PRODUCTION OF COHERENT X-RAYS WITH A FREE ELECTRON LASER BASED ON AN OPTICAL WIGGLER

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Abstract

The interaction between high-brilliance electron beams and counter-propagating laser pulses produces X-rays via Thomson back-scattering. If the laser source is long and intense enough, the beam electrons can bunch on the scale of the emitted X-ray wavelength and collective effects can occur. In this case, the FEL instability can develop and the system behaves like a free-electron laser based on an optical undulator. Coherent X-rays can be irradiated, with a bandwidth very much thinner than that of the corresponding incoherent emission. We analyse with a 3-D code the transverse effects in the emission. A generalized form of the Pellegrini criterion is validated on the basis of the numerical evidence.

INTRODUCTION

A Thomson back-scattering set-up can be considered in principle as a source of intense X-ray pulses which is at the same time easily tunable and highly monochromatic. Due to recent technological developments in the production of high brilliance electron beams and high power CPA laser pulses, it is now even conceivable to make steps toward their practical realisation [1-2].

The radiation generated in the Thomson back-scattering is usually considered incoherent and calculated by summing at the collector the intensities of the fields produced in single processes by each electron [3]. If the laser pulse is long enough, however, collective effects can establish and become dominant. The system in this range of parameters behaves therefore like a free-electron laser, where the static wiggler is substituted by the optical laser pulse [4].

From the point of view of the theoretical description of the process, the possibility of generate coherent X radiation can be demonstrated with the same set of one-dimensional equations that are used in the theory of high-gain free-electron laser amplifier. However, many aspects of the process are connected with the finite transverse geometry of the electron beam and of the laser and, in order to give a quantitative evaluation of the radiation pulses it is obviously necessary to consider 3D equations [5].

In this paper, we present some particularly interesting data relevant to the solution of the 3-D equations with a discussion of their importance and some conditions for operating in a FEL mode.

MODEL EQUATIONS AND RESULTS

We can write our model equations as:

$$\frac{d}{d\bar{t}}\bar{r}_j(\bar{t}) = \rho\frac{\mathbf{P}_j(\bar{t})}{\bar{\gamma}_j(\bar{t})} \tag{1}$$

$$\frac{d}{d\bar{t}}P_{jz}(\bar{t}) = -\frac{\bar{a}_{L0}^2}{2\rho\gamma_0^2}\frac{1}{\bar{\gamma}_j}\left[\frac{\partial}{\partial\bar{z}}|g|^2\right]_{\bar{x}=\bar{r}_j} - \frac{2}{\bar{\gamma}_j}\text{Real}\left[(g^*\bar{A})_{\bar{x}=\bar{r}_j}e^{i\theta_j(\bar{t})}\right]+.... \tag{2}$$

$$\frac{d}{d\bar{t}}\mathbf{P}_{j\perp}(\bar{t}) = -\frac{\bar{a}_{L0}^2}{2\rho\gamma_0^2}\frac{1}{\bar{\gamma}_j}\left[\bar{\nabla}_\perp|g|^2\right]_{\bar{x}=\bar{r}_j} - \frac{4\eta}{1+\frac{k_L}{k}}\frac{1}{\bar{\gamma}_j}\text{Im}\left[(\nabla_\perp(g^*\bar{A}))_{\bar{x}=\bar{r}_j}e^{i\theta_j(\bar{t})}\right]+.... \tag{3}$$

$$(\frac{\partial}{\partial\bar{t}}+\frac{\partial}{\partial\bar{z}})\bar{A}(\bar{x},\bar{t})-i\eta\bar{\nabla}_\perp^2\bar{A} = b = \frac{1}{N_S}\frac{V_b}{V_b(t)}\sum_S\frac{g(\bar{r}_S(\bar{t}),\bar{t})}{\bar{\gamma}_S(\bar{t})}e^{-i\theta_S(\bar{t})}+... \tag{4}$$

where:

$$\theta_j(\bar{t}) = \frac{k}{2\rho k_L}((1+\frac{k_L}{k})\bar{z}_j(\bar{t})+(\frac{k_L}{k}-1)\bar{t}) \tag{5}$$

and

$$\gamma_j^2 = 1 + \gamma_0^2\rho^2 P_{jz}^2 + \bar{a}_{L0}^2(|g|^2)_{\bar{x}=\bar{r}_j(\bar{t})}+... \tag{6}$$

In the preceding equations the laser of parameter $\bar{a}_{L0} = \frac{e}{mc^2}a_{L0}$ has wavelength $\lambda_L = 2\pi/k_L$, σ_L is the r.m.s. spot radius averaged on the laser intensity, g(xyzt) is the envelope, $\omega_L = ck_L$ the angular frequency . γ_0 is the average value of γ over all electrons of the beam at t=0, $\bar{\gamma}_j = \gamma_j/\gamma_0$, $\mathbf{P}_j = \mathbf{p}_j/\gamma_0\rho$, $\eta = \frac{k_L}{k}\rho$.

