

# OPTIMIZING MULTITURN INJECTION WITH SPACE CHARGE AND LINEAR COUPLING

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## Abstract

We present an optimization procedure using the MIMAC-code for multiturn injection into the SIS including injection optics, septum, fully self-consistent 2D space charge calculation as well as skew quadrupoles. Linear coupling is shown to lead to approximately 20% higher maximum efficiency by using also the vertical acceptance. The most effective use of linear coupling is obtained for optimization towards minimum loss including an injection delay at the beginning of the orbit bump, which brings the efficiency close to the design goal at very low loss level, as required for the future GSI RIB facility. We thus predict by simulation that we can store  $1.7 \cdot 10^{11}$  of  $U^{28+}$  with a septum loss of only 6% (12% including full space charge, but not yet fine-optimized working point) compared with the usual losses of about 30%.

## 1 INTRODUCTION

In the context of a future Radioactive Beam Facility at GSI the generation of up to  $10^{12}$  uranium ions per second requires a substantial reduction of injection losses. A promising method for the SIS as booster synchrotron is the use of linear coupling, which was studied first at the CERN PS booster [1]. For high intensity beams ( $\sim 4 \cdot 10^{12}$  p.p.p. injected) the injection efficiency increased, for a particular setting of the machine parameters, by  $\sim 15\% \dots 20\%$ . These PS booster results are influenced favourably by the relatively large vertical machine acceptance ( $A_x = 220\pi$  mm mrad,  $A_y = 95\pi$  mm mrad). Experimental investigation of linear coupling theory for small emittance linac beam has also been performed at the AGS Booster [2] where turn-by-turn measurements were used to study the coupled injection scheme. In our study we apply the optimization procedure to the SIS high current synchrotron in order to fill its acceptance, which is vertically relatively small. In horizontal multiturn injection with closed orbit bump the main loss occurs on the septum, depending on the number of turns  $N_t$  used for injection, and the horizontal tune  $q_{x0}$ . Linear coupling by a skew quadrupole excites a difference resonance and transfers the horizontal amplitude into the vertical plane, which helps to move particles faster away from the septum. The amount and velocity of emittance exchange depends on the distance  $\delta = q_{x0} - q_{y0} - n$  of the working point from the difference resonance. The possible reduction of horizontal emittance is limited by the available vertical acceptance  $A_y$ , which determines the allowed skew strength for given  $\delta$ . Here it is assumed that the skew quadrupoles are practically constant over the injection time, and are turned off only on a

slow time scale. A schematic evolution of the effect of linear coupling on beamlet emittances (ignoring the septum and space charge) is shown in Fig. 1 relatively far from the resonance ( $\delta = -0.37$  in a), b)) and close to the resonance ( $\delta = -0.01$  in c), d)). In both examples the skew strength was determined by the condition to limit the vertical emittance by the vertical acceptance ( $A_y = 50$  mm mrad). The helpful faster exchange with larger distance from the resonance is obvious. This condition can be written as

$$\delta \cdot N_{ex} = \sqrt{\epsilon_{x,0}/A_y - 1} \quad (1)$$

where  $\epsilon_{x,0}$  is the single particle emittance of the last beamlet injected, and  $N_{ex}$  the skew strength expressed in units of the number of turns per emittance oscillation assuming  $\delta = 0$ . We will use this relation to set the skew strength, where the integrated skew strength  $J$  is given by  $J = 2\pi/(N_{ex} \sqrt{\beta_x \beta_y})$ .

## 2 OPTIMIZATION PROCEDURE FOR SIS

The linac beam has unnormalized (KV-) emittances  $\epsilon_x = \epsilon_y = 4\pi$  mm mrad,  $I = 10$  mA,  $Z = 28$ ,  $A = 238$  at energy 11.4 MeV/u. In the simulation a new beamlet of macroparticles is added at each passing near the septum. All particles satisfying the condition  $x > x_{sept.} = 59.5$  mm are scraped, and the acceptance is simulated by removing all particles with Courant Snyder emittance (with respect to the bumped closed orbit) bigger than  $A_x = 200\pi$  mm mrad or  $A_y = 50\pi$  mm mrad. The horizontal acceptance limits the useful injection to 27 turns. The simulation has been performed with the MIMAC-code, which is a library that allows 2D symplectic tracking of beams including lattice nonlinearities. The symplecticity is preserved by using the micromap technique [3]. The self-consistent 2D space charge calculation is performed with a FFT based Poisson solver [4] using a rectangular beam pipe. Performing tests on the accuracy of the space charge calculation with a KV beam of 20000 particles,  $\epsilon_x = 200\pi$  mm mrad,  $\epsilon_y = 50\pi$  mm mrad,  $I = 160$  mA, a  $64 \times 64$  grid over a boundary box with size of 0.477 m, and with 20 timesteps per turn we found beamlets of 500 macroparticles should give sufficient accuracy of the space charge calculation. We first investigate the influence of the horizontal tune  $q_{x0}$  on the efficiency when space charge and linear coupling are neglected. A scan of  $q_{x0}$  keeping  $q_{y0} = 3.29$  (Fig. 1e) shows the efficiency defined as ratio of particles inside the ring to injected particles. The minima are related to the resonance condition  $q_{x0} n = m$ . The maximum efficiency

