SURVEY MESHES FOR SYNCHROTRON MAGNET ALIGNMENT

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Abstract

For the precise alignment of the synchrotron magnets several survey meshes based on the triangulation method for quadrupoles are investigated. Errors accompanied by the measurements are processed as an eigenvalue problem to solve the matrix equations which depend on the survey meshes. An analyzed spatial distribution of the magnet misalignments including both the real and pseudo displacements can be decomposed into many eigenmodes. Relations between displacement modes and the rms COD are discussed from the viewpoint of the beam dynamical aspect.

1 INTRODUCTION

The synchrotron magnets must be aligned precisely on the designed orbit to ensure the stable beam acceleration. After an early installation of magnets is made in a few mm accuracy depending on the ground survey, the precise alignment of 0.1 mm accuracy follows using special tools such as distinvar, distometer, offset measuring device and etc. Recently a laser tracker is going to take a place of these tools. It tracks light of an incorporated laser interferometer measuring both the angle and the distance between the home position and the cat's eye reflector. Using either the former tools or the latter laser tracker, are obtained the survey data suitable for the reduction of the magnet mis-alignment.

Several survey meshes are introduced at the different laboratories. However, the survey mesh shall be selected what kind of survey data is useful to determine the magnet position. The ring structure composed of the different magnet lattices will affect on the survey mesh. In the determination of the magnet positions the quads are most important because their mis-alignments give the direct deflections to the beam.

In a large synchrotron most part of the ring is composed of the same lattice except for the experimental insertions. This kind of synchrotron is classified here as an asymmetric ring of which shape is different from the circle. To the contrary the ring which holds the circle is classified as a symmetric ring. The degree of the asymmetry is continuous from the true circular ring to the highly asymmetric ring. The present work treats three kinds of the survey meshes for the different degrees of asymmetry.

2 SURVEY PROBLEM

If N quads make a whole ring and their N positioning target points are on the designed orbit, at least two survey variables S_i and P_i $(i = 1, 2, ..., N)$ are necessary to each quad. These variables are expressed as a function of R_i and Θ_i.

$$S_i = F(R_i, \Theta_i) \text{ and } P_i = G(R_i, \Theta_i), \tag{1}$$

where the subscript $\boldsymbol{i}$ of R and Θ is considered for all possible range. For an example, if S_i is a distance between the adjacent quads, $S_i = F(R_i, R_{i+1}, \Theta_i)$. Differentiating the above relations to the first order,

$$\begin{aligned} s_i \equiv \Delta S_i &= \sum_i \frac{\partial F}{\partial R_i}\Delta R_i + \sum_i \frac{\partial F}{\partial \Theta_i}\Delta\Theta_i \\ &= \sum_i \frac{\partial F}{\partial R_i} r_i + \sum_i \frac{\partial F}{\partial \Theta_i}(\theta_{i+1} - \theta_i) \end{aligned} \tag{2}$$

and

$$\begin{aligned} p_i \equiv \Delta P_i &= \sum_i \frac{\partial G}{\partial R_i}\Delta R_i + \sum_i \frac{\partial G}{\partial \Theta_i}\Delta\Theta_i \\ &= \sum_i \frac{\partial G}{\partial R_i} r_i + \sum_i \frac{\partial G}{\partial \Theta_i}(\theta_{i+1} - \theta_i) \end{aligned}, \tag{3}$$

where (s_i and p_i) are the deviations from the design which are determined from the precise survey and (r_i and θ_i) the quad displacements to the radial and azimuthal direction, respectively. Deriving relations (2) and (3) for all quads,

$$\begin{pmatrix} p_1 \\ : \\ p_N \\ s_1 \\ : \\ s_N \end{pmatrix} = \begin{pmatrix} a_{11} & \cdot\cdot & a_{1N} & c_{11} & \cdot\cdot & c_{1N} \\ : & & : & : & & : \\ a_{N1} & \cdot\cdot & a_{NN} & c_{N1} & \cdot\cdot & c_{NN} \\ b_{11} & \cdot\cdot & b_{1N} & d_{11} & \cdot\cdot & d_{1N} \\ : & & : & : & & : \\ b_{N1} & \cdot\cdot & b_{NN} & d_{N1} & \cdot\cdot & d_{NN} \end{pmatrix} \begin{pmatrix} r_1 \\ : \\ r_N \\ \theta_1 \\ : \\ \theta_N \end{pmatrix}, \tag{4}$$

