

APPLICATION OF FREQUENCY MAP ANALYSIS TO THE STUDY OF THE SOLEIL AND ESRF BEAM DYNAMICS*

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Abstract

The Frequency Map Analysis (FMA) [1] is a refined numerical method based on Fourier techniques which provides a global view of the dynamics of multi-dimensional systems and which was successfully applied to accelerator dynamics starting with the Dumas and Laskar work [2].

Two third generation synchrotron light sources, the SOLEIL Project and the ESRF are studied with this method. A full computation of the dynamic aperture (DA) is performed with a discussion of its inner complex structure (resonances) in correlation with its associated frequency map (FM) through diffusion (long term stability criterion). We also underline that despite the large DAs of both machines, the FM shapes are fully distinct due to the high sensitivity of the dynamics to the sextupole strengths.

1 INTRODUCTION

The lattices of SOLEIL [3] and the ESRF [4] are built up of strong focusing quadrupoles generating large chromatic aberrations. Sextupole magnets enable to compensate the chromaticity but induce geometric and nonlinear chromatic aberrations exciting resonances that may lead to unstable motions. To improve the performance of such light sources, accelerator dynamicists try to reduce the resonance influence but unfortunately the prediction of resonance strengths is a difficult task.

Here after a brief description of both light sources, frequency maps and dynamic apertures are computed. The principal resonances are revealed, a one-to-one correspondence between the configuration space and the frequency space is performed. We show that the shape of a FM is highly sensitive to sextupole strengths and very different from one machine to another. Moreover time variations of the betatron tunes give some additional information for the global dynamics of the beam.

2 METHOD

2.1 Frequency map analysis

The study of the global dynamics of a beam is realized with the numerical frequency map analysis [1][5]. The transverse dynamics is modeled by a 2+1 degrees of freedom system, the tunes are normalized by the revolution frequency. Resonances appear for integer linear combinations of the fundamental tunes: $p\nu_x + q\nu_y + kM = 0$ where ν_x

and ν_y are the transverse tunes and M is the inner periodicity of the ring ¹. $|p| + |q|$ is called the resonance order.

FMA constructs the so-called *frequency map* $F^T : (x, y) \rightarrow (\nu_x, \nu_y)$ from the space of initial conditions to the tune space over a finite time span T by searching for a quasiperiodic approximation of the transverse motion. The main properties of the numerical frequency map F^T are:

- independent of the initial momenta (x'_0, y'_0)
- fast convergence in $\frac{1}{T^4}$ (with a Hanning window)
- invariant by time translation for a regular solution (KAM solution): otherwise the time variation of the tunes called *orbit diffusion* gives a stability criterion of the trajectory
- On a set of KAM trajectories, F^T is a regular function. The study of the regularity of this map gives information about resonances and nonlinear behaviors.

2.2 Map construction

Throughout this work only the transverse dynamics is taken into account which is justified by the very low longitudinal frequency ($\nu_s^{soleil} = 0.006$ whereas $\nu_x^{soleil} = 18.28$).

To study such a system we use a surface of section i.e. at a given longitudinal position (typically $s = 0$) we look at the *return map*. The coordinates used are the canonical transverse positions (x, y) and momenta (x', y') . Given a set of initial conditions $(x_0, y_0, x'_0 = y'_0 = 0)$ the particle trajectory is numerically computed over 2000 turns ². For a surviving particle, we plot it in the configuration space defining the dynamic aperture and we compute its transverse tunes with the FMA over the first 1000 turns and then again over the last 1000 turns. The logarithmic tune difference gives a *diffusion index* [2] coded by a color from dark for very stable orbits to light color for very unstable ones. Due to the fast convergence of the method, the diffusion is a good long-term stability criterion.

3 THE SOLEIL PROJECT

3.1 Lattice

The SOLEIL cell is based on a modified Chasman-Green structure [6]. Table 1 sums up the main characteristics of the storage ring and the lattice functions are given by Fig. 1. The SOLEIL lattice is supposed perfect, with a full 4-fold periodicity and zero chromaticities. Moreover the particles are on momentum for this work.

