DISCRETE AND CONTINUOUS VELOCITIES WITHIN THE MICHELSON INTERFEROMETER

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Abstract

The trajectories for electromagnetic propagation within the Michelson interferometer may be described by four parameters (space, time, wavelength and phase). We show by a simple scheme that there are discrete solutions for the relative velocity and also that there is a minimum value for the relative velocity that can be measured with the interferometer.

1 INTRODUCTION

The Michelson interferometer has always been important in the study of electromagnetic propagation. It is usually assumed that there are only continuous solutions for the space and time measurements, given by the Lorentz equation. In this work, considering four parameters to define the propagation (space, time, wavelength and phase) we show that discrete solutions may be achieved. In addition, we show that there is always a minimum velocity that can be measured by the interferometer.

2 TIME MEASUREMENT

2.1 Time measurement by light rays

Consider the geometrical scheme shown in Fig.1. **O'** and **O** are at a distance L from a perfect plane mirror and **O'** is moving from right to left with constant velocity v. **A** is the space-point where the coincidence instant position **O** $\equiv$ **O'** in which a simultaneous emanation of light occurred. **B** is the **O'** position after a time interval $\Delta\tau$, measured by **O'**. The light propagation and the space are considered to be isotropic in all directions.

The light signal will be detected by **O** after a minimum interval of time $(\Delta t = 2Lc^{-1})$ that light takes to travel the path **A-C^0** normal to the mirror surface, and back.

The light can reach **O'** at the space-point **B** by different angle reflections at the mirror, as for instance by the rays **A-C^1-B**, **A-C^2-B**, etc, only if **O'** is traveling at a velocity $\mathrm{v} < c$. The intensity of each ray is the same [1]. For higher velocities, i. e. $\mathrm{v} > c$ no light ray will reach **O'**, even the direct ray.

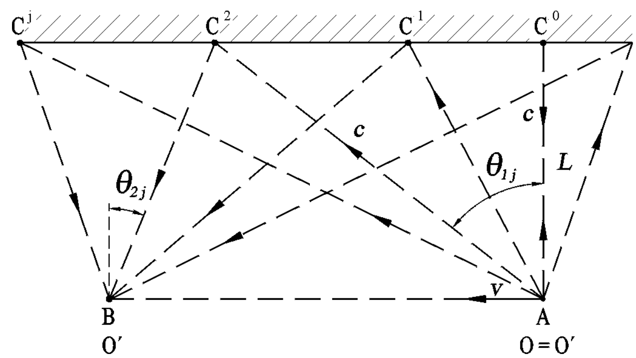

Figure 1: The time delay from the emitted and reflected rays is compared by two observers.

Two equations can be formulated to calculate the interval of time $\Delta\tau$ that light will be detected by **O'** at point **B**. The first equation is obtained by considering the time spent for the light to travel its path by any mirror reflection path **A-C^j-B,**

$$c\Delta\tau = L\left(\frac{1}{\cos\theta_{1j}} + \frac{1}{\cos\theta_{2j}}\right) \qquad (1)$$

where the subscript (j) denotes a particular reflection path given by the angles θ_{1j} and θ_{2j}. The second equation is given by the time that observer **O'** spends in traveling the path **A-B**:

$$\mathrm{v}\Delta\tau = L\left(\tan\theta_{1j} + \tan\theta_{2j}\right) \qquad (2)$$

Thus, combining eqns (1) and (2),

$$\frac{\mathrm{v}}{c} = \frac{\tan\theta_{1j} + \tan\theta_{2j}}{\sec\theta_{1j} + \sec\theta_{2j}} \qquad (3)$$

Fig. 2 shows a parametric plot of this transcendental equation. One can easily see that besides the trivial condition, i.e. $\theta_{1,j} = \theta_{2,j} = \theta$ an infinite number of solutions given by angles $\theta_{1,j} \neq \theta_{2,j}$ are possible.

2.2 *Phaseless time measurement*

Regardless of the phase and with $\theta_{1,j} = \theta_{2,j} = \theta$, we may write from equations (1) and (2)

$$\Delta\tau_Z = \frac{2L}{c\cos\theta} \qquad \text{and} \qquad \theta = \arcsin\frac{\mathrm{v}}{c} \qquad (4)$$

where the subscript (z) is introduced in the above equation to indicate the time measurement independently of the phase ($\Delta\tau_z$). Therefore, the ratio of the time intervals as measured by **O** and **O'** is given by

$$\frac{\Delta\tau_z}{\Delta t} = \frac{1}{\cos\theta} = \sqrt{\frac{1}{1-\left(\frac{\mathrm{v}}{c}\right)^2}} \qquad (5)$$

which is the well-known Lorentz equation.

