including space for the injection and ejection fast kicker rise and fall times), a repetition frequency $f_r$, the *reduced damping time* $\tau_r = \tau_{x,y}/C$ has to satisfy in both transverse planes $\tau_r \leq 1/(n_\tau l_s f_r)$, $n_\tau$ being the number of damping times necessary to damp the incoming emittance to the acceptable level $\epsilon_{target}$.

The present study shows that the total amount of synchrotron radiation loss per turn is equal to $2\gamma m_0 c/\tau_r$. The choice of $\tau_r$, the arc field $B_{arc}$ and the wiggler field $B_{wig}$ fix the total wiggler length, taking into account the radiation losses in the arcs. The ratio $R_{ra}$ was found to be

$$R_{ra} = \frac{3}{2} \frac{m_0}{e\gamma^2 \pi B_{arc} r_e \tau_r}$$

where $e$ and $r_e$ denote the electron charge and the classical electron radius. $R_{ra} = 1$ means no wiggler and $R_{ra} >> 1$ represents a wiggler dominated ring.
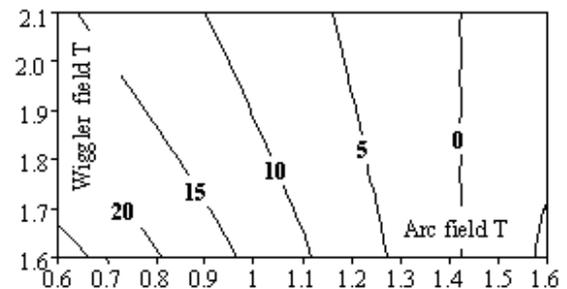


Figure 1: Wiggler field *vs.* arc bend field for a given reduced damping time of 44 $\mu$s/m, for different wiggler lengths (m), at 2 GeV

The total radiation loss in the presence of wigglers with a given field $B_{wig}$, for a fixed value of $\tau_r$, is independent of the ring structure. For given arc field $B_{arc}$ and wiggler field $B_{wig}$ values, the length of the wiggler $l_{wig}$ may be adjusted to produce the required $\tau_r$. This result is independent of the value specified for the normalised target emittance

$\gamma\epsilon_{target}$. Fig. 1 shows curves of $B_{arc}$ and $B_{wig}$ combinations leading to different wiggler lengths for a given $\tau_r$.

The emittance reduction by the wiggler will be $Q_e = 1/R_{ra} = arc\ loss\ /\ total\ loss$, but at the same time the wiggler will create horizontal emittance through the non-zero local dispersion function. The emittance production, calculated using [3], is shown in Fig. 2 as a function of the ratio $R_{ra}$. Aiming at a very low ring equilibrium emittance, and thus a small wiggler contribution to the emittance, requires a short wiggler period.
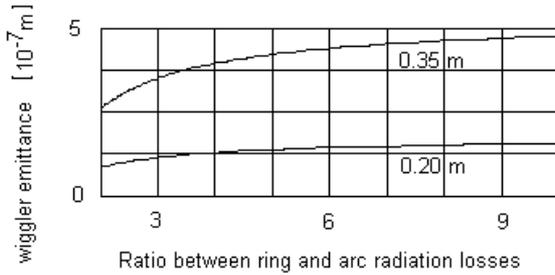


Figure 2: Wiggler contribution to the horizontal emittance for $B_{wig} = 1.73$ T, $\bar{\beta}_x = 2.5$ m at 2 GeV and $\tau_x = 35$ $\mu$s/m for wiggler wavelengths 0.35 m and 0.20 m

Furthermore, increasing the wiggler length or the ratio $R_{ra}$ to large values will not reduce the ring equilibrium emittance indefinitely, as it will become dominated by the wiggler contribution to the emittance.

Once the reduced damping time has been defined it may be seen (Fig. 3) that the energy spread is only weakly dependent on the ratio $R_{ra}$
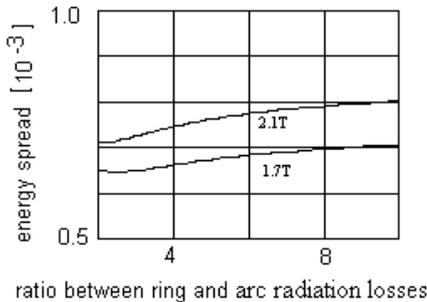


Figure 3: Relative energy spread $vs.$ ring - wigglers radiation loss ratio $R_{ra}$ for two wiggler fields and a constant reduced wiggler damping time $\tau_r = 35$ $\mu$s/m at 2 GeV

