SIMULATIONS OF COHERENT BEAM-BEAM MODES AT THE LHC

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Abstract

The transverse coherent oscillations of the two colliding LHC proton beams are studied via multi-particle tracking, using the beam-beam force of a Gaussian distribution with variable barycenters and rms sizes. In addition to head-on collisions, our simulation optionally includes the effect of long-range collisions and an external impedance. Simulation results are the coherent and incoherent oscillation frequencies, the emittance growth of either beam, and evidence for the existence or absence of Landau damping. We find that with equal beam sizes and equal tunes Landau damping is lost for current ratios larger than 60%. However, Landau damping of the coherent dipole oscillations can be restored by separating the tunes, provided the external impedance is sufficiently small.

1 MODEL

We simulate the collision of two strong proton beams. Our system of normalized variables is $x = X/\sigma_{oX}$, $v_x = \beta X'/\sigma_{oX}$, $y = Y/\sigma_{oY}$, $v_y = \beta Y'/\sigma_{oY}$ where $\sigma_{oX} = \sigma_{oY} = \sigma$ are the nominal horizontal and vertical rms sizes, and β the beta function at the interaction point. The prime denotes the derivative with respect to longitudinal position s. Each of the beams is represented by a set of N macroparticles, whose trajectories are followed over n turns, assuming linear betatron motion (rotation with tune Q_x on the horizontal plane (x, v_x) and tune Q_y on the vertical plane (y, v_y)) from one IP to the next. At the IP, each particle in the bunch experiences a deflection in the field of the counter-rotating beam with barycentres at $(\bar{x}^{(i)}, \bar{y}^{(i)})$ and squared transverse sizes $M_{xx}^{(i)} =< (x^{(i)} - \bar{x}^{(i)})^2 >$, $M_{yy}^{(i)} =< (y^{(i)} - \bar{y}^{(i)})^2 >$. This deflection is computed assuming that the opposing beam has a Gaussian shape. Thus, for $M_{xx}^{(i)} > M_{yy}^{(i)}$ we apply a horizontal beam-beam kick $\Delta v_x(n) = \frac{r_p N_p^{(i)}}{\gamma} \frac{\beta}{\sigma^2} F_x(x-\bar{x}^{(i)}, y-\bar{y}^{(i)}, M_{xx}^{(i)}, M_{yy}^{(i)})$ with r_p the classical proton radius, $N_p^{(i)}$ the bunch population, γ the relativistic Lorentz factor and F_x is given by $F_x(x, y, M_{xx}, M_{yy}) = \sqrt{\frac{2\pi}{(M_{xx}-M_{yy})}} \Im \left[W\left(\frac{x+iy}{\sqrt{2(M_{xx}-M_{yy})}}\right) - \sqrt{\frac{2\pi}{(M_{xx}-M_{yy})}} e^{(-\frac{x^2}{2M_{xx}} - \frac{y^2}{2M_{yy}})} W\left(\frac{x\sqrt{M_{yy}/M_{xx}} + iy\sqrt{M_{xx}/M_{yy}}}{\sqrt{2(M_{xx}-M_{yy})}}\right)\right]$, where W denotes the complex error function (if $M_{yy}^{(i)} > M_{xx}^{(i)}$ we substitute x by y on both sides of the two equations, and vice versa). The vertical beam-beam force is described by the real part of the same expression. In these maps the superindex (i) indicates variables of the counter-rotating beam.

At the LHC, there are about 16 parasitic encounters on each side of an IP, with a minimum transverse separation of $L_x = 7.5$ and $L_y = 7.5$ (in units of σ_x). The long-range beam-beam kick is then $\Delta v_x(n) = +n_{par} \frac{2r_p N_p^{(i)}}{\gamma} \frac{\beta}{\sigma^2} \left\{ \frac{(x-\overline{x}^{(i)}-L_x)}{R^2} \left[1 - \exp\left(-\frac{R^2}{M_{xx}^{(i)} + M_{yy}^{(i)}}\right)\right]\right\} - n_{par} \frac{2r_p N_p^{(i)}}{\gamma} \frac{\beta}{\sigma^2} \left\{ -\frac{1}{L_x} \left[1 - \exp\left(-\frac{L_x^2}{M_{xx}^{(i)} + M_{yy}^{(i)}}\right)\right]\right\}$ where $R^2 = (x - \overline{x}^{(i)} - L_x)^2 + (y - \overline{y}^{(i)})^2$. An equivalent expression will be used for vertical long range collisions with separation L_y. In the simulations we assume the following typical LHC parameters: fractional betatron tunes of $Q_x \approx Q_y = 0.32$, bunch population $N_p^{(1)} = 1.05 \times 10^{11}$ for beam 1, and $N_p^{(2)} = r \times N_p^{(1)}$ for the second beam with r varying between 0 and 1, proton beam energy of 7 TeV, unperturbed horizontal and vertical rms beam sizes at the primary collision point $\sigma = 16 \times 10^{-6}$ m, and an IP beta function $\beta_{x,y} = 0.5$ m. The beam-beam parameters are defined by $\xi_{x,y}^{(i)} = \frac{N_p^{(i)} r_p \beta x,y}{2\pi\gamma\sigma_{x,y}(\sigma_x+\sigma_y)}$ with $i = 1$ for beam 1, and $i = 2$ for beam 2. With the above LHC parameters we find $\xi \approx 0.0034$. For equal beam sizes, the ratio of the beam currents, $r = N_p^{(2)}/N_p^{(1)} = \xi^{(2)}/\xi^{(1)}$, determines the behaviour of the system [1]. In the simulation, the initial coordinates (x, v_x, y, v_y) for two groups of N macroparticles representing the two beams are selected from a Gaussian random distribution in each variable with $< x >=< v_x >=< y >=< v_y >= 0$ and $< x^2 >=< v_x^2 >=< y^2 >=< v_y^2 >= 1$.