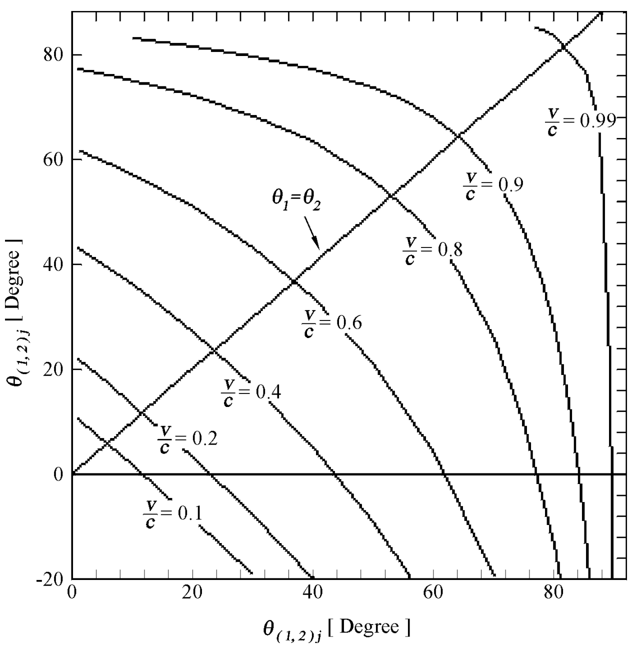

Figure 2: Parametric solution of Eqn (4). Solutions for $\theta_{1,j} = \theta_{2,j}$ (straight line) and $\theta_{1,j} \neq \theta_{2,j}$ (curved lines) are shown.

2.2 *Time and phase measurement*

For the phase measurement, one also needs to assume the condition that the initial phase (ϕ_i) measured by the two observers at the coincidence instant position **O ≡ O'** is the same. As a consequence, the final phase measured by **O** and **O'** (ϕ_f and ϕ_f', respectively) will be determined by the respective path lengths of the light trajectories.

First let us consider only $\phi_f = \phi_f'$ regardless the initial phase ϕ_i. Owing to the difference between the optical path **A-C⁰-A** and any other optical path **A-Cʲ-B**, **O'** will detect the light ray with the same phase of **O** only if

$$L\left(\frac{1}{\cos\theta_{1j}} + \frac{1}{\cos\theta_{2j}}\right) = 2L + m\lambda \qquad (6)$$

$$m = 1,\ 2\ \ldots$$

where λ is the wavelength of light (the $m = 0$ condition is excluded since it leads to the trivial case of $\mathrm{v} = 0$).

Writing the mirror distance in wavelength λ units ($L = k\lambda$, $k \in \mathrm{R}$) equation (6) becomes

$$\left(\frac{1}{\cos\theta_{1j}} + \frac{1}{\cos\theta_{2j}}\right) = 2 + \frac{m}{k} \qquad (7)$$

For "synchronized phase detection" the observer **O'** must be at point **B** at the same instant the reflected light ray reaches him. Combining equations (3) and (7), the angles $(\theta_{1j},\ \theta_{2j})$ are be given by

$$\theta_{1j} = \arccos\left[\alpha + \frac{\mathrm{v}}{c}\sqrt{(\alpha)^2 - \frac{1}{1-\frac{v^2}{c^2}}}\right]^{-1} \qquad (8)$$

and

$$\theta_{2j} = \arccos\left[\alpha - \frac{\mathrm{v}}{c}\sqrt{(\alpha)^2 - \frac{1}{1-\frac{v^2}{c^2}}}\right]^{-1} \qquad (9)$$

where $\alpha = 1 + \frac{m}{2k}$.

The general solution yields two trajectories of equal lengths with angles $(\theta_{1j}, \theta_{2j})$ and $(\theta_{1j+1} = \theta_{2j}\ ,\ \theta_{2j+1} = \theta_{1j})$. The ratio of time intervals measured by **O** and **O'** can be written as

$$\frac{\Delta\tau_P}{\Delta t} = \frac{c}{2\mathrm{v}}\left(\tan\theta_{1j} + \tan\theta_{2j}\right) \qquad (10)$$

where the subscript (P) in $\Delta\tau_P$ indicates a time measurement by **O'** in phase with **O**.

