TRANSPORT OF ELECTRON BEAMS WITH LARGE ENERGY SPREAD IN A PERIODIC LONGITUDINAL MAGNETIC FIELD

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Abstract

The transport of an electron beam with an energy spread of up to 75% is discussed. In the paraxial approach the particle motion in a periodic field with a non-zero average component is described by a Mathieu-type equation. For specific energies in the beam spectrum the solution of this equation shows resonances between the period of the focusing field and the spatial Larmor period of the particles. The powers of resonances drop with their harmonic number. Non-resonance transport is possible only at low average focusing field; then for all the energies the Larmor period is longer than the period of the focusing field. Practically, for short beam lines, transport is possible also at higher focusing fields. Low powers of resonance in this case do not significantly increase the beam radius along the length of the beam line. Calculations, made for the transport section between decelerator and depressed collector of the FOM fusion free-electron maser (FEM), where the electron energy in the beam varies from 50 to 350 keV, has shown that highly efficient transport is possible if the harmonic number is larger than 3 over the whole energy range.

1 INTRODUCTION

In modern radiation sources, based on one-pass free electron laser (FEL) interaction, electron beams of high power are used. The beam is accelerated up to an operational energy, which depends on the desirable frequency of the emitted radiation, then it passes through an undulator, transforming part of its energy to the radiation, after that the residual beam energy is recuperated with a recovery system. As after the FEL interaction the beam obtains a considarable energy spread (up to several percent of the operational energy), the process of energy recovery is performed in two stages. Firstly, the beam energy is reduced to an intermediate value, after that the energy is recuperated with a depressed collector. In order for the collector to be able to accept a beam with large energy spread it is developed as a multistage device. The level of the intermediate energy is defined by the possibility of transportation of the beam with a large energy spread from the exit of the decelerator to the depressed collector.

In the 1 MW Fusion-FEM [1] the FEL interaction takes place at an energy of 2 MeV. The energy spread at the undulator exit at an output power of 1 MW is 10%. The energy spectrum of the beam at the exit of the undulator, calculated with the GPT code [2] is shown in Fig. 1. Before recovering energy in the depressed collector [3], the beam is decelerated to 225 keV with an electrostatic decelerator. The energy spread at the exit of the decelerator becomes as high as 75%.

Thus, highly efficient beam transport between the decelerator and depressed collector is one of the main problems in the design of the recovery system. In this work we discuss the beam transport in a periodical magnetic focusing system.

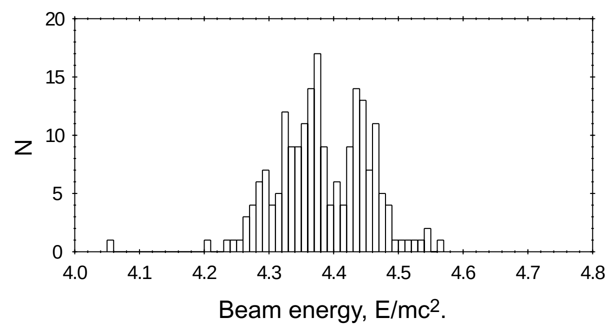

Figure 1: Energy spectrum of the electron beam at the FEM unfulator exit.

2 BEAM TRANSPORT WITH LONGITUDINAL GUIDING FIELD

The simplest optical system, which allows to transport beams with a large energy spread, is a longitudinal magnetic field which is provided by a solenoid. In a long solenoid with magnetic field B_o the particle trajectory in the paraxial approach is defined by the equation:

$$r'' + \left(\frac{eB_o}{2mc\beta\gamma}\right)^2 r = 0. \quad (1)$$

The evolution of the trajectory then is defined by its radius r_o and divergence r'_o at the entrance of the solenoid

$$r(z) = r_o\cos(kz) + \frac{r'_o}{k}\sin(kz), \quad (2)$$

$$r'(z) = -kr_o\sin(kz) + r'_o\cos(kz), \quad (3)$$

with $k = eB_o/2mc\beta\gamma$. The maximum trajectory radius is then

$$r_m = \sqrt{{r_o}^2 + \frac{{r'_o}^2}{k^2}}. \quad (4)$$

The envelope radius of the beam with an emittance ϵ may be found by averaging r_m over the initial distributions of r_o and r'_o. The rms value of r_m is then

$$< r_m^2 >=< r_o^2 > \cdot \left[1 + \frac{< {r'_o}^2 >}{< r_o^2 >} \cdot \left(\frac{2mc\beta\gamma}{eB_o}\right)^2\right], \quad (5)$$

where $< r_o^2 >= \hat{\beta}\epsilon$, $< r_o'^2 >= \hat{\gamma}\epsilon$, $\hat{\beta}$ and $\hat{\gamma}$ are the Twiss parameters of the beam.