is  $\sim 63\%$  at the tunes  $q_{x0} = 4.23, 4.29$ . The dependence on  $q_{y0}$  so far is weak. Including the skew quadrupole and taking  $q_{x0} = 4.29$  we now explore  $q_{y0} > 3.29$ . Taking the skew strength as defined by Eq. 1 the efficiency of multiturn injection first increases with  $\delta$  as the skew moves the beamlet away from the septum ( Fig. 1f). This figure can be interpreted in the following way: For  $q_{x0} = 4.29$  a beamlet reaches the septum (one turn in the phase space) in  $\sim 7$  turns, hence the maximum benefit of the linear coupling is expected if the emittance is exchanged in 7 turns. Since the number of turns needed for the maximum emittance exchange is  $0.5\sqrt{1 - A_y/\epsilon_{x0}}/\delta$ , we expect the maximum effect to happen for  $\delta = 0.061$ , i.e. for  $q_{x0} = 3.35$ , which is confirmed by the simulation result of Fig. 1f. For bigger values of  $\delta$  the exchange is too fast and the efficiency drops. Since the emittance exchange is proportional to the single particle emittance of the injected particles (which increases during injection with decreasing orbit bump), we expect that a significant reduction of beam loss is obtained by delaying the injection with respect to the orbit bump. We have simulated the delayed injection for  $q_{x0} = 4.29, q_{y0} = 3.35$  in Fig. 1g, which shows the number of effectively stored turns versus the number of injected turns. The upper dashed line shows the turns stored and the lower one the turns lost for a delayed multiturn injection without linear coupling with a saturation at 16.5 stored turns and 10.5 turns lost. For increasing delay (decreasing number of injected turns) the phase space angular width of injected beamlets gets smaller and the upper dashed line approaches the  $45^\circ$  loss-free reference line (no loss below 6 turns). For only 21 turn delayed injection still 16 turns are stored ( $1.7 \cdot 10^{11}$  of  $U^{28+}$ ) with a loss of 5 turns (24%). This beneficial effect of delay is significantly enhanced if the skew quadrupole is included. For 16 stored turns the loss is reduced to below 1 turn (6%). We mention that the optimization of  $q_{y0}$  is found nearly independent of the delay. Space charge changes the efficiency dependence on  $q_{x0}$  as well as on  $\delta$  (as discussed in Ref. [3]). Since space charge varies during multiturn injection, it is appropriate to refer to effective tunes  $q_{x,eff}, q_{y,eff}$  for this problem. We expect  $q_{x,eff}$  has a similar efficiency resonance behaviour as  $q_{x0}$ , and that the distance from the difference resonance  $\delta_{eff} = q_{y,eff} - q_{x,eff} + 1$  is as without space charge. For accurate optimization we repeat the scan over  $q_{x0}$  keeping  $q_{y0} = 3.29$  but including now space charge. Fig 1h shows that space charge shifts the peak from  $q_{x0} = 4.29$  (without space charge) to  $q_{x0} = 4.34$ . Using this  $\Delta q_{x,eff} = 0.05$  and assuming  $\Delta q_x/\Delta q_y \simeq \sqrt{A_y/A_x} = 0.5$  (justified towards the end of the multiturn injection) we expect that  $\Delta q_{y,eff} = 0.1$ . With this interpretation we obtain an effective distance from the resonance as  $\delta_{eff} = (q_{x0} - \Delta q_{x,eff}) - (q_{y0} - \Delta q_{y,eff}) + 1$ . Using this formula we can compute the skew strength (using Eq. 1 for  $N_{ex}$ ) and calculate by simulation the dependence of the efficiency versus  $q_{y0}$  as shown in Fig. 1i. The maximum is indeed found for  $q_{y0} = 3.45$  (efficiency 67 %) as is shown in Fig. 1i, which confirms the

above estimated  $\Delta q_{y,eff} = 0.1$ . For the scheme of the delayed injection the fine optimization is more difficult since the space charge effect depends on the length of the injected pulse. This means that the optimized working point  $q_{x0} = 4.34, q_{y0} = 3.45$  for the full 27 injected turns has to move towards  $q_{x0} = 4.29, q_{y0} = 3.35$  along a proper path, which is not obvious and quite time consuming to calculate. We have adopted a simplified approach and chosen a linear path (linear in injected beam) connecting these two points follow. The efficiency is shown in Fig. 1l, which shows a similar behaviour as in the case without space charge. In particular, for 16 stored turns the septum loss is reduced by a factor 3 (12%)! We have good reasons to believe that further fine-optimization of the tune for each injection delay brings the efficiency/loss curves close to the zero space charge case.