For the radiation a single mode expression is assumed, with frequency ω, wavelength λ, amplitude

$$A = -i(\frac{\omega_b^2 a_{L0}}{4\sqrt{2}\omega\omega_L\gamma_0\rho})\frac{e\bar{A}}{mc^2} .$$

The FEL parameter is:

$$\rho = \frac{1}{\gamma_0}\left(\frac{\omega_b^2 a_{L0}^2}{16\omega_L^2}(1+\frac{\omega_L}{\omega})\right)^{\frac{1}{3}} \qquad (7)$$

Furthermore $\bar{t} = 2\rho\omega_L t$ and $\bar{x} = 2\rho k_L x$.

We have developed a three-dimensional code that solves the set of equations (1)-(4), based on a fourth-order Runge-Kutta for the particles and on a finite-difference scheme for the radiation field. An example of solution is provided by a beam with an energy of about 15 MeV (a factor 2 lower than the typical Sparc-PlasmonX case), corresponding to $<\gamma>=30$, with a mean radius $\sigma_0=10$ micron, a total charge of 1 nC, a length $L_b= 200$ μm, corresponding to a beam current of I=1.5 KA. The laser pulse considered in this case has wavelength $\lambda_L=0,8$ μm. Furthermore, the focal spot has radius w_0 of about 50 μm with $a_{L0}=0.8$ so that the radiation has $\lambda= 3,64$ Angstrom and the $\rho = 4.38 \cdot 10^{-4}$. The gain length is about 145 μm, the appearance and the saturation of the collective effects (taking place in 7-12 gain lengths) being contained in 5 psec, a time of the same order of the duration of the laser pulse. The quantum parameter q is 0.5. The energy spread $\Delta\gamma/\gamma$ is $1.\cdot 10^{-4}$ and the initial normalized transverse emittance has been varied from 0 up to 2.

In Fig. 1 the growth of the averaged collective potential amplitude is shown in time ((b), curve (1)), as well as the bunching factor ((a)). The parameters of this calculation are: a flat laser profile inside a region with $w_0=50$ micron, the laser parameter $a_{L0}=0.8$, a value of the initial emittance of $\varepsilon_n=0,88$ μm, a detuning of $\Delta\omega/\omega= -210^{-4}$. The saturation level of the radiation is reached at t=4 psec at $<|A|^2>_{peak}=0,275$, with a total number of photons of $1,86\cdot 10^{10}$, against the $2\cdot 10^8$ provided by the incoherent process. The peak brilliance, for this example, is $3,7\cdot 10^{25}$ photons/(sec mm^2 mr^{2}0.1%), while the coherent power is 15.5 MW.
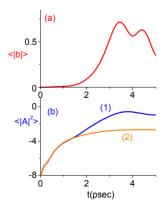
Figure 1: Averaged bunching (a) and logarithm of the radiation intensity (b) versus t in coherent (1) and incoherent (2) case for: $\lambda_L=0.8$ μm, $a_{L0}=0.8$, $\Delta\gamma/\gamma=10^{-4}$, $\Delta\omega/\omega=-210^{-4}$, $\varepsilon_n=0.88$.

In Fig. 2 the spectrum of the radiation is reported versus $\Delta\omega/(\omega\rho)$ for (a) $\varepsilon_n=0,44$ micron and (b) $\varepsilon_n=0,88$ micron.
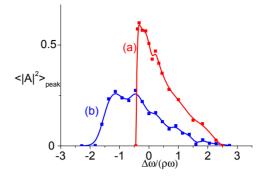
Figure 2: $<|A|^2>_{peak}$ versus $\Delta\omega/(\omega\rho)$ for the case of Fig. 5 and (a) $\varepsilon_n=0,44$ μm and (b) $\varepsilon_n=0,88$ μm.

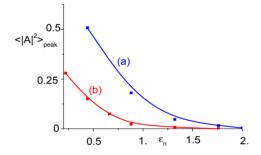We can note an enlargement of a factor two of the bandwidth with increasing emittance.

Figure 3: $<|A|^2>_{peak}$ versus ε_n for the case of fig 5, with $\Delta\omega/\omega=0$, and for: (a) flat laser profile with $w_0=50$ μm and $a_{L0}=0.8$ and (b) Gaussian laser profile with $a_{L0}=0.8$ and $\sigma_L=106$ μm.

In Fig. 3 the dependence of the saturation radiation on the emittance is presented. Curve (a) is relevant to the situation of flat laser pulse with $w_0=50$ μm, while curve (b) shows the more critical situation of Gaussian profile. In this case the quantity σ_L is 106 μm with $a_{L0}=0.8$, increasing consequently the laser power.