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¹4-fold periodicity for SOLEIL and 16-fold for the ESRF

²the damping time is circa 4000 turns for SOLEIL

Table 1: SOLEIL main parameters

Energy (GeV)	2.5
Energy spread σ_E	$9.24 \cdot 10^{-4}$
Circumference (m)	337
Emittance ϵ_x (nm.rad)	3
Working point ν_x, ν_y	18.28, 8.38

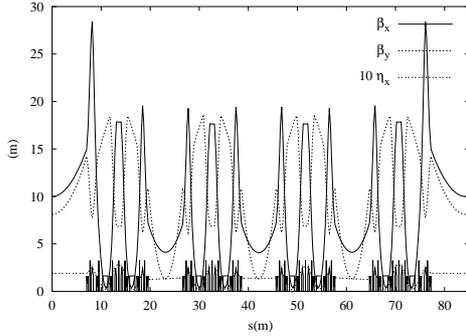


Figure 1: Optical functions for a SOLEIL superperiod.

3.2 Dynamics

Following the preceding construction scheme, the DA and the FM for the standard SOLEIL optics are computed using DESPOT [7] as tracking code.

As the first nonlinear contribution of the amplitude to the frequency tune shift is quadratic, a square root step is chosen for the initial conditions. Moreover we have a more precise determination of the dynamics around the borders of the DA i.e. in the vicinity of the resonances limiting the stability area of the transverse motion. In this way an *exhaustive* computation of the dynamic aperture and of its *inner structure* are obtained.

We notice a large dynamic aperture (Fig. 2-b): $x_{max} = 40mm$, $y_{max} = 24mm$ but these dimensions have to be slightly reduced to 35mm in x (9th-order resonance island) and 20mm in y .

On the FM (Fig. 2-a), 3 kinds of areas can be identified:

- *regular areas* where tune space points are regularly spaced with very low diffusion (dark color). The motion is quasiperiodic.
- *straight lines* with rational slope which are resonance lines. In their neighborhood there is either a lack of points if the resonance is crossed through a hyperbolic point or an accumulation of points for the elliptic case.
- *irregular areas* where all structure is lost with high diffusion (light color): particles will be either lost or their motion may lead to chaotic behaviors.

The FM is rather clean except at high amplitudes where the dynamics is dominated by several resonances: the 7th-order coupled resonance $5\nu_x + 2\nu_y - 4 \times 27 = 0$ ($x \approx 25mm$) and the resonance node between a 7th-order $3\nu_x +$

$4\nu_y - 4 \times 22 = 0$ and the a 9th-order $9\nu_x - 41 \times 4 = 0$ which seem the most dangerous for the dynamics.

In addition we may notice that the frequency map is folded a bit just around the working point and at high amplitude which may induce other complicated dynamical features.

By using orbit diffusion, a *one-to-one correspondence* between a point of the dynamic aperture and a point of the tune space is obtained.

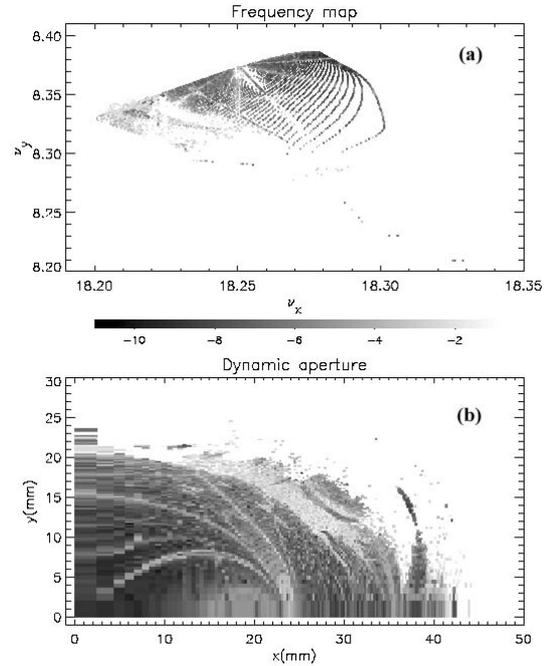


Figure 2: SOLEIL frequency map (a) and dynamic aperture (b) at $s = 0$, $\beta_x = 10m$ and $\beta_y = 8m$.

