CALCULATION OF RF EIGENMODES USING S-PARAMETERS OF RESONATOR PARTS *

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Abstract

Eigenmode analysis is a fundamental method to characterize the interaction of beam and accelerator components. Several codes are able to calculate eigenmodes but all of them act on the complete structure which is under consideration. Thus they are restricted to a certain complexity of the resonator depending on the computational capabilities. We present a method that overcomes this restriction utilizing a description of structure components in terms of their wave scattering properties. The eigenmodes are found as the solutions of an eigenproblem of small dimension, once the S-parameters of each structure section are known. Their calculation can be performed in parallel. Sections of identical shape have to be calculated only once. Furthermore it is possible to specify a frequency range to search for eigenmodes without the knowledge of all modes with lower frequency. The method is explained in detail in the paper; its capabilities are illustrated using a prototype example which allows to compare with the direct eigenmode calculation.

1 INTRODUCTION

The rf analysis of accelerator components is often based on the knowledge of their eigenmodes. Length and complexity of some structures demand even in modern computational environment a significant amount of resources to perform direct eigenmode calculations. This paper presents a method called Coupled S-Parameter Calculation (CSC) which allows to split the whole section into several subsections that are easy to handle. CSC determines both the eigenfrequencies and the corresponding field distributions using the scattering parameters of each section.

First a general procedure to calculate the coupling between the external ports of an arbitrarily structured system of scattering sections is described. Resonators which are entirely closed by definition are handled as a special case that has no external coupling. Then the problem reduces to the repeated solution of low dimensional eigenvalue equations parameterized by frequency. Occurrence of eigenvalue 0 indicates resonance frequencies. The eigenvectors belonging to eigenvalue 0 are the amplitude patterns of all the waveguide modes in the system that are needed to calculate the corresponding field distributions.

The method is demonstrated using an example that allows for comparison with direct eigenmode solution using MAFIA 4.20 [1] running on a 2 GB RAM workstation.

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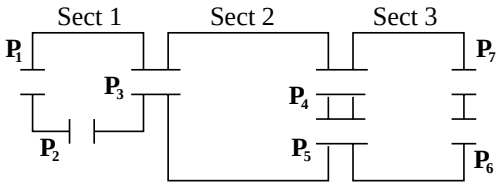

Figure 1: A rf-system of three sections with external (P_1, P_2, P_6, P_7) and internal ports (P_3, P_4, P_5)

2 THEORY

Scattering- or S-parameters are used to describe signal reflection and transmission between each of the ports of rf-components. For an n-port structure they can be represented by the $(n \times n)$-matrix $\mathbf{S}$ where the entry S_{ij} describes the transmission of a signal from port j to port i. The S_{ij} are complex functions of the frequency. With $\vec{a} = (a_1, \ldots, a_n)^T$ and $\vec{b} = (b_1, \ldots, b_n)^T$ describing the input and output signal amplitudes (resp.) the S-matrix $\mathbf{S}_k$ of the k-th section forms the relation

$$\vec{b}_k = \mathbf{S}_k \ \vec{a}_k = \begin{pmatrix} s_{11} & \cdots & s_{1n} \\ \vdots & & \vdots \\ s_{n1} & \cdots & s_{nn} \end{pmatrix}_k \vec{a}_k \qquad (1)$$

The method is not restricted to single moded waveguide ports. Every mode has its individual scattering parameters and therefore increases the dimension of the matrix system by one.

2.1 Open Structures

If $\vec{a}$ and $\vec{b}$ hold the signals of all sections simultaneously both can be arranged in a way that $\mathbf{S}$ becomes a diagonal block matrix of the $\mathbf{S}_k$

$$\mathbf{S} = \begin{pmatrix} \mathbf{s}_1 & & \mathbf{0} \\ & \mathbf{s}_k & \\ \mathbf{0} & & \mathbf{s}_N \end{pmatrix} \qquad (2)$$

The incident signals forming $\vec{a}$ are sorted now in a different manner, dividing them into a vector $\vec{I}$ of signals incident at the external ports and a vector $\vec{A}_{int}$ of signals that are scattered from neighbouring sections. The correlation between this new vector of signal quantities $\begin{pmatrix} \vec{A}_{int} \\ \vec{I} \end{pmatrix}$ and $\vec{a}$ is expressed by a permutation matrix $\mathbf{F}$.

$$\vec{a} = \mathbf{F} \begin{pmatrix} \vec{A}_{int} \\ \vec{I} \end{pmatrix} \qquad (3)$$

Figure 2: Geometry used to compare the direct eigenmode calculation with CSC.