### 3 CONCLUSION

The proposed scheme shows that an effective injection improvement is possible if  $q_{x0}, q_{y0}$  are optimized with the correct (vertical acceptance limited) skew strength. Space charge can be taken into account by further fine-optimization for given number of injection turns. Experimental verification of this demanding optimization is planned for the SIS.

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Figure 1: a), b) show multiturn injection with a working point far from resonance ( $\delta = -0.37$ ) and skew strength defined by Eq. 1; c), d) closer to resonance with  $\delta = -0.01$ ; e) efficiency as function of  $q_{x0}$  without linear coupling and no space charge ( $q_{y0} = 3.29$ ); f) dependence of efficiency with  $q_{y0}$  moving away from the resonance ( $q_{x0} = 4.29$  and no space charge); g) efficiency for retarded multiturn injection ( $q_{x0} = 4.29, q_{y0} = 3.35$  and no space charge); h) efficiency versus  $q_{x0}$  including space charge ( $q_{y0} = 3.29$ ); i) same as f) including space charge (10 mA of  $U^{28+}$ ) and shifted working point  $q_{x0} = 4.34$ ; l) delayed multiturn injection with linear working point correction (starting from  $q_{x0} = 4.34, q_{y0} = 3.45$ ).

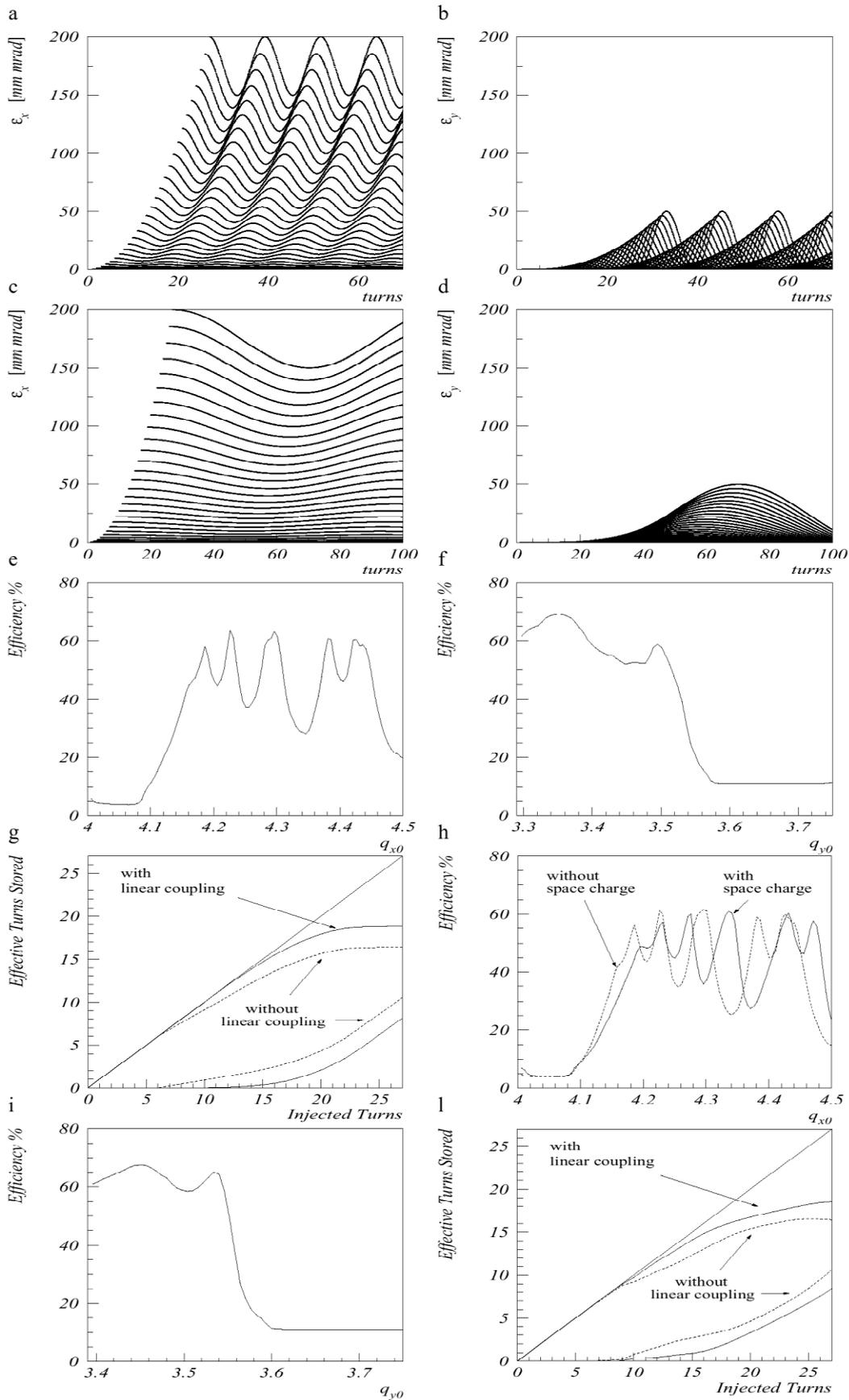


Figure 1