PECULIARITIES OF THE DOPPLER EFFECT FOR MOVING RADIATIVE PARTICLES IN DISPERSIVE MEDIUM AT EXTREME CONDITIONS

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Abstract