CORRECTION OF THE SYSTEMATIC b3 ERROR WITH THE RESONANCE-FREE LATTICE IN THE LHC

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Abstract

The effect of the sextupole component b3 in the LHC dipoles on the resonance-free lattice has been investigated. It is shown that its dynamic aperture, without b3 spool piece correction, is close to that of the nominal LHC lattice version 6.0 with spool piece correction. A prerequisite is the addition of a few chromaticity sextupoles in the dispersion suppressors. Under this condition an increase of the b3 component by a factor of two can probably be accepted. Furthermore, a systematic relative gradient error up to one per mil can be tolerated without changing this result.

1 INTRODUCTION

A resonance free lattice (RFL) [1] can be used in LHC to overcome possible problems associated with unexpected large multipole components in the main dipoles. This lattice is made out of blocks of cells periodic both in linear and nonlinear components, with suitable phase advances such that many resonance driving terms are cancelled to first order in multipole strength.

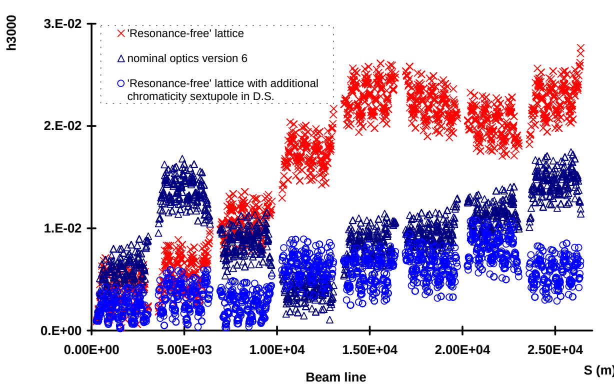

Figure 1: Comparison of the 3^{rd} order resonance driving term **h3000** (*random seed 1*) excited along the LHC ring for the resonance–free lattice and the nominal optics version 6 in the absence of b_3 spool pieces. The former is shown with and without extra chromaticity sextupoles in the D.S..

Some interesting results have been obtained for the case of the octupole component b_4 [2]. The present paper deals with the case of the b_3 sextupole component. Its uncertainty (component constant over one octant of the machine) could become large due to the manufacturing process. This is why it could be helpful to master such a geometric effect.

A procedure similar to that developed in [2] has been followed. In section 2 the resonance driving terms are examined. Section 3 is devoted to tracking studies to be compared with the resonance analysis. A first estimation of the effect of gradient errors in the dipoles is given.

2 RESONANCE ANALYSIS

A comparison of the 3^{rd} order resonance driving term excited along the ring has been done for the RFL and for the nominal optics version 6 using the code SODD [4]. To test more clearly the efficiency of the resonance–free lattice in suppressing the driving term, the $\mathbf{b}_3$ spool pieces that are normally used to correct the sextupole components of the main dipoles are excluded. The resonance–free lattice is much worse than the nominal optics version 6 as shown in Figure 1. This can be ascribed to the chromaticity sextupoles which are present in only 23 cells rather than 25 cells in each arc as required for the RFL [1]. Therefore 12 sextupoles were added in the dispersion suppressors (D.S.) to arrive at 25 cells with chromaticity sextupoles in each octant. As expected, we could achieve a large reduction of the 3^{rd} order resonance driving terms with this modification. This effect is shown in Figure 1 for a particular set of random errors.

Table 1: Resonance driving terms associated with 60 different error distributions and two different optics, no b3 spool piece corrector.

	res. term	aver.	max.
Nominal optics	h3000	0.0885	0.1378
	h1020	0.2166	0.3477
	h1002	0.0699	0.3758
Res.-free optics	h3000	0.3034	0.3875
	h1020	0.0813	0.1654
	h1002	0.7049	0.8299
Nominal optics + sext. in DS	h3000	0.0401	0.0918
	h1020	0.1378	0.2847
	h1002	0.1122	0.2616
Res.-free optics + sext. in DS	h3000	0.0935	0.1710
	h1020	0.0782	0.1789
	h1002	0.2995	0.4143
Nominal sext. in DS b2=-1.4	h3000	0.0540	0.1021
	h1020	0.0859	0.1952
	h1002	0.1536	0.2758
res.free sext. in DS b2=-1.4	h3000	0.0696	0.1063
	h1020	0.0678	0.1299
	h1002	0.1011	0.2137

A summary of the third order resonance driving terms is shown in Table 1 for 60 sets of error distributions. The dramatic differences associates with the presence of extra chromaticity sextupoles in the dispersion suppressor appears clearly when one compares upper and middle part of the table. Therefore we expect a big improvement in the

Table 2: Long–term dynamic aperture (100,000 turns) for the nominal LHC optics V6 and the resonance–free lattice with the error table **9901** at the nominal working point (0.28,0.31). Average and minimum value over 60 random seeds are given at the phase space angles $\phi = 15°, 45°$ respectively for each case.

