SYNCHRONOUS WAVES IN THE LHC BEAM SCREEN WITH RIBBED SURFACE

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Abstract

The problem of the wake field generated by a relativistic particle travelling in a long beam tube with rough surface has been revisited by means of a standard theory based on the hybrid modes excited in a periodically corrugated rectangular waveguide. Slow waves synchronous with the particle can be excited in the structure, producing wake fields whose frequency and amplitude depend on the depth of the corrugation. The main features of the longitudinal wake, especially relevant for very short bunches, and its possible effect on proton bunches in the LHC beam screen with ribbed surface are discussed.

1 INTRODUCTION AND MOTIVATIONS

The effect of surface roughness is a rather new subject, arisen in the design of machines with extremely short bunches of the order of tens of microns. In this case, in fact, the surface roughness may be a source of wake fields which might significantly increase the beam emittance and the energy spread. Recently, a corrugation of the LHC beam pipe has been proposed in order to reduce the reflectivity of the walls, and therefore reduce the heat load on the dipole beam screen due to photoelectrons accelerated by the proton beam [1].

The low frequency coupling impedance due to the wall surface roughness has been estimated in Ref. [2], while at high frequency a surface wave synchronous with the beam can be excited, and its interaction with the beam may lead to beam degradation [3]. The roughness is replaced by a thin dielectric layer at the waveguide walls, which supports the surface wave.

In this paper we review the problem of the wake fields produced by an ultra-relativistic charge travelling inside a beam tube with a periodic corrugation making use of a standard theory based on the hybrid modes propagating in the waveguide. After having described the model (sec. 2), we first derive the dispersion relation for the fields and study the frequency where the synchronous wave can be excited. Then, through the reciprocity principle we get the amplitude of the fields excited by the charge (sec. 3). From the longitudinal field E_z we calculate the coupling impedance and the wake fields, using them to estimate the threshold for the single bunch longitudinal instability (sec. 4).

2 MODEL

Let us consider the periodically corrugated rectangular waveguide sketched in Fig. 1, with a (b) the dimension along the x-axis (y-axis). We model the wall roughness as a series of periodic (with period L) rectangular obstacles of height h and thickness t. The beam travels on the z-axis; we assume $t \ll L$, $L \ll \lambda$ and neglect ohmic losses in the material.

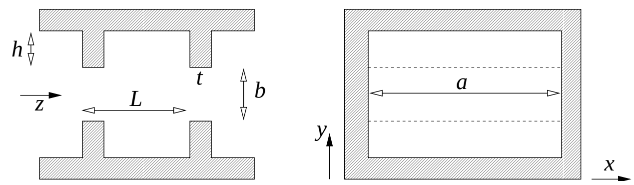

Figure 1: Schematic view of the waveguide and notations adopted.

Following [4], we consider a rectangular pipe with corrugations in only two opposite sides. The periodicity of the geometry along z allows the use of Floquet's theorem which implies a field solution independent of the period L (obtained from a single cell).

Throughout the paper, we use (ξ_x, ξ_y, ξ_z) for the propagation constants of the field in a rough waveguide, (k_x, k_y, k_z) for a smooth waveguide, and (K_x, K_y) for the wave number of the field inside the corrugation.

3 THEORY

To find the longitudinal wake function per unit length we follow the method applied in [5], that we summarise for reader's convenience. First, the homogeneous problem is solved, thus finding the modes propagating in the corrugated waveguide; then, applying the reciprocity principle, we derive the fields generated by a relativistic point charge.

Having assumed $L \ll \lambda$, the fields inside the corrugation do not depend on the z variable (that is $K_z = 0$); since the corrugation is only on the two faces normal to the y-axis, $K_x = k_x = n\pi/a$ (with $n = 1, 2, \dots$). The fields in the internal region of the waveguide can be derived from a magnetic Hertz potential directed along the x-axis, such that E_z is non vanishing on the beam axis. A y-directed Hertz potential would produce an E_x vanishing on the corrugation, because of continuity over the boundary, just like in a smooth conducting rectangular waveguide. On the contrary, a z-directed Hertz potential would not give all the possible field configurations. Imposing the continuity of E_z and H_x over the boundary, we get (c is the speed of light in vacuum):

$$k_{xn}^2 + \xi_y^2 + \xi_z^2 = (\omega/c)^2 \,, \qquad (1)$$

$$K_{yn} \tan(K_{yn} h) = \xi_y \cot(\xi_y b/2) \,. \qquad (2)$$

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Eq. (2) is usually referred to as dispersion equation; for each value of $K_{yn} = \sqrt{(\omega/c)^2 - k_{xn}^2}$, it gives an infinite number of solutions.