2 COLLISION WITH EQUAL TUNES

2.1 π- and σ-modes for round beams

First we consider the strong-strong case, $r = 1$, and head-on collisions of two bunches.The statistical fluctuation in the macroparticle distribution is sufficient to excite the coherent modes. Fourier analysing the motion of the barycentre of one bunch reveals two coupling modes. One is located at Q; the other has a lower frequency. In Fig. 1 the simulated frequency spectrum of one beam $S_A(w)$ is plotted on a logarithmic scale, as determined by an FFT.

We can identify the two dipole coherent modes σ (at the unperturbed tune) and π. Theoretical studies based on the linearized Vlasov theory predict a tune shift between the σ and π-modes equal to $Y \times \xi$. For the case of round beams this factor is predicted to be $Y = 1.21$.The shift obtained in our simulations corresponds to $Y \approx 1.1$. The difference is probably due to the simplifying assumptions of our model, where the beam-beam forces are calculated assuming that the beams are of Gaussian shape.

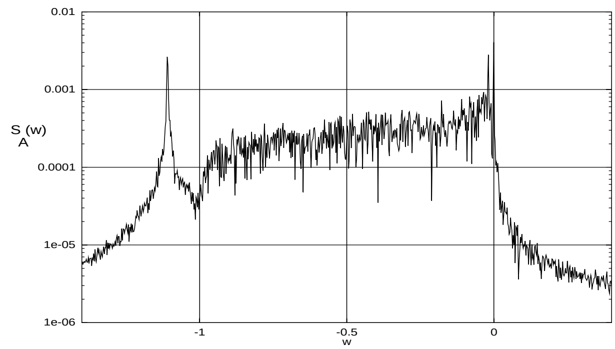

Figure 1: Frequency spectrum of the bunch centroid motion (over 2^{17} turns, $N = 10^4$ macroparticles) for round beams with $\xi_x = \xi_y = \xi = 0.0034$, and $Q = 0.32$. The horizontal axis gives the distance w to the unperturbed tune Q in units of ξ, *i.e.*, w= $(\nu - Q)/\xi$. The vertical axis is the corresponding amplitude on a logarithmic scale. The π- and σ-oscillation modes are clearly visible.

Between the π and the σ-mode in Fig. 1 we also see the *continuum*. This is related to the incoherent oscillation frequencies of individual particles. Due to the non-linearity of the beam-beam interaction particles with different amplitudes experience different focusing force. The result is an incoherent tune spread, which extends from 0 to $-\xi$.

We have found a size dependence on the tune. The size variations as a function of tune can be fitted by the standard dynamic-beta effect $\frac{\beta}{\hat{\beta}} = \sqrt{1 - \left(\frac{2\pi\xi_0}{\sin(2\pi Q)}\right)^2 - 2\pi\xi_0 \cot(2\pi Q)}$ with some modification for the strong-strong case. Integrating the beam-beam force over the Gaussian distribution, the effective beam-beam force is one half that experienced by a single particle near the beam centre. We evaluate then the dynamic beta effect replacing in this equation ξ_0 by $\Xi = \xi_0/2$, see Fig. 2.