## 3 ARC CELL OPTIMISATION

As proposed in [2] the arcs are made of TME cells. The dispersion and $\beta_x$ functions in the bending magnets and at the potential sextupole locations are chosen larger than those required for the minimum emittance. For a given ratio $\epsilon_r$ of the actual emittance to the minimum possible emittance,

the maximum $D_x$ value is selected, as a large momentum compaction is required to maximise the impedance threshold. In this case the lattice parameters in the centre of the bending magnet are entirely determined [2] and given by

$$\beta_x = \epsilon_r \frac{l_{bend}}{2\sqrt{15}} \qquad D_x = (1+\frac{2}{\sqrt{5}}\sqrt{\epsilon_r^2-1})\frac{\theta\ l_{bend}}{24}$$

where $\theta$ and $l_{bend}$ are the deflection angle and the length of the bending magnet. The symmetry of the TME cell lattice functions with respect to the centre of the bending magnet imposes equally the horizontal cell phase advance $\mu_x$:

$$tan(\frac{\mu_x}{2}) = \epsilon_r \frac{\sqrt{3}}{\sqrt{\epsilon_r^2-1} - \sqrt{5}}$$
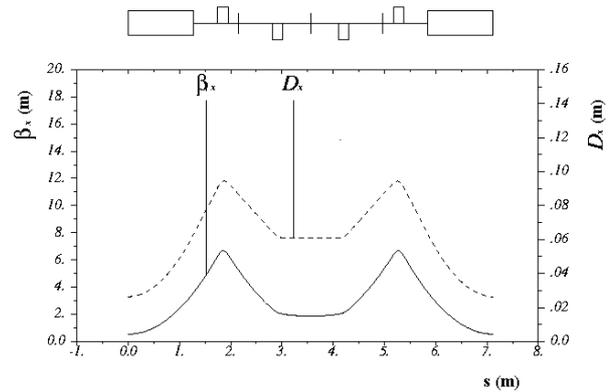


Figure 4: Horizontal lattice functions of the arc cell

The horizontal lattice functions of the arc cell are shown in Fig. 4. Using the above expressions the momentum compaction factor of a ring containing only regular TME cells depends only on the cell length $l_{cell}$, the bending magnet parameters and the emittance detuning ratio:

$$\alpha = \frac{1}{12}\frac{l_{bend}}{l_{cell}}\theta^2\ (1+\sqrt{\frac{\epsilon_r^2-1}{5}})$$

## 4 MOMENTUM COMPACTION AND TURBULENCE IMPEDANCE THRESHOLD

The *number of regular arc cells* required to produce the target emittance and the reduced damping time, taking into account the wiggler effects, can now be evaluated. It is a function of the wiggler characteristics (field, period), of the emittance detuning chosen and, obviously, of the target emittance and the beam momentum. Fig. 5 shows the number of cells (assumed to be even) $vs.$ the ratio $R_{ra}$.

The *arc cell* and *straight section* lengths are modelled as follows. The ring used in the analytical calculations has a race-track shape. It consists of two $180°$ arcs, made of bending magnets of length $l_{bend}$ in regular arc cells of length $l_{cell} = 2l_{bend}+$ a constant space required for the focusing part of the cell. Two long straight sections house
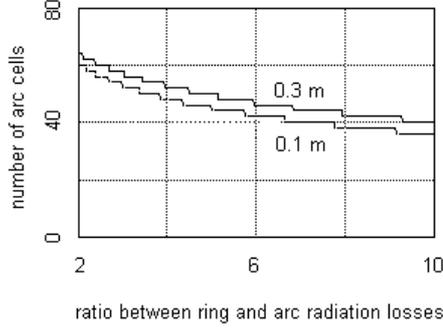
Figure 5: Number of arc cells required to produce the target emittance $\gamma\epsilon_{target} = 1.9\,10^{-6}$ m for wiggler wavelengths 0.3 m and 0.1 m, for $\epsilon_r = 3.9$, $B_{wig} = 1.7$ T, $\tau_r = 35\ \mu$s/m at 1.98 GeV

the injection/ejection system and the RF cavity, with a total length assumed to be the sum of a fixed part of 18 m and a "variable" part of 1.8 times the wiggler length. The wigglers are distributed among the two straight sections in order to minimise the ring dimensions.

Using this geometry the expression for the momentum compaction shown in Section 3 may be corrected for the presence of the two long straight sections. As $\sigma_e$ has a weak dependence on $R_{ra}$ (see Fig. 3), the turbulence impedance threshold, calculated with the usual formula $(Z/n)_{thr} = \alpha\ (2\pi)^{3/2}\ E\ \sigma_e^2\ \sigma_s\ /\ (N_b\ e^2\ c)$, will show approximately the same behaviour as $\alpha$ (Section 3). In Fig. 6 the variation of $(Z/n)_{thr}$ and of the arc field required to produce a constant $\gamma\epsilon_{target}$ and $\tau_r$ is shown $vs.\ R_{ra}$.