Case	**Dynamic Aperture (100,000 turns)**			
	$\phi = 15°$		$\phi = 45°$	
	Minimum	**Average**	**Minimum**	**Average**
Case A1: Nominal optics v6	11.5	12.9	11.8	14.4
Case A2: **Case A1** without $\mathbf{b}_3$ correction	10.7	12.5	10.0	12.7
Case A3: **Case A1** with additional Chro. Sext. in D.S. and b_3 correction	11.3	13.0	12.2	14.2
Case A4: **Case A1** with additional Chro. Sext. in D.S. and without b_3 correction	10.5	12.9	10.7	13.4
Case B1: resonance–free lattice	11.8	13.5	13.5	15.3
Case B2: **Case B1** without $\mathbf{b}_3$ correction	8.1	9.7	8.2	10.3
Case B3: **Case B1** with additional Chro. Sext. in D.S. and b_3 correction	12.2	13.8	14.2	15.7
Case B4: **Case B1** with additional Chro. Sext. in D.S. and without b_3 correction	12.0	13.0	11.1	14.1

dynamic aperture when those extra sextupoles are switched on. However, it is less clear how the nominal and RFL behave since there is no clear overall difference in the resonance terms.

Up to now gradient errors have been ignored. In the LHC three different gradient errors are expected:

- Systematic with alternating sign from inner to outer channel with no effect on the tunes
- Uncertainty, i.e. constant over one arc and random change from arc to arc
- Random (from magnet to magnet)

We have to study their effect on the RFL, since gradient errors change the cell phases. The results for the systematic errors are shown in the lower part of Table 1. For the RFL there is a general improvement of the resonance terms, which results in an improvement of the short–term dynamic aperture (1000 turns) if a systematic error of -1.4$\times 10^{-4}$ is introduced (the insertions have been rematched accordingly). A reversal of the sign of b_2 has the opposite effect for most of the terms. Adding an uncertainty of 7×10^{-4} in the dipoles and 10×10^{-4} in the quadrupoles does not change the results in a dramatic way, i.e. the maximum increase of the resonance terms is 20%.

3 LONG–TERM DYNAMIC APERTURE

The dynamic aperture is expressed in terms of the transverse r.m.s. beam size σ, the LHC normalised emittance is 3.75μm at 1σ. Particle motion samples different resonances depending on the ratio between horizontal and vertical oscillation amplitudes, with $A_x = \sqrt{\beta_x \cdot \epsilon_x}$ and $A_y = \sqrt{\beta_x \cdot \epsilon_y}$ with ϵ_x, ϵ_y the horizontal and vertical transverse emittances, respectively. To obtain a realistic estimate for the dynamic aperture one has to vary this ratio, expressed as a phase space angle: $\phi = \arctan\left(\sqrt{\frac{\epsilon_y}{\epsilon_x}}\right)$.

Table 3: The Multipole components of the main LHC dipoles at injection energy. Unit: 10^{-4} relative field error at a radius of 17 mm.

Error Table 9901						
(Persistent & Geometric)						
	Mean		**Uncertainty**		**Random**	
n	**b**	**a**	**b**	**a**	**b**	**a**
3	-9.70	-0.082	1.376	0.867	1.474	0.479
4	0.22		0.344	0.130	0.513	0.513
5	0.89	0.007	0.436	0.418	0.428	0.341
6	-0.011		0.057	0.057	0.088	0.165
7	-0.16	0.017	0.053		0.219	0.078
8	-0.00				0.043	0.084
9	0.36	-0.006	0.028		0.071	0.115
10						0.012
11	0.57	0.002				

As a bare minimum one has to study "round beams" (equal horizontal and vertical emittance) and the case of mainly horizontal motion (horizontal emittance much larger than the vertical emittance). The tracking is performed over 10^5 turns in the full six–dimensional phase space at 75% of the bucket half size, (i.e. ($\frac{\delta p}{p_0} = 0.00075$) using the tracking code SixTrack [5]. The amplitude has been varied in steps of $\frac{1}{15}\sigma$ to determine the minimum and average dynamic aperture for the 60 random seeds. It is necessary to use two values for the dynamic aperture since the minimum is a possible worst case, with a 95% probability that the true dynamic aperture is above this value, and the average dynamic aperture serves to compare the overall quality of the different lattices. The uncertainty of the minimum value is about 0.5σ while it is some 0.2σ for the average dynamic aperture. Table 2 lists the long–term dynamic aperture for both the nominal optics version 6 and the RFL, each lattice in four different configurations. The best guessed multipole errors of table **9901** listed in Table 3 have been included.