where the 2Nx2N matrix is obtained for the above relations. If 3 variables (S_i, P_i and Q_i) are considered, the matrix becomes 3Nx2N and the left column vector has 3N components. In any case the matrix equation (4) is expressed simply as

$$\mathbf{p} = (A)\mathbf{r} \tag{5}$$

and solved by the least squares method as

$$\mathbf{h} \equiv (A^*)\mathbf{p} = (A^*A)\mathbf{r}, \tag{6}$$

where A^* is the transposed matrix of A and A^*A the 2Nx2N matrix. The solution is given by

$$\mathbf{r} = (A^*A)^{-1}\mathbf{h}, \tag{7}$$

if the matrix is not singular. Otherwise, the rank reduction must be considered as is often the case.

From the relation (6) the precise magnet alignment problem can be treated as an eigenvalue problem in which the individual eigenmode is superposed to form a whole magnet mis-alignment structure.

Replacing the matrix A^*A with H, the eigenvalue λ_i is given by the following equation,

$$(H)\mathbf{v}_i = \lambda_i \mathbf{v}_i, \tag{8}$$

where $\mathbf{v}_i$ is the eigenvector belonging to the eigenvalue λ_i. The magnet displacements r_i and θ_i consist of a linear combination of the eigenvectors as follows,

$$\mathbf{r} = \sum_k c_k \mathbf{v}_k, \tag{9}$$

where c_k is the coefficient. From (6), (8) and (9),

$$\sum_k (H) c_k \mathbf{v}_k = \mathbf{h}. \tag{10}$$

Therefore, the linear coefficient of (9) is

$$c_k = (\mathbf{v}_k^T \mathbf{h}) / \lambda_k, \tag{11}$$

and the j-th component of the displacement vector $\mathbf{r}$ is

$$r_j = \sum_k \frac{(\mathbf{v}_k^T \mathbf{h})}{\lambda_k} v_{kj}, \tag{12}$$

where v_{kj} is the j-th component of the eigenvector $\mathbf{v}_k$ [1].

3 SURVEY MESHES

Three meshes which are treated here are depend on the survey variables,

(Case#1) short chord and perpendicular,
(Case#2) long chord and perpendicular, and
(Case#3) short chord and 2 perpendiculars,

as shown in Fig.1.

The matrix equations for 3 cases are derived for the ideal symmetric ring and the asymmetric ring TRISTAN, which has been converted to KEK-B factory, assuming that both rings have the same diameter and the same number of quads (392 quads). The eigenvectors for the largest and smallest eigenvalues are shown for the asymmetric 3 cases in Fig.2 where each eigenvector is plotted to the order of the quad arrangements and a left (right) half corresponds to the radial (azimuthal) displacement. The symmetric 3 cases for the same number of magnets are also shown in Fig.3.