For finite values of h, solving numerically Eq. (2) and plugging the obtained value for ξ_y in Eq. (1), we get the propagation constant ξ_z; its behaviour with the frequency is usually called dispersion diagram. The result for a simpler square waveguide of side a is shown in Fig. 2 (h/a=0.1 and h/a=0.01). The dispersion curve of a smooth waveguide at

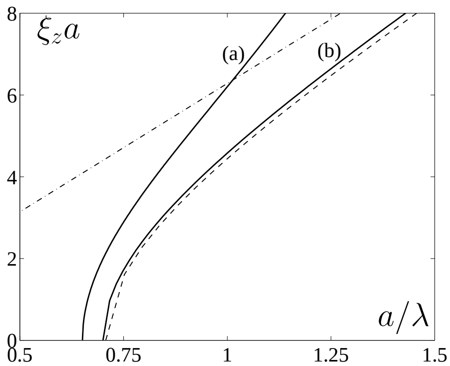

Figure 2: Dispersion diagram for a square waveguide of side a with corrugation on two opposite faces. The solid line (a) is obtained from Eq. (2), for h/a = 0.1, while the line (b) is for h/a = 0.01. The dashed one corresponds to the unperturbed TM_{11} mode of the smooth waveguide and the dot-dashed line rappresents a relativistic beam.

very high frequencies tends to the propagation constant of a relativistic beam $\xi_z = \omega/c = 2\pi/\lambda$ (dot-dashed line), but it never crosses it. However, due to the corrugation, there may be a crossing of the curves at particular frequencies where the waveguide mode and the beam are synchronous and a coherent exchange of energy is then possible. It is clear from Fig 2 that the smaller is h, the higher is the crossing frequency. If the crossing frequency is inside the bunch spectrum, characterised by the typical frequency $c/2\pi\sigma$ (σ being the r.m.s. bunch length), the coupling with this synchronous (surface) mode could be quite efficient, resulting in beam degradation. In ultra-short bunch machines, this may become an important effect. On the contrary, in an LHC-like geometry ($a = 3.6$ cm, $h = 30$ μm and $\sigma \approx 7$ cm) the bunch spectrum extends approximately up to $a/\sigma \approx 0.5$, i.e. well below the crossing point $a/\lambda \approx 10$. It can be shown [5] that for $h \ll \lambda$, a and high energy particles ($\gamma \to \infty$) the crossing frequencies are:

$$\overline{f_n} = \frac{c}{2\pi}\sqrt{k_{xn}^2 + \frac{k_{xn}}{h}\coth\left(k_{xn}\frac{b}{2}\right)}. \quad (3)$$

For very small h, the second term in the square root dominates resulting in a behaviour $\propto 1/\sqrt{h}$, analogous to the case of a pipe covered with a dielectric layer of thickness h [6].

The modes propagating in the corrugated waveguide are called hybrid modes [4], since they are a superposition of the standard TE and TM modes (along the z-axis). The hybrid modes in general do not satisfy an orthogonality condition; physically this means that they are coupled to each other. Since the coupling coefficients are proportional to the height of the corrugation h, for very small depths of the corrugation the modes are practically decoupled.

Once the homogeneous problem is solved, the modes of the structure are known, the field generated by a point charge can be found by means of the Lorentz reciprocity principle [5]; for an infinitely long structure, the field has a resonant behaviour. The (specific) longitudinal coupling impedance is proportional to E_z on the beam axis [7]:

$$\frac{\partial Z(\omega)}{\partial z} = 4\pi^2 Z_0 \frac{h}{a}\frac{1}{ab}\tanh\left(\frac{\pi}{2}\frac{b}{a}\right) \times \frac{\delta(\omega/c - \xi_{z1}) + \delta(\omega/c + \xi_{z1})}{\sinh(\pi b/a)/(\pi b/a) - 1}, \quad (4)$$

where δ is the Dirac function, Z_0 is the free-space characteristic impedance and ξ_{z1} is obtained by solving Eqs. (2) for $n = 1$. It is straightforward now to get the wake function for unit length and for a point charge [7], i.e.

$$\frac{\partial w(\tau)}{\partial z} = \frac{H(\tau)}{\pi}\int_{-\infty}^{\infty}\frac{\partial Z(\omega)}{\partial z}e^{j\omega\tau}\mathrm{d}\omega, \quad (5)$$

where τ is the time distance of the trailing charge from the leading one and $H(\tau)$ is the Heaviside function; eventually we get:

$$\frac{\partial w(\tau)}{\partial z} = w_0(a,b,h)\cos\left(2\pi\overline{f_1}\,\tau\right), \quad (6)$$

where

$$w_0(a,b,h) = 8\pi\frac{Z_0\,c}{ab}\,\frac{h}{a}\,\frac{\tanh(\pi b/2a)}{\sinh(\pi b/a)/(\pi b/a) - 1}. \quad (7)$$

To get the wake function for a bunch, one has to perform the convolution of Eq. (6) with the current density. For instance, the wake function is

$$\frac{\partial W(\tau)}{\partial z} = w_0(a,b,h)\int_0^{\infty}\frac{e^{-(t-\tau)^2/2\sigma^2}}{\sqrt{2\pi}\sigma}\cos\left(2\pi\overline{f_1}\,t\right)\,\mathrm{d}t \quad (8)$$

for a gaussian bunch of r.m.s bunch length σ and this integral can not be done analytically.