Introducing equations (3) and (7) into equation (10),

$$\frac{\Delta\tau_P}{\Delta t} = 1 + \frac{m}{2k} = \alpha \qquad (11)$$

One may observe that, unlike what happened in equation (5) the mirror distance (k) or the number of wavelengths (m) is not eliminated in this equation.

3 COMPARISON OF PHASE AND PHASELESS MEASUREMENTS

From equations (11) and (8) (Lorentz equation) we have

$$\frac{\Delta\tau_p}{\Delta\tau_z} = \alpha\sqrt{1-\left(\frac{\mathrm{v}}{\mathrm{c}}\right)^2} \qquad (12)$$

The two solutions are coincident only for

$$\frac{\mathrm{v}}{\mathrm{c}} = \sqrt{1-\frac{1}{\alpha^2}} \qquad (13)$$

and for this specific case

$$\theta_1 = \theta_2 = \mathrm{arc\ cos}\,\alpha^{-1} \qquad (14)$$

4 CONCLUSIONS

Since both observers will measure a phase reversion (π) due to the mirror reflection let us consider the phase measurement with $\phi_f = \phi'_f = \phi_i + \pi$. For the static observer **O** the "resonant" condition is attained only when

$$k = \frac{n}{2} \qquad (n = 1, 2, \ldots) \qquad (15)$$

Imposing this condition on equations (13) and (14) one obtains,

$$\frac{\mathrm{v}}{\mathrm{c}} = \sqrt{1-\left(1+\frac{m}{n}\right)^{-2}} \qquad (16)$$

and

$$\theta_1 = \theta_2 = \arccos\left(1+\frac{m}{n}\right)^{-1} \qquad (17)$$

Thus, the possible values of velocities are related to the integers m and n. A diagram of the distribution of the velocities that are solutions of the Lorentz equation and also phase-locked solutions for the two independent observers, is sketched in Fig.3. The discreteness decreases when the mirror-observer distance or the reflection angle is increased (from $\theta = 0^\circ$ to $\theta \to 90^\circ$), which is equivalent to an increase in the m number. It is important to observe that for each condition there is a minimum value of velocity that may be measured, given by m = 1.

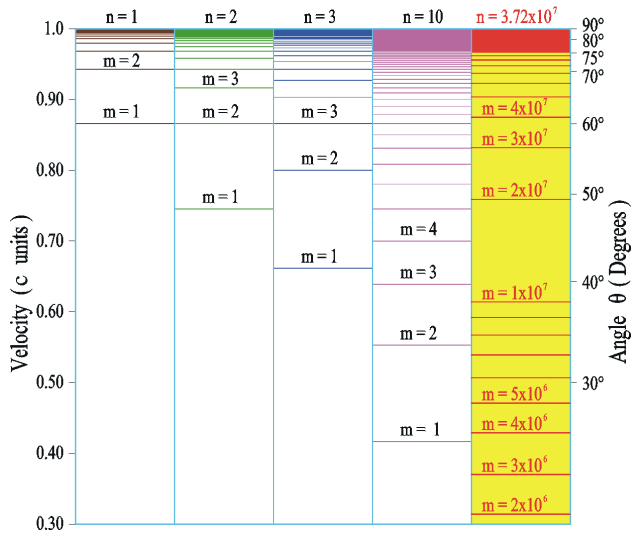

Figure 3: Diagram of the moving observer's O' velocities that are in accordance to the Lorentz equation and to the phase-locked measurements.

The last column of Fig.3 corresponds to the typical experimental parameters used in the Michelson-Morley experiment. The optical path, corresponding to the "mirror to observer distance" was 11 meters and the wavelength, λ = 590 nm [2], so we have $n = 2L\lambda^{-1} = 3.72\times10^7$. This set-up leads to a "quasi-continuum" range of solutions, shown in Fig.3 in yellow. Discrete solutions will be characteristic only of high m values, as is sketched by the red lines.

It is important to emphasize that using these typical parameters, we obtain directly from equation (16) that the minimum detectable velocity is $\mathrm{v} = 2.3\times10^{-4}\,c$, which is a value above of the average value of the Earth velocity around the Sun ($\mathrm{v} = 0.98\times10^{-4}\,c$).

In conclusion, by applying a simple but strong scheme of space, time and phase matching in a Michelson interferometer it was possible to predict its velocity discreteness and to calculate its minimum measurable velocity.

ACKNOWLEDGEMENTS

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[1] R. P. Feynman, "QED - The Strange Theory of Light and Matter", Gradiva 1988.

[2] A. A. Michelson and E. W. Morley, Phil. Mag. (5), 24, 449, 1887.