From the comparison between the **Case A2** and **B2** in Table 2 , we know that the RFL has a dynamic aperture much smaller in the case that the $\mathbf{b}_3$ correction is excluded. This is consistent with the above analysis concerning the 3^{rd} order resonance driving term. In the case of additional chromaticity sextupoles added in the D.S. for the RFL, even without $\mathbf{b}_3$ correction the dynamic aperture has reached almost the same level as for the nominal optics version 6 with $\mathbf{b}_3$ correction (compare **Case B4** and **A1** respectively). For the modified RFL the $\mathbf{b}_3$ correction is much less effective (compare **Case B4** with **B3** and **B1**). For the nominal optics version 6, we also have placed additional chromaticity sextupoles in the (D.S.). In this case the dynamic aperture becomes smaller without $\mathbf{b}_3$ correction (compare **Case A4** with **A1**) while in the case with $\mathbf{b}_3$ correction the dynamic aperture does not show any improvement (compare **Case A3** with **A1**).

In the light of these dynamic aperture results we now look at the importance of the resonance terms from Table 1. As expected the additional sextupoles improve the dynamic aperture considerably in the case of the RFL. However, the small improvement of dynamic aperture associated with the RFL compared with the nominal one is not apparent from the values of the resonance driving terms, except directly on resonance as shown in tune scans.

These have been produced for the nominal LHC lattice version 6 and the RFL, in both cases with the additional chromaticity sextupoles in the D.S. and without the b_3 spool piece correction, i.e. **Case A4** and **Case B4** respectively. There is some apparent reduction for both cases compared to the situation including the b_3 spool piece correction. However the RFL leads to a gain of about 10% in dynamic aperture far from strong low order resonances as seen on Figure 2. In the third order resonance regions the dynamic aperture ratio depends strongly on tune changes in the range of 0.01 and seems to reflect the difference in driving terms. For the case of the 4^{th} order resonances, the RFL is definitely worse because of the systematic excitation of the $\{$Qx+3Qy$\}$ resonance.

4 CONCLUSIONS

The effect of uncompensated $\mathbf{b}_3$ uncertainty has been studied both for the standard LHC lattice and for the resonance–free lattice. The tune scan shows that stability domains in the tune diagram are still separated by the 3^{rd} and 4^{th} order resonances. However, the dynamic aperture for this lattice is improved, although not very significantly.

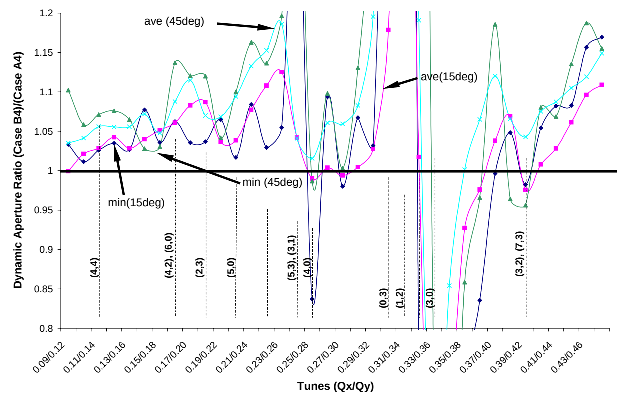

Figure 2: Ratio of short–term dynamic aperture (1,000 turns) (**Case B4**)/(**Case A4**) v.s. tune. Average and minimum ratio over 60 random seeds are given at the phase space angles $\phi = 15°, 45°$ respectively for each pair of tunes, which are separated by 0.03.

Additional chromaticity sextupoles in the D.S. are necessary to cancel the contribution to the 3^{rd} order resonance driving terms from all chromaticity sextupoles in each arc. For this modified resonance–free lattice, the b_3 spool pieces correction system is much less needed, or alternatively it allows for potentially larger systematic $\mathbf{b}_3$ errors.

The effect of random gradient errors depends greatly on their sign. With the nominal systematic gradient errors, the situation is improved.

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