The amplitude $w_0(a,b,h)$ of the sinusoidal function for an LHC-like geometry ($a = 3.6$ cm, $b = 4.3$ cm and $h = 30$ μm) is $w_0 \simeq 0.3\ V\ pC^{-1}\ m^{-1}$ and the crossing frequency $\overline{f_1}$ is 83 GHz (and $h \ll \lambda$, as assumed). The possible effect of such a wake on the single bunch longitudinal stability is discussed in the next section.

4 SINGLE BUNCH LONGITUDINAL INSTABILITY

In the previous section we have shown that the wake field due to the synchronous mode has a resonant behaviour and the quality factor Q of such a resonance is infinite (since an infinite interaction length is assumed). This is clearly an approximation and, to evaluate the threshold of the longitudinal instability, we will consider the standard resonator model [9] with a shunt resistance R_s given by

$$R_s = \frac{Q w_0 L_0}{2\pi \overline{f_1}} = 1.6\ 10^4 Q \quad \Omega \tag{9}$$

and a finite Q. The LHC parameters we will use are given in Table 1.

Table 1: LHC parameter list

mom. compaction	α_c	$3.47\ 10^{-4}$	
machine length (Km)	L_o	26.66	
rev. freq. (Hz)	f_o	$11\ 10^3$	
energy (GeV)	E_o	$7\ 10^3$ (top)	450 (inj.)
bunch length (mm)	σ	75 (top)	130 (inj.)
energy spread (10^{-4})	$\sigma_{\varepsilon o}$	1.1 (top)	4.7 (inj.)

The impedance at frequencies well below the resonator frequency (for example at the frequency $f_c = c/2\pi\sigma$ associated with the bunch length) is inductive and such that

$$|Z/n| \approx \frac{R_s}{Q} \frac{f_o}{\overline{f_1}} \approx 2\ \mathrm{m}\Omega, \tag{10}$$

i.e. two order of magnitude smaller than the LHC impedance budget (n is the harmonic number $n = f/f_o$). At the resonator frequency $\overline{f_1}$, the impedance $|Z/n|$ is real and equal to that of Eq. (10) multiplied by the Q factor.

An estimate of the longitudinal instability threshold (for mode numbers of the order of $\overline{f_1}/f_c = 130$) can be done by using the Boussard criterion [8], derived from the coasting beam theory, which we write here in the form

$$N_{th} = \frac{(2\pi)^{(3/2)}\ (E_0/e)\ \alpha_c\ \sigma\ \sigma_{\varepsilon 0}^2}{c\ e\ |Z/n|}, \tag{11}$$

with E_0 the nominal energy, e the electron charge, α_c the momentum compaction, $\sigma_{\varepsilon 0}$ the energy spread, and Z the coupling impedance at a frequency corresponding to harmonic number n. For the top energy of 7 TeV we get

$$N_{th} = \frac{7.22 \times 10^{11}}{\left|\frac{Z/n}{\Omega}\right|} \tag{12}$$

protons per bunch, while in the case of injection energy (450 GeV) the threshold is a factor 2 higher.

In the most pessimistic case in which the perturbation of the unstable oscillation mode has the same resonant frequency of the wake $\overline{f_1}$, we obtain a threshold of

$$N_{th} = \frac{3.67 \times 10^{14}}{Q} \tag{13}$$

protons per bunch, depending on the value of Q. Quality factors Q higher than 2×10^3 (equivalent to interaction lengths longer than 6 m) are therefore dangerous for the LHC whose ultimate intensity is 1.6×10^{11} protons per bunch.

On one hand, a more accurate stability analysis including azimuthal [9] and radial [10] mode coupling would then be in order. On the other hand several mechanisms, not included in the previous derivation, will limit the quality factor Q, including ohmic losses and geometric imperfections. In particular, pumping slots in the LHC beam screen induce mixing of the modes propagating in the beam pipe and attenuation of the synchronous surface wave, presumably after a distance of few meters [11].

5 CONCLUSION

In this paper we have derived the longitudinal wakes due to a periodic corrugation in a rectangular beam pipe, interesting for the design of the LHC beam screen. For a point charge, the amplitude of the sinusoidal wake function is proportional to the corrugation depth h and the oscillation frequency is proportional to $1/\sqrt{h}$. For $h \to 0$ the frequency of the wake function goes to infinity, while its amplitude vanishes. A possible way to evaluate the single bunch instability threshold has been proposed, by means of the Boussard criterion; the resulting threshold depends on the Q value of the synchronous field, that has still to be evaluated.

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