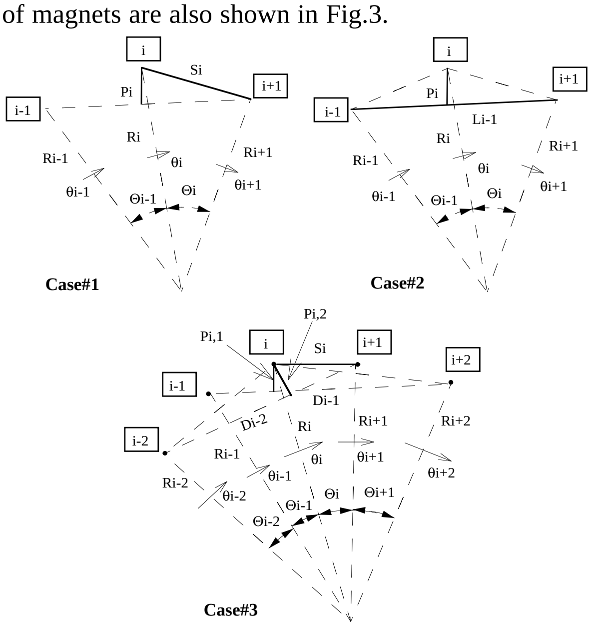

Figure 1: Survey variables for 3 cases

The characteristic patterns of displacements are obtained. If the lattice symmetry is lost as in the case of a colliding machine, the radial and azimuthal displacements can be treated separately for larger eigenvalues but both displacements are combined each other for lower ones. Whereas in the symmetric quad arrangement, the case#1 shows the strong coupling between the radial and azimuthal displacements for larger eigenvalues. The case#2 has less independent modes than the others by one as shown in Fig.2 and Fig.3 because this case has 4 variables to be pre-fixed to define the global coordinate on which all other variables are mapped.

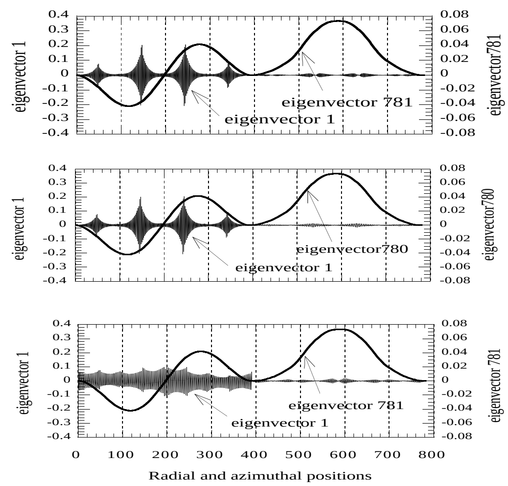

Figure 2: Asymmetric cases, the eigenvectors for the largest (numbered as 1) and smallest (as 781 or 780) eigenvalues. Left and right half correspond to the radial and azimuthal displacement, respectively, to the order of quad arrangements. Case#1: upper, Case#2: middle, and Case#3: lower

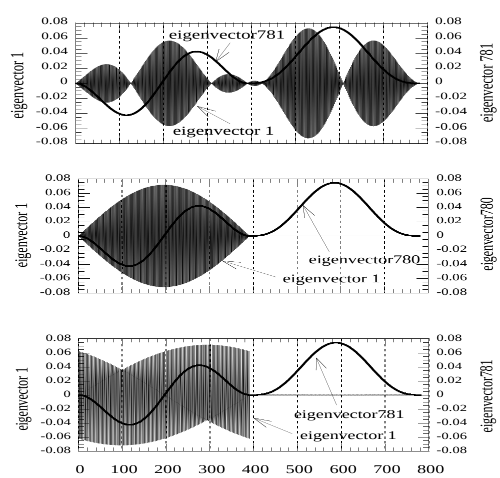

Figure 3: Symmetric cases, the figure caption is same as in Fig.2. Case#1: upper, Case#2: middle, and Case#3: lower

For lower eigenvalues every case has the same eigen-vectors. The lowest one gives one sinusoidal variations along the synchrotron ring to both radial and azimuthal directions. If the eigenvalue increases, the number of oscillations increases. These behaviors are obtained without considering the particle beam motion in the ring. As each eigenvector gives the pattern of the magnet mis-alignment, if both oscillation modes become similar the magnet alignment error will give the beam the forced oscillation and the beam will be lost in result.

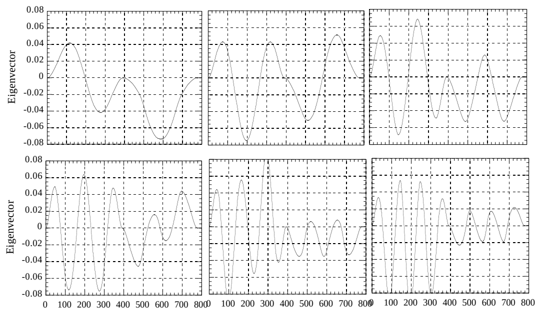

Figure 4: Eigenvectors for lowest 6 eigenvalues

The relation (9) or (12) means that the contribution of eigenvectors which are less significant to the beam motion may be omitted from the solution. In general, the smaller the eigenvalue the larger it contributes to the magnet displacement as given by (12) and it is desirable to neglect the eigenvectors for small eigenvalues. Fig.5 shows the simulation results of the case#1 with and without considering the eigenvector elimination for the same input data. In this example smallest 10 eigenvectors are eliminated for the dotted line. This kind of displacement appears in the solution as pseudo-components which have relatively large coefficients of (11) for smaller eigenvalues.