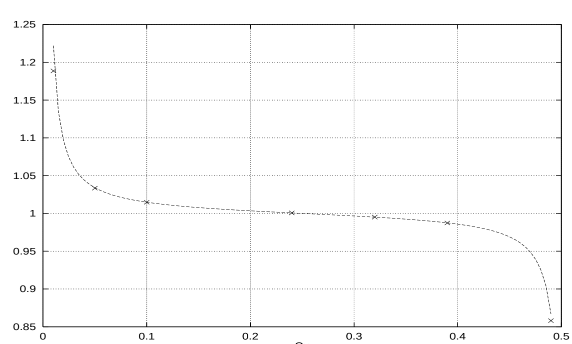

Figure 2: Comparison of the tracking results with the dynamic beta effect. Vertical axis: beam size ratio σ^2/σ_0^2, horizontal axis fractional tune. Points: data obtained from tracking; curve: $\sigma^2/\sigma_0^2 = \hat{\beta}/\beta$ dynamic beta theoretical prediction with $\Xi = \xi_0/2$.

2.2 Landau damping and emittance growth

If the frequency of the π-mode lies within the incoherent tune spread its energy is absorbed by individual particles with similar oscillation frequencies. This phenomenon is known as Landau damping . The fraction of energy which is absorbed by the continuum leads to an irreversible emittance growth. Decreasing the beam-beam parameter ratio r there is a point at which the discrete π-mode joins the continuum [1, 2]. We launch the two beams with an initial horizontal offset $d = 0.2$ (in units of σ_x) and study the spectrum of the π-mode at consecutive intervals (not shown). For the case $r = 1$ the π-mode is well outside the continuum and the amplitude (after some initial loss of energy which is transfered to the continuum) stays constant. For $r = 0.6$, the π-mode lies at $w = -0.9$ (inside the continuum) the amplitude of the π-mode decreases in time. For the case $r = 0.3$ the π-mode is well inside the continuum and rapidly damped. These results confirm the prediction that for current ratios $r \leq 0.6$ the π-mode frequency falls in the incoherent tune spread of the weaker beam [1] and is therefore Landau damped. As a consequence an initial π-mode oscillation will disappear and the beam emittance will grow until the π-mode energy has been completely absorbed. Examples are the two upper curves in Fig. 3. As expected, the final emittance is larger than for $r = 1$, namely $\Delta\epsilon^I/\epsilon_0 \approx 0.01$.

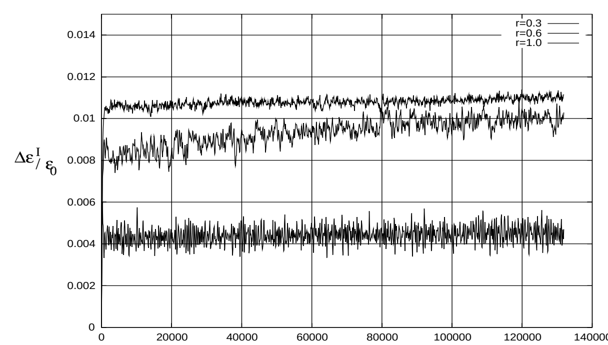

Figure 3: Irreversible emittance growth $\Delta\epsilon^I/\epsilon_0$ (vertical axis) as a function of time (horizontal axis, time in turns) for three different current ratios, $r = 0.3$, $r = 0.6$ and $r = 1$. The beams are perturbed by an initial offset of $d = 0.2$ (in units of σ). For $r \leq 0.6$ the frequency of the π-mode lies in the continuum, and, thus, the mode is Landau damped, and the intrinsic emittance grows until the π-mode energy has been fully absorbed. For $r = 1$ the π-mode is not Landau damped, and carries part of the kick energy. There is an initial emittance growth which is significantly smaller.

2.3 Long range collisions

The vertical and horizontal tunes are chosen equal ($Q_x = Q_y = 0.32$). Since the tune shifts from long range collisions have opposite signs in the two transverse planes, for the LHC an alternating crossing scheme was proposed, where the beams are separated in orthogonal planes at the two main IPs. This reduces the overall incoherent tune shift and tune spread by cancellation of the tune shift between IPs. We consider two closely spaced bunches per beam, and two interaction regions (beam 1=$[a, b]$, beam 2=$[c, d]$). First the bunches are collided head-on (a-c and b-d). We then apply a phase advance of 90° to reach the long range collision region. There the bunch pairs (a-d) and (b-c) are collided with a horizontal separation of L_x and a beam-beam parameter which is n_{par} times stronger than for the primary collision, representing the accumulated effect of $n_{par} = 32$ parasitic collisions around each IP. Subsequently, we advance the phase of the beams to reach the other interaction region and evaluate the head-on collisions (a-d) and (b-d). This is followed by another phase advance of 90° to the long range collision point, where again long range collisions of the pairs (a-d) and (b-c) are applied but this time with a vertical separation of L_y.

The spectrum of the bunch motion is illustrated in Fig. 4. Coherent modes still survive outside the continuum. Collision schemes with and without alternating crossing were compared in Ref. [2], with similar conclusions.