4 THE ESRF

4.1 Lattice

Table 2: ESRF main parameters

Energy (GeV)	6
Energy spread σ_E	$1 \cdot 10^{-4}$
Emittance ϵ_x (nm.rad)	4
Chromaticities $\xi_k = (\frac{d\nu_k/\nu_k}{dp/p_0})_{k=x,y}$	0.1, 0.4
Working point ν_x, ν_y	36.44, 14.39

The ESRF lattice has a 16-fold periodicity [9]. The storage ring lattice is a Chasman-Green structure with distributed dispersion, alternating low and high beta straight sections. The operation of the machine requires a significant over-compensation of the chromaticity, as indicated in Table 2 for multibunch mode. The lattice functions are

given by Fig. 3. The tracking was performed with the MAD program [8] for on momentum particles.

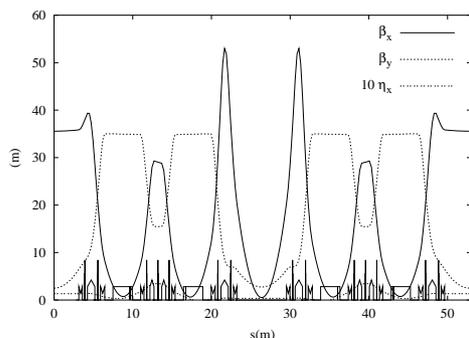


Figure 3: ESRF optical functions

4.2 Dynamics

The ESRF FM (Fig. 4-a) is more complicated than the SOLEIL one. The associated DA (Fig. 4-b) is very large but once again these dimensions are overestimated: the DA is full of resonance islands. Actually the map can be divided into two parts:

A first part around the working point extending up to -20mm in x and 7mm in y with low diffusion, nearly free of resonances except the 5th-order resonance ($3\nu_x - 2\nu_y - 5 \times 16 = 0$) reached at $x \approx -20\text{mm}$ inducing a large resonance island in the DA.

On the second part, the FM changes of signature. In addition some regular areas last but many resonances appear: the integer resonance $\nu_x - 36 = 0$ is reached at $x = -27\text{mm}$, in fact all particles between -40mm and -30mm in x are captured in the resonance island. Beyond, resonances from low order (4,5) to high order (8,...,21) are encountered and the global stability is spoiled with great amplitude tune shifts ($\Delta\nu_x \approx 0.5$, $\Delta\nu_y \approx 0.15$) and high diffusion.

Even if for a perfect lattice some orbits are stable beyond the integer resonance, dipolar defaults will excite it and the dynamic aperture would be reduced by a factor 2.

5 CONCLUSION

We want to emphasize that too often DA dimensions presented are too large because tune shifts are usually computed without coupling (e.g. $\nu_x = f(x)|_{y=0}$) whereas when y dependence is taken into account, sextupole coupling destroys the dynamics mainly at high amplitude (excitation of coupling resonances).

Frequency map analysis gives a footprint of the beam. The global dynamics is revealed in a way often neglected by standard lattice optimization code. Besides very slight modification in the sextupolar strengths induces large modifications of the shape of the frequency map and the global stability. For similar dynamic apertures, the frequency maps are very different.

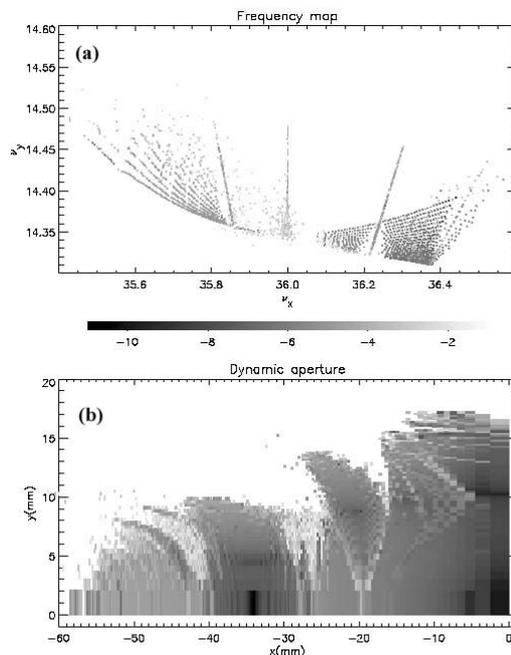


Figure 4: ESRF frequency map (a) and dynamic aperture (b) at $s = 0$, $\beta_x = 36\text{m}$, $\beta_y = 2.5\text{m}$

To end these results are too much optimistic since we only studied perfect lattices. With measured defaults stability area is reduced often by a factor 2 (see for instance the ALS case [10], and the first experimental FM [11]).

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