THE COMPUTER SIMULATION OF THE PARTICLE DYNAMICS IN THE STORAGE RING WITH STRONG COUPLING OF TRANSVERSE MODES

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Abstract

The Low Energy Particle Toroidal Accumulator (LEPTA) with circulating positron beams is proposed for the electron cooling of positrons and generation of antihydrogen and positronium in flight [1]. The peculiarities of the LEPTA are the section structure of its lattice cells, the longitudinal guiding magnetic field and the quadrupole spiral field, which used to form a closed orbit. The longitudinal magnetic field provides both the positron magnetisation and, as a consequence, long lifetime of the circulating positrons. However, it leads to the strong coupling between horizontal and vertical freedom degrees and provokes additional resonances.

The particle dynamics simulation in the LEPTA was performed with especially elaborated computer code BETATRON based on BOLIDE package (Beam Optic Library & Interface Development Environment). The description of the numerical algorithm, examples of simulation results, the structure of BOLIDE package are presented.

1 INTRODUCTION

Linear particle dynamics in the sectional structure can be investigated using matrix method. The 6x6 dimensional transformation map of an optic element can be obtained by analytical solution of the motion equation of the particle, or in the case, when the motion equation has no analytical solution, by integration of motion equation with Runge-Kutt method. The analytically obtained coefficients of the transformation maps of several optic elements of the LEPTA ring are presented in [2]. Now in our calculations of matrixes of optic elements the longitudinal momentum variation is not taken into account. The motion equations are written in the variables

$$\mathbf{X} = \begin{bmatrix} x \\ P_x / P \\ y \\ P_y / P \\ s \\ \Delta P / P \end{bmatrix}, \qquad (1)$$

where x, y are the particle transverse coordinates, s - longitudinal coordinate along the equilibrium orbit, P_x, P_y – corresponding transverse momenta, ΔP - longitudinal momentum deviation, P is the longitudinal momentum of the particle. The variables (1) are conventionally used for tracking procedure, but in our case the use of them for an analytic investigation of the lattice parameters is difficult. The presence of the longitudinal and spiral quadrupole magnetic fields leads to the strong coupling of a particle motion in two transverse directions. Direct investigation of the ring transformation map is impossible because the map is not simplectic for these variables.

Teng's theory [7] permits to consider transverse motion as a projections of two uncoupled modes. The version of this theory is realized in MAD program [4]. However the use of the calculated Edwards-Teng functions is inconvenient due to presence of “break points” [5]. Another problem of Teng's method is that the motion invariants obtained in new-gained phase space have not an obvious physical interpretation.

Preferrable method for solution of this task is the searching for eigenvectors of transformation matrix written for variables canonically conjugated in presence of the longitudinal and spiral quadrupole magnetic fields. This method was realised for real number space [4] and further investigated for complex number space in [6]. The following section briefly describes the version of the last method realised in our calculations.

2 NUMERICAL ALGORITHM

Non-canonical transformation matrix $\mathbf{T}$ is connected to canonical matrix $\mathbf{T^C}$ and particle coordinates $\mathbf{X}$ to canonical ones $\mathbf{X^C}$ by the following expressions:

$$\mathbf{T^C} = \mathbf{M} \cdot \mathbf{T} \cdot \mathbf{M^{-1}}, \ \mathbf{X^C} = \mathbf{M} \cdot \mathbf{X},$$

$$\mathbf{M} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -1/2\rho & 0 \\ 0 & 0 & 1 & 0 \\ 1/2\rho & 0 & 0 & 1 \end{bmatrix} \qquad (2)$$

where index ^{C} means canonically conjugated, ρ is the particle Larmor radius in the longitudinal magnetic field.

On first step of the calculation algorithm the transformation matrix of the whole ring is written as canonical one at initial point of the ring s1 and it's eigenvectors $\mathbf{V}_{k=1..4}$ (s1) are calculated. Then each eigenvector should be normalised by the next expression:

$$\mathbf{Y}_k = -\mathbf{V}_k \sqrt{0.5(\mathbf{V}_k^T \mathbf{S} \mathbf{V}_k^*)}, \qquad (3)$$

where index ^{T} - means transposed matrix, index * - means complex conjugated, $\mathbf{S}$ is 2n*2n generalisation of unit simplectic matrix $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$. From obtained normalised eigenvectors we choose two, which satisfy the condition:

$$\mathbf{Y}_k^T \mathbf{S} \mathbf{Y}_k^* = -2i \,, \qquad (4)$$

The solution of motion equation can be written in the following form:

$$\mathbf{X}^{\mathbf{C}}(s) = 0.5\left(I_1\mathbf{Y}_1(s) + I_2\mathbf{Y}_1^*(s) + I_3\mathbf{Y}_2(s) + I_4\mathbf{Y}_2^*(s)\right), \qquad (5)$$

where $I_2 = I_1^*, I_4 = I_3^*$, here I_1 and I_3 are the invariants of the motion corresponding to two eigenmodes of oscillations. The vector of invariants **I** can be obtained by the formula $\mathbf{I} = \sqrt{2} \cdot \mathbf{Z}^{-1}(s1) \cdot \mathbf{X}^{\mathbf{C}}(s1)$, where **Z**($s$1) is 4x4 matrix formed from chosen eigenvectors in the following form $\mathbf{Z}(s1) = \left[\mathbf{Y}_1 \;\; \mathbf{Y}_1^* \;\; \mathbf{Y}_2 \;\; \mathbf{Y}_2^*\right]$ and $\mathbf{X}^{\mathbf{C}}(s1)$ are the initial particle coordinates.