A second permutation matrix $\mathbf{K}$ mediates both the feedback in the system (i.e. the fact that the outgoing signal of one section often is the incident signal of another port) and the signal order permutation inverse to (3). Thus it couples similar to (3) all scattered signals to a vector carrying first the internal signals and second the vector $\vec{R}$ of signals going outwards the system:

$$\begin{pmatrix} \vec{A}_{int} \\ \vec{R} \end{pmatrix} = \mathbf{K}\,\vec{b} \tag{4}$$

Combining (1) to (4) results in the following matrix-vector-equation:

$$\begin{pmatrix} \vec{A}_{int} \\ \vec{R} \end{pmatrix} = \mathbf{K}\,\mathbf{S}\,\mathbf{F} \begin{pmatrix} \vec{A}_{int} \\ \vec{I} \end{pmatrix} = \mathbf{G} \begin{pmatrix} \vec{A}_{int} \\ \vec{I} \end{pmatrix}, \tag{5}$$

summarizing the matrix product in a new matrix $\mathbf{G}$. The matrices $\mathbf{F}$ and $\mathbf{K}$ are determined by the systems topology whereas the rf properties are carried by $\mathbf{S}$. $\mathbf{G}$ has an internal block structure determined by the dimensions of $\vec{A}_{int}$:

$$\begin{pmatrix} \vec{A}_{int} \\ \vec{R} \end{pmatrix} = \begin{pmatrix} \mathbf{G}_{11} & \mathbf{G}_{12} \\ \mathbf{G}_{21} & \mathbf{G}_{22} \end{pmatrix} \begin{pmatrix} \vec{A}_{int} \\ \vec{I} \end{pmatrix}, \tag{6}$$

which leads to the following system of equations:

$$\vec{A}_{int} = \mathbf{G_{11}}\vec{A}_{int} + \mathbf{G_{12}}\vec{I} \tag{7}$$

$$\vec{R} = \mathbf{G_{21}}\vec{A}_{int} + \mathbf{G_{22}}\vec{I} \tag{8}$$

Thus the coupling between signals coming from and going to the system is given by

$$\vec{R} = \left[\mathbf{G_{21}}\ (\mathbf{1} - \mathbf{G_{11}})^{-1}\ \mathbf{G_{12}} + \mathbf{G_{22}}\right] \vec{I}, \tag{9}$$

which means that the overall S-parameter matrix $\mathbf{S}_{tot}$ yields as:

$$\mathbf{S}_{tot} = \mathbf{G_{21}}\ (\mathbf{1} - \mathbf{G_{11}})^{-1}\ \mathbf{G_{12}} + \mathbf{G_{22}} \tag{10}$$

2.2 Resonators

In the case of a resonator problem there are no open ports. Then holds $dim(\vec{R}) = dim(\vec{I}) = 0$ and only the coupling between the internal ports remains. Further it follows from (7) that

$$(\mathbf{1} - \mathbf{G}_{11}(\omega_0))\,\vec{A}_{int} = \vec{0} \tag{11}$$

has to be fulfilled. Eq.(11) is only valid for discrete frequencies ω_0, which are the resonance frequencies aimed for.

In case of frequencies that satisfy (11) the vector $\vec{A}_{int}$ contains the amplitudes of the waveguide modes that compose the resonant field in the entire resonator. $\vec{A}_{int}$ is found as the eigenspace - usually one vector - of the eigenvalue 0 which is the core of the matrix $(\mathbf{1} - \mathbf{G}_{11}(\omega_0))$. The fields themselves are computed afterwards in separated runs with incident wave amplitudes given from $\vec{A}_{int}$.

A software system has been set up that performs both the broadband S-parameter determination of the elementary sections (MAFIA-T, Microwave Studio [1]), the CSC calculation (Mathematica [2]) and the field calculations (MAFIA-T, MAFIA-W, Microwave Studio).

3 NUMERICAL RESULTS

To verify the procedure a test geometry was modelled (see Fig. 2) which is split into five subsections. The S-parameters of each single subsection were calculated using the MAFIA time domain solver T3 in a frequency range of $1.2 \ldots 1.75$ GHz.

Simultaneously the eigenfrequencies of the whole structure were computed with MAFIAs eigenmode solver E to verify the solution. As shown in Tab.1 the frequencies found by the CSC technique match very well those calculated directly.

Table 1: Comparison of eigenfrequencies found by CSC and by MAFIA-E in the frequency range of $1.2 \ldots 1.75$ GHz.

CSC	MAFIA E-mod.	relative error
1.21208 GHz	1.210308 GHz	1.46×10^{-3}
1.24005 GHz	1.240023 GHz	2.18×10^{-5}
1.34807 GHz	1.347277 GHz	5.88×10^{-4}
1.38342 GHz	1.382202 GHz	8.81×10^{-4}
1.44258 GHz	1.442681 GHz	-6.99×10^{-5}
1.46395 GHz	1.463867 GHz	5.67×10^{-5}
1.50325 GHz	1.502614 GHz	4.23×10^{-4}
1.53157 GHz	1.540615 GHz	-5.91×10^{-3}
1.60483 GHz	1.603293 GHz	9.58×10^{-4}
1.65345 GHz	1.652757 GHz	4.19×10^{-4}
1.68786 GHz	1.682436 GHz	3.21×10^{-3}