Let's estimate the envelope radius for a practical Fusion FEM beam with 1-D emittance ϵ=100 mm·mrad and $\sqrt{< r_o^2 >}$=10 mm which travels in a field B_o=100 mT. If the initial condition for the beam radius and divergence are taken at the beam waist, $\sqrt{< r_o'^2 >}$=10 mrad. Direct calculations show that the contribution to the beam radius of the second, energy dependent, term in Eq. 5 for the region 50–350 keV is an order of less than 10^{-3}. It means that the beam radius is defined by its initial radius r_o only and does not depend on its energy. This fact allows us to use a longitudinal magnetic field for the transport of a beam with a large energy spread.

Unfortunately, the period of the beam oscillation strongly depends on the energy and at the long solenoid the beam loses its initial coherence. That leads to a valuable dispersion of the beam divergence at the solenoid exit. Another drawback of the "elegant" solution of the guiding field is that the long coil which produces this field makes it difficult to introduce other beam line components.

3 ARRAY OF THIN MAGNETIC LENSES

An alternative solution of the beam transport is the use of an equidistant array of thin axial symmetric lenses. For an array with period S in the paraxial approximation, particle motion is stable if $0 \leq S \leq 4f$, where f is the focal distance of the lens [4]. For a beam with energy spread, this relation must be valid for all energies in the spectrum.

The focal distance of the solenoidal magnetic lens is defined by

$$\frac{1}{f} = \left(\frac{NI}{\beta\gamma}\right)^2 \cdot K, \quad (6)$$

where NI is the number of ampere turns and the coefficient K depends on the lens design. As far as the focal distance $f \sim (\beta\gamma)^2$, the requirements for S must be satisfied for lower energies. Filling in the FEM-case with say S=0.5 m f_{min} must be more than 0.125 m.

Following [4] the maximum beam radius R_m and the minimum beam radius (in the waist) R_w are found from the formulas

$$R_m = \sqrt{\epsilon S}\left[\frac{4(f/S)^2}{4(f/S)-1}\right]^{1/4}, \quad (7)$$

$$R_w = R_m\sqrt{1 - \frac{S}{4f}}. \quad (8)$$

The plots of R_m and R_w are shown in Fig. 2. As may be seen for f/S >1 the dependence of both the radiuses

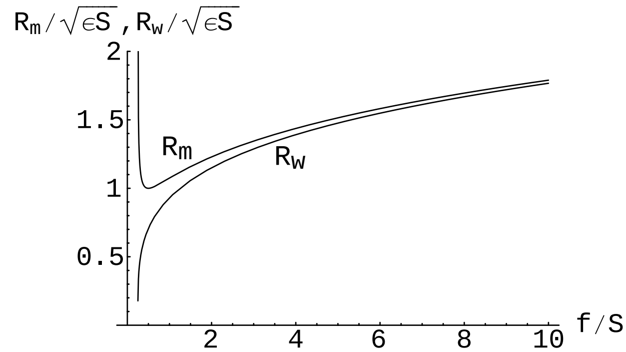

Figure 2: Dependence of the maximum beam radius R_m (upper curve) and the beam radius in the waist R_w (lower curve) on the ratio f/S.

on the focal distances is very weak. Moreover, the difference between R_m and R_w is small. For an emittance ϵ=100 mm·mrad and a period S=0.5 m the calculations give us $R_m \approx R_w = 1.7\sqrt{\epsilon S}$=12 mm. So, the beam line with periodic focusing also may be used to transport a beam with large energy spread.

4 MIXED GUIDING FIELD

Unfortunately the longitudinal (along the beam trajectory) size of a real magnetic lens is an order of magnitude of the distance between them. So, such a lens can not be considered as a thin one. In this section we study the beam behavior in a mixed field, which is formed by overlapping fields of an array of real magnetic lenses as shown in Fig. 3.

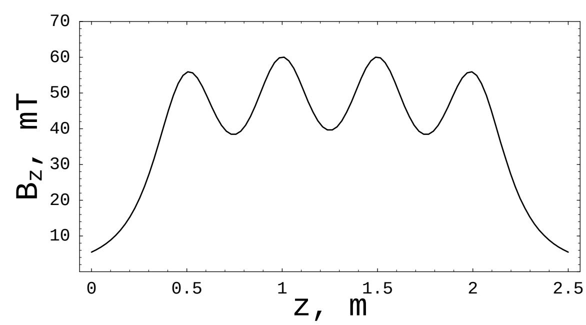

Figure 3: Profile of the longitudinal magnetic field formed by an array of real magnetic lenses.