The lattice functions describing the beam parameters like the Courant-Snyder ones can be calculated from **Z** matrix as following:

$$\begin{aligned} \beta_{m,n} &= \mathbf{Z}_{2m-1,n} \cdot \mathbf{Z}_{2m-1,n} \\ \gamma_{m,n} &= \mathbf{Z}_{2m,n} \cdot \mathbf{Z}_{2m,n} \\ \alpha_{m,n} &= \mathbf{Z}_{2m-1,n} \cdot \mathbf{Z}_{2m,n} \end{aligned} \qquad (6)$$

The index m=1,2 refers to x and y transverse coordinates correspondingly, the index n=1,3 to two eigenmodes. The dependence of **Z** matrix on longitudinal coordinate is found from matrix **T** transforming the particle co-ordinates from initial point s1 to current point s2 :

$$\mathbf{X}(s2) = \mathbf{T}(s2,s1)\cdot\mathbf{X}(s1),\ \mathbf{Z}(s2) = \mathbf{T}^{\mathbf{C}}(s2,s1)\cdot\mathbf{Z}(s1) \qquad (7)$$

The complex motion invariant I can be presented in the form: $I = |I|e^{i\varphi}$, the $|I|$ is proportional to the volume value of the four dimensional ellipsoid formed by the particle phase trajectory and φ is the initial phase of the particle oscillations. The ellipsoid parameters can be described using matrix **A,** which is calculated from the motion invariant values and normalised matrices corresponding to each mode of oscillations by the formulae:

$$\begin{aligned} \mathbf{A} &= \left\{\mathbf{A}^1 \cdot (I_1 \cdot I_2) + \mathbf{A}^3 \cdot (I_3 \cdot I_4)\right\} \cdot 2, \\ \mathbf{A}^n &= \sum_{i,j=1}^{4} \left\{\mathrm{Re}(\mathbf{Z}_{i,n}) \cdot \mathrm{Re}(\mathbf{Z}_{j,n}) + \mathrm{Im}(\mathbf{Z}_{i,n}) \cdot \mathrm{Im}(\mathbf{Z}_{j,n})\right\}, \end{aligned} \qquad (8)$$

The coefficients of the matrix **A**

$$\mathbf{A} = \begin{bmatrix} <xx> & <xP_x^{\mathbf{C}}> & <xy> & <P_y^{\mathbf{C}}x> \\ <P_x^{\mathbf{C}}x> & <P_x^{\mathbf{C}}P_x^{\mathbf{C}}> & <P_x^{\mathbf{C}}y> & <P_x^{\mathbf{C}}P_y^{\mathbf{C}}> \\ <yx> & <yP_x^{\mathbf{C}}> & <yy> & <yP_y^{\mathbf{C}}> \\ <P_y^{\mathbf{C}}x> & <P_y^{\mathbf{C}}P_x^{\mathbf{C}}> & <P_y^{\mathbf{C}}y> & <P_y^{\mathbf{C}}P_y^{\mathbf{C}}> \end{bmatrix}$$

are equal to the mean square of the half-axis of the ellipsoid projection onto corresponding direction. For example $\mathbf{A}_{1,1}$ and $\mathbf{A}_{3,3}$ describe the size of the ellipsoid in (x, y) plane.

The mathematics roach described here gives us possibility to investigate nonlinear particle dynamics for strong coupling of transverse modes using Hamilton formalism. For this purpose the particle Hamiltonian is to be rewritten in the variables $|I_1|$, φ_1, $|I_3|$, φ_3, that are equivalent to variables "action" – "phase" for uncoupled motion. In this form of the motion equation the width and power of non-linear resonances can be investigated using standard method of overaging. The part of the program dedicated to non-linear dynamics calculation is under development now.

3 CALCULATION USING THE COMPUTER CODE "BETATRON"

The BETATRON code elaborated for particle dynamics calculation provides the following functions:

- input the optic structure of the ring;
- calculation of the transformation matrices for all optic elements;
- single and multy particles tracking in the variables (1);
- calculation of **Z** and **A** matrices in each point of the ring (Fig.1 shows the phase trajectory of the particle obtained by tracking and ellipse of parameters calculated using **A** matrices);
- the following lattice functions for strong-coupled motion are calculated: horizontal D_x and vertical D_y dispersions (Fig.2), α, β - function for each plane and each invariant (Fig.3);
- the stability diagram is calculated as dependence of the ring transformation map eigenvalues on particle energy and gradient of the quadrupole spiral field [3].