$(Z/n)_{thr}$ and $\alpha$ increase rapidly with $R_{ra}$, and the required arc bend field decreases. This allows an increase of $l_{bend}$ and larger values for $D_x$ and $\beta_x$, which eases the chromaticity correction.
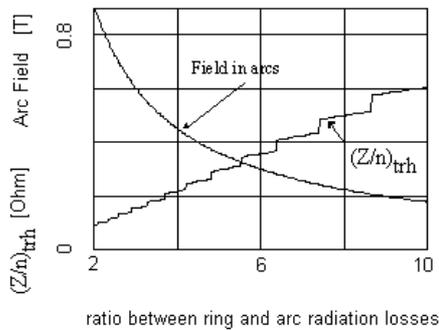


Figure 6: Variation of $(Z/n)_{thr}$ and the arc bending field $vs.$ the loss ratio $R_{ra}$ assuming $\sigma_s = 3\ mm$, $4.2\,10^9$ particles per bunch, $\gamma\epsilon_{target} = 1.9\,10^{-6}$ m, $\tau_r = 35\ \mu$s/m, $B_{wig} = 1.7$ T, $\epsilon_r = 3.9$ at 1.98 GeV

For $\gamma\epsilon_{target}$ varying from 0.43 to $1.7\,10^{-6}$ m, $\epsilon_r$ from 1.6 to 7.7, $\tau_r$ from 26 to 53 $\mu$s/m at 1.98 GeV, numerical calculations with MAD have confirmed to the analytical approach. Both $\gamma\epsilon_{target}$ and $\epsilon_r$ could be reproduced to within 3 %, and $\alpha$, $\tau_r$ and $\sigma_e$ to within 10 %.

## 5 A POSSIBLE RING FOR THE CLIC 0.5 TEV AND 1 TEV OPTIONS

The optimum beam energy depends on optics considerations, but also on Intra-Beam Scattering and polarisation. The polarised electron option requires beam energies of $(n + 0.5) * 0.44$ GeV so as to stay away from depolarising resonances. The $\gamma^3$ dependence of the normalized equilibrium emittance and the importance of IBS problems at low energy lead to the choice of 1.98 GeV. Table 1 shows a tentative parameter list for a Damping Ring capable of fulfilling the requirements of both the 0.5 TeV and the 1.0 TeV options of the CLIC project [4].

Table 1: Tentative Damping Ring parameters

| Parameter | Unit | 0.5 TeV | 1.0 TeV |
|---|---|---|---|
| $\gamma\epsilon_x$ | $10^{-6}$ m | 1.7 | 1.1 |
| Bunch Train Length | m | 38.1 | |
| Charge per Bunch | | $4.2\,10^9$ | |
| Repetition rate | Hz | 200 | 150 |
| $n_\tau$ | | 5 | |
| $\tau_r$ | $\mu$s/m | 26 | 35 |
| Wiggler field | T | 2.1 | 1.7 |
| Emittance detuning | | 7.7 | 3.9 |
| Mom. compaction | $10^{-4}$ | 12.8 | 7.8 |
| $(Z/n)_{thr}$ | $\Omega$ | 0.34 | 0.19 |
| $R_{ra}$ | | 4.9 | 3.7 |
| Energy | GeV | 1.98 | |
| Number of Arc Cells | | 52 | |
| Circumference | m | 348 | |
| Wiggler length | m | 37 | |
| Arc field | T | 0.3 | |

## 6 CONCLUSION

An analytical model of a Damping Ring, using TME cells, was used to study the relations between the target emittance, the reduced damping time, the turbulence threshold, the energy spread, the momentum compaction, the field in the arcs and in the wigglers, the emittance detuning of the lattice and the cell length. It is a powerful tool for the exploration of the Damping Ring parameters. The results are confirmed by simulations with the MAD program.

A set of design parameters is proposed for the 1.0 TeV CLIC case, which can be detuned to fulfil the requirements of the 0.5 TeV case, keeping the same magnets and geometry.

## REFERENCES

[1] F.C. Iselin, "The MAD Program", CERN/SL/90-13, 1996

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[3] A. Ropert, $in$ CAS 1989, CERN 90-03, 1990, p.188

[4] The CLIC Study Team, "CLIC, a 0.5 to 5 TeV $e^+/e^-$ Compact Linear Collider", CERN PS/98-009 (LP)