We note considerable emission in violation of the Pellegrini criterion [29] for a static wiggler. In fact, in case (a) of Fig 3, for instance, the emittance largely exceed the value $\gamma\lambda/4\pi$, that in this case is about $9\cdot 10^{-4}$ μm. We can justify this result by considering that the line width in a situation dominated by emittance effects can be written as [30]

$$\frac{\Delta\lambda}{\lambda} \approx \frac{\gamma^2\theta^2}{1+a_{L0}^2} \approx \frac{\varepsilon_n^2}{\sigma_0^2} \qquad (8)$$

In order to have considerable emission, we must assume that the linewidth $\Delta\lambda/\lambda < \alpha\rho$, with α a numerical factor not very much larger than 1. Hence, we can write for the emittance

$$\varepsilon_n \leq \sqrt{\alpha\rho}\ \sigma_0 \qquad (9)$$

Considering the definitions of the gain length Lg $=\lambda_L/(4\pi\rho)$ and that of the radiation Rayleigh length $Z_R=2\pi\sigma_R^2/\lambda$, we can express the factor ρ in terms of the ratio Z_R/L_g, obtaining $\rho=\dfrac{Z_R}{L_g}\dfrac{\lambda\lambda_L}{8\pi^2\sigma_R^2}$. Supposing furthermore that the electron beam and the radiation overlap, so that $\sigma_0=\sigma_R$, and remembering the resonance relation in its simpler expression $\lambda_L=4\gamma^2\lambda$, we obtain for an optical undulator

$$\varepsilon_n \leq \sqrt{\alpha}\sqrt{\frac{Z_R}{L_g}}\frac{\lambda}{\sqrt{2\pi}}\gamma \qquad (10)$$

where $\alpha=\dfrac{\delta\omega}{\omega\rho}$.

The usual Pellegrini criterion can be obtained for a static wiggler assuming $Z_R=L_g$ and considering the resonance condition for the static undulator.

Taking into account the fact that in our situation $Z_R/L_g=$ 1.18 10^4, and estimating α=2, we can predict considerable emission up to an emittance value of ε_n=0,3 μm (corresponding to a value of ε_{nx}=0,15 mm), not far from the results of Fig. 8.

The last Fig. 8 shows the most critical effect, i. e. the dependence of the growth of the signal on the transverse energy distribution of the laser in the case of a Gaussian pulse for ε_n=0,44 μm, $\Delta\omega/\omega$=-1.10^{-4}, a_{L0}=0.8. In fact, in this case, a spot size with a radius smaller than 75 micron does not permit the instauration of the instability. The collective signal in this condition, therefore, does not grow.
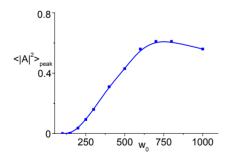

Figure 4: $<|A|^2>_{peak}$ versus w_0 for the case for the Gaussian laser profile for ε_n=0,44 μm, $\Delta\omega/\omega$=-1.10^{-4}, a_{L0}=0.8.

A possible remedy could be the development and use of a flat energy distribution of the laser beam obtained with a guided propagation.

This example is characterized by a choice of the electron beam with larger emittance than in the first case, not far from the best actual experimental values, but with a larger current. However, the requirements on the total energy of the laser and on the stability of the energy transverse profile are in this case particularly demanding.

Another critical issue is the variation in the laser intensity $\Delta=\Delta a_{L0}/a_{L0}$, that leads to a broadening in the spectrum of

$\Delta\lambda/\lambda=2a_{L0}^2\Delta/(1+a_{L0}^2)$. For a FEL operation of the Thomson source, the condition $\Delta\leq\rho\dfrac{(1+a_{L0}^2)}{a_{L0}^2}$ must therefore be added. Assuming a laser pulse duration of $c\tau$= 10 L_g, we can derive a further threshold condition on the laser pulse energy U of the form U>18.6 λ_L/Δ^2, that in our case means U>0.15/ Δ^2J.

CONCLUSIONS

Considerable coherent X-rays radiation is possible as a result of the collective interaction between an electron beam and a counter-propagating laser pulse. If the laser pulse is long enough the FEL instability can develop and a regime of collective effects can establish. The result is an emission at least two order of magnitudes larger than the incoherent one and with a thinner and more peaked spectrum However, the brilliance and the power delivered are a few orders of magnitude smaller than these same quantities for a static wiggler FEL in the X-ray range.

Other critical issues for the appearance of collective effects are connected with: (i) the current density carried by the electron beam which has to be large enough, (ii) the emittance of the packet which has to be not too much larger than that provided by the generalized Pellegrini criterion, (iii) the transverse distribution of the laser pulse which cannot have a sensible variation on the region occupied by the electrons, (iv) the fluctuations of the laser intensity and (v) the large laser energy needed .

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