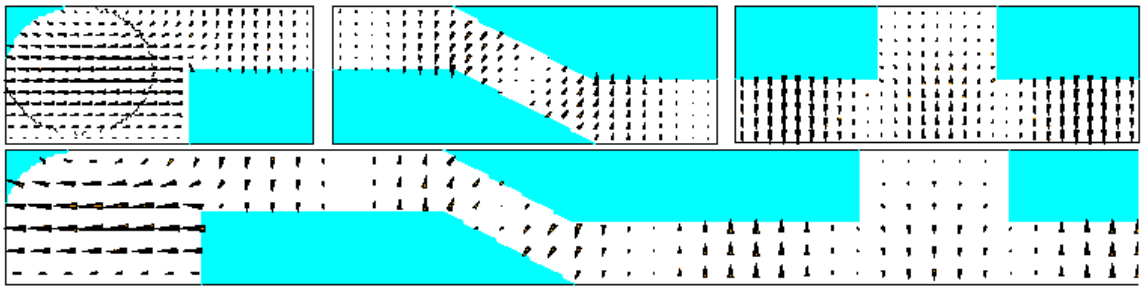

Figure 3: Electric field of the $f = 1.50325$ GHz-eigenmode computed using CSC (upper) and MAFIA-E (lower). (Jumps in vector size are artificially introduced by different display scalings).

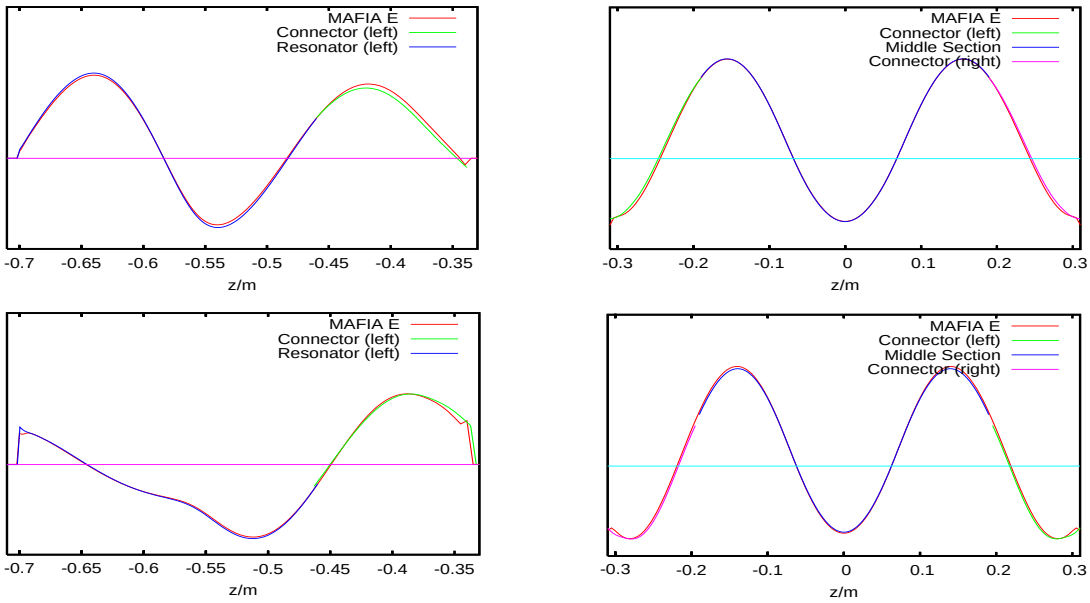

Figure 4: E_y-component of the eigenmodes with $f = 1.44258$ GHz (first row) and ($f = 1.50325$ GHz) (second row) along two path segments perpendicular to the port plane comparing CSC and MAFIA-E.

The field distributions of the eigenmodes in all subsections were also computed by MAFIA T3. Monochromatic waves of the resonance frequency are excited at every port with amplitudes and phases given by CSC. The field is monitored after it stabilized to the steady state, which yields the resonant field pattern in the according resonator section. Fig. 3 shows examples of field patterns found by CSC and compared with MAFIA-E. The fields match like the frequencies very precisely the results of the direct eigenmode calculation.

4 CONCLUSIONS

The CSC-technique presented here is a method to split the calculation of rf properties of complex structures into several small runs. After dividing the structure in sections the S-parameters of every section are calculated separately. Combination of them yields either the S-matrix of the complete structure if it has open ports or its eigenfrequencies and eigenmodes if it is a closed resonator.

The advantages of this technique are the possibility to calculate the S-parameters of every subsection parallel on different machines, to exploit possible symmetries or repetitions of subsections and to specify frequency ranges for the eigenmodes searched for.

If a direct calculation is possible, the overall effort of this procedure is essentially higher, but CSC gives the possibility to compute eigenmodes of structures which cannot be handled directly, or to perform optimization iterations calculating only modified sections.

5 REFERENCES

[1] MAFIA V4.20 and Microwave Studio V2.0 , CST GmbH, Büdinger Straße 2a, D-64289 Darmstadt, Germany.

[2] Mathematica V4.0.1, Wolfram Research Inc, 100 Trade Center Drive, Champaign, IL 61820-7237, USA