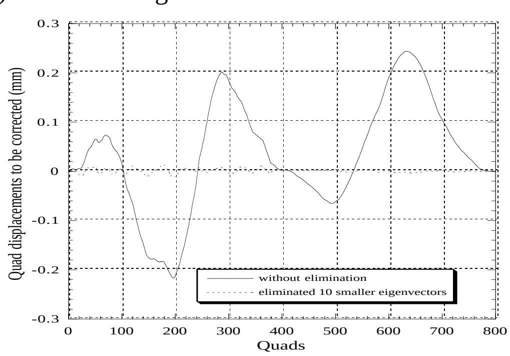

Figure 5: Simulation results of case#1 with and without considering the eigenvector elimination

4 EFFECT OF EACH EIGENVECTOR ON THE CLOSED ORBIT

The quad mis-alignment effect on the closed orbit can be treated for each eigenmode in the linear motion regime. It can be converted directly to the mis-alignment of the individual quad and its effect is estimated by the beam simulation code. An example for the 700-th eigenvalue for the asymmetric case#1 is given in Fig.6 where the deviations of the perpendicular and short chord length are given by the corresponding eigenvector and the radial and azimuthal positional errors are calculated.

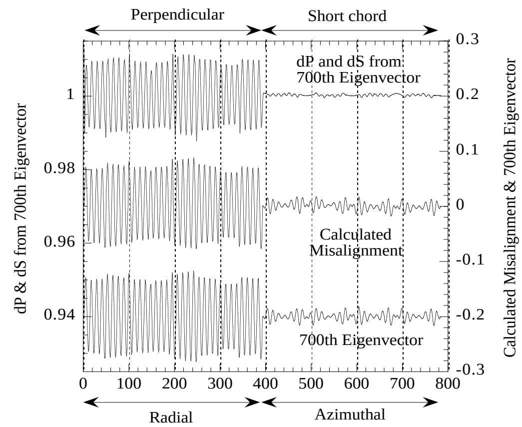

Figure 6: The 700-th eigenvector and misalignment obtained from deviations of the perpendicular and short chord lengths when the eigenvector for the 700-th eigenvalue is assumed as a misalignment mode

As the deviations of survey variables for each eigenvalue reflect the corresponding eigenvector as shown in Fig.6, each eigenmode can be treated as a quad displacement mode separately in the beam simulation code [2]. The rms radial displacement error is adjusted as 0.1 mm for the fair evaluation of the individual eigenmode. The resultant rms COD is given in Fig.7 for all eigenmodes of the asymmetric case#1. As contributions by small eigenvalue modes are significantly low, these modes may be neglected for the magnet re-alignment.

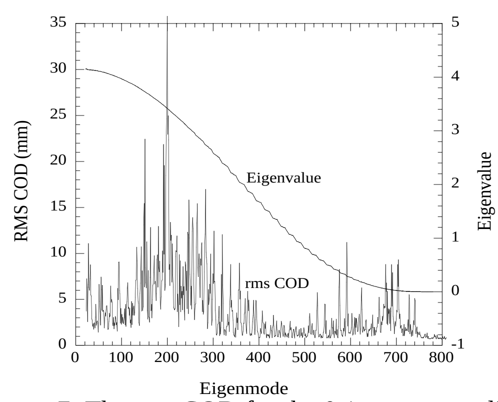

Figure 7: The rms COD for the 0.1 mm rms radial eigenmode offset of quads for the asymmetric case#1

REFERENCES

[1] J.H. Wilkinson, “The Algebraic Eigenvalue Problem,” Oxford Univ. Press, 1965.

[2] A.S. King, M.J. Lee and W.W. Lee, “MAGIC, A Computer Code for Design Studies of Insertions and Storage Ring,” SLAC-183, 1975. It was modified greatly to apply for the COD calculation due to the quad displacements.