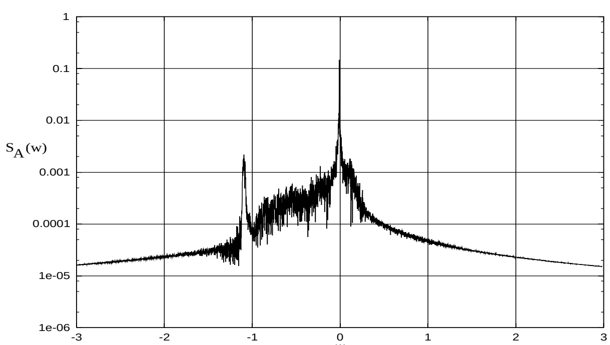

Figure 4: Spectrum in the case of head-on and long range collisions with alternating crossing when each bunch collides head-on at two interaction points, and undergoes long range collisions with $n_{par} = 32$ bunches behind each IP. The horizontal axis the tune distance to the unperturbed betatron frequency in units of the new incoherent tune shift $2 \times \xi$: w= $(\nu - Q)/2\xi$.

3 SEPARATED TUNES

A. Hofmann pointed out that the coherent frequency shifts can be reduced by separating the tunes of the two beams. We simulate this situation operating beam 1 with $Q^{(1)}_{x,y} = 0.32$ and beam 2 with $Q^{(2)}_{x,y} = 0.31$ for $\xi = 0.0034$. External impedances cause additional coherent tune shifts. We model the effect of the ring impedance applying in the vertical plane only, every turn, a localized kick that depends linearly on the bunch centroid position $\Delta v_y = -4\pi \Delta Q_{Z,y} < y >$. This results in a coherent tune shift of the centroid motion (dipole mode), but has no effect on the tunes of individual particles (incoherent spectrum). The continuum of beam 2 extends from $(0.31 - \xi = 0.3066)$ to 0.31, and that of beam 1 from $(0.32 - \xi) = 0.3166$ to 0.32. In the horizontal plane both coupling modes are now inside the incoherent spread of one or the other beam, and Landau damping is restored. But in the vertical plane, and due to the impedance tune shift, the coherent modes emerge from the incoherent spectrum, see Fig. 5.

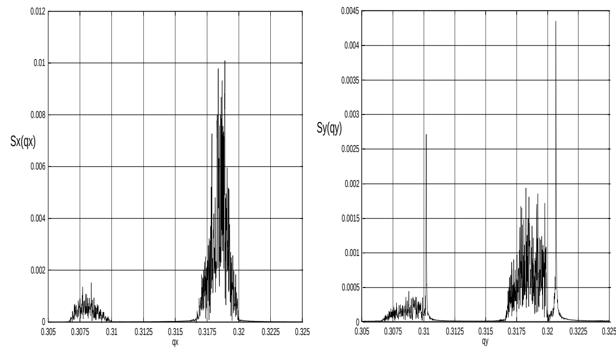

Figure 5: Spectrum in the case of head-on collision of bunch 1 with tune $Q^{(1)}_{x,y} = 0.32$ and bunch 2 with tune $Q^{(2)}_{x,y} = 0.31$. Horizontal axis: tune; vertical axis: the corresponding oscillation amplitude for the horizontal (left) and vertical (right) motion of beam 1. In the horizontal plane Landau damping is restored, but in the vertical plane an additional coherent tune shift of $+20 \times 10^{-4}$ pushes the vertical coherent modes away from the continuum.

4 CONCLUSIONS

We have confirmed that, for equal beam sizes and current ratio $0 < r \leq 0.6$, the π-mode lies within the continuum and is Landau damped. Its energy is transferred to the continuum, leading to an irreversible finite emittance growth. For equal beam-beam parameters of the two beams, we find a π-mode tune shift of -1.1 in units of ξ, sufficiently large to place it outside of the continuum and to lose Landau damping [1]. We have observed a decrease of the beam size with increasing fractional tune, which is explained by the dynamic beta effect.

In the case of two equally strong beams with head-on and long range collisions, coherent modes exist outside of the continuum, even with alternating crossing at two IPs. However, if the betatron tunes of the two beams are sufficiently different, and if the impedance coherent tune shift is lower than $\Delta Q = 2 \times 10^{-3}$, the frequencies of the coherent modes are shifted towards the continuum of one or the other beam and Landau damping can be restored.

REFERENCES

[1] Y.I. Alexahin, *On the Landau damping and decoherence of transverse dipole oscillations in colliding beams.* Particle Accelerators, Volume 59, p. 43 (April 1999).

[2] M. P. Zorzano, F. Zimmermann. *Coherent beam-beam oscillations at the LHC.* LHC project report 314, 1999.