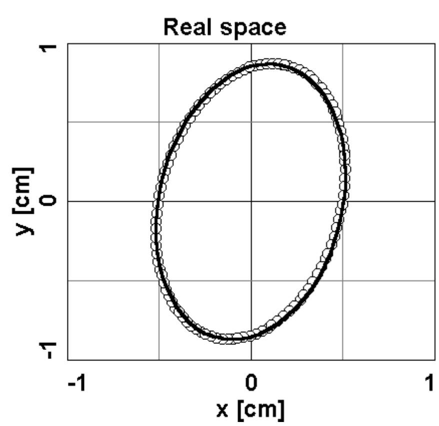

Figure 1. The particle trajectory in the transverse plan of real space at the initial point of storage ring (s=0). Circle points are the result of tracking, solid line - phase ellipse calculated using **A** matrices.

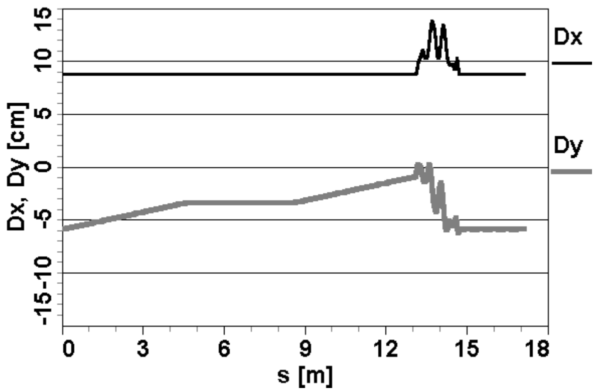

Figure 2. Horizontal and vertical dispersion functions.

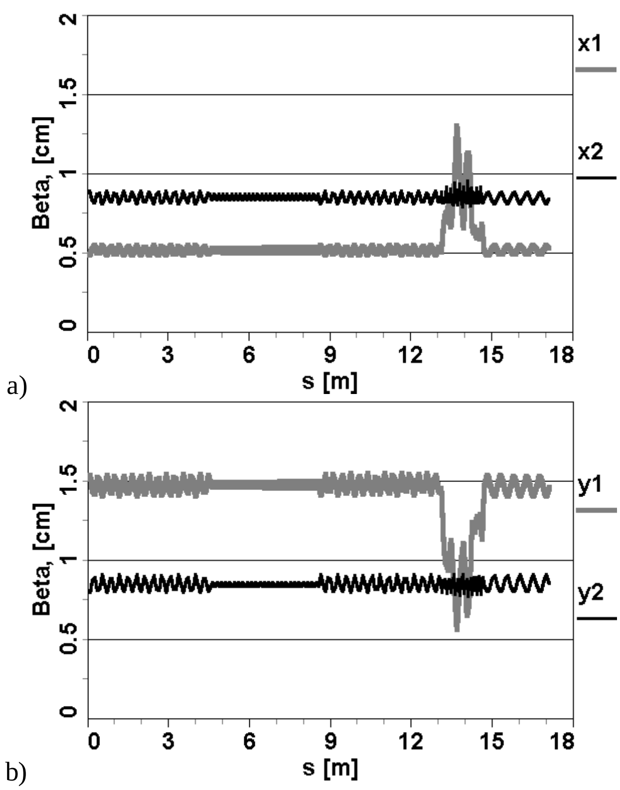

Figure 3. β_x (a) and β_y (b) functions for two invariants of motion.

4 PROGRAM STRUCTURE AND INTERFACE

BETATRON code has the friendly interface to introduce different optical structures of storage ring, to set the particle parameters and to calculate particle dynamics, lattice functions, beam stability diagrams. Optical structure of the storage ring is visualised as tree-type hierarchy. Every node of this tree contains multiplication of optical element's matrixes of node's branch. That scheme allows fast recalculation of whole transformation matrix, if parameters of some elements are changed, i.e. only edited branch will be recalculated.

BETATRON program based on BOLIDE [8] system, which is the kit of program modules developed for quick creation of new applications in Borland C++ Builder (Delphi) for physical and mathematical calculations. BOLIDE includes the following VCL modules: 2d- and 3d- graphs, management by parallel multithread processes and data arrays.

Discriminating particularity of given package is the way of visualisation of data on screen. For data output the special separate process is used, which is continuously working in program controlled by timer events. Calculation processes do not perform directly any data output on screen by themselves. This scheme allows to avoid the losing of the interface control and decreasing of calculation speed during the visualisation of massive data arrays.

The mathematical part of application based on BOLIDE is created in ANSI C++ standard. For connection between interface part and mathematical code special kit of macro commands was elaborated. For programmers this structure allows easy to use their own mathematical algorithms without any changes when another compilers or operation systems are used.

ACKNOWLEGEMENTS

This work is supported by RFBR grant #99-02-17716.

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