The periodical longitudinal field on the axis of an array of real lenses may be presented as a Fourier series

$$B_z(z) = B_o\left[1 + \sum_{n=1}^{\infty} b_n \cos nk_o z\right], \quad (9)$$

where $k_o = 2\pi/S$, and S is the period of the array. Substituting this field into the paraxial Eq. 1 we obtain

$$r'' + k^2\left[1 + \sum_{n=1}^{\infty} a_n \cos nk_o z\right] r = 0, \quad (10)$$

where a_n are Fourier coefficients of the magnetic field square. For the largest Fourier components it transforms to a Mathieu-type equation

$$r'' + k^2 (1 + a_1 \cos k_o z) r = 0. \quad (11)$$

The criterion of stability for this equation is well known and in our terms can be written as

$$k \neq \frac{k_o}{2} l \qquad (l = 1, 2, 3, \ldots). \quad (12)$$

It means that for a particle with momentum

$$mc\beta\gamma = \frac{eB_o S}{2\pi l} \quad (13)$$

the motion in the beam line is unstable. In order to provide stable transport of the beam with energy spread in the infinite periodic beam line, the magnetic field B_o must satisfy the following condition

$$B_o < \frac{2\pi}{S} \cdot \frac{mc(\beta\gamma)_{min}}{e}, \quad (14)$$

where $(\beta\gamma)_{min}$ corresponds to the minimum energy in the spectrum. For our beam line with S=0.5 m and a minimum energy of 50 keV the average longitudinal field should not exceed 24 mT. However, direct numerical experiments have shown that in short beam lines with a small number of periods beam transport is also possible with higher values of the field. It results from the fact that powers of resonances drop with their harmonic number l. In order to satisfy these conditions the field amplitude B_0 must be higher than the one corresponding to the 3-4-th order of resonances for the maximum beam energy. For a maximum energy of 350 keV it corresponds to B_o=116 mT.

As an example we have performed a numerical simulation of the beam transport covering the energy range 50 - 350 keV in a field shown in Fig. 3. The field is produced by 4 lenses with a center-to-center distance of 0.5 m. This array is a simplified model of the Fusion-FEM transport section between decelerator and depressed collector. In Fig. 4 we show trajectories of particles with different energies which travel in the field B_o = 120 mT along a distance of 2 m. As may be seen the amplitudes for all the energies do not rise much higher than their initial values given by Eq. 5.

5 CONCLUSIONS

We considered three electron optical systems for transport of a beam with a large energy spread. The best way to transport in the required energy range is just a longitudinal magnetic field. In this scheme for the given energy range, and the practical values of magnetic field the beam radius does not depend on energy, though the dispersion of angles at the exit of the solenoid is quite high. Unfortunately, the practical realization of this optics meets serious problems in the case of the Fusion-FEM.

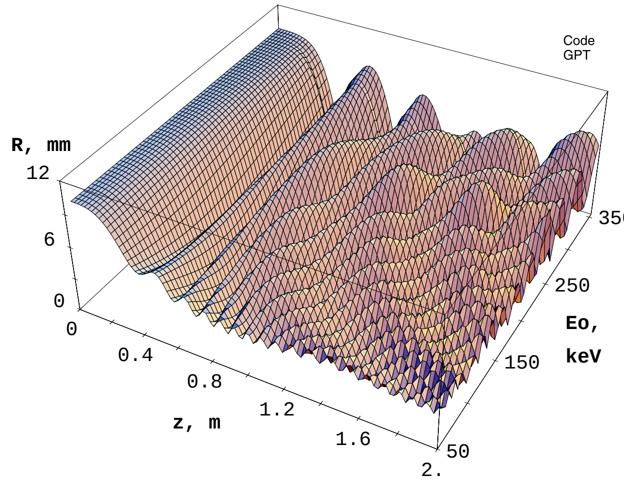

Figure 4: Trajectories of the particles with different energies in a mixed guiding field. The longitudinal axis corresponds to the longitudinal position, the transverse axis - the particle energy, vertical - to the radius of the trajectory.

A periodic array of thin lenses can also transport a beam with large energy spread. Moreover it has good dispersion properties for long focusing distances. However, realization of this array for the Fusion-FEM energy range is difficult, because real lenses can not be considered as thin ones.

An alternative and practical optical system is obtained by combination of the two previous cases. It may be realized as an array of real magnetic lenses having the same direction of the magnetic field. However, the array period and the field amplitude must satisfy non-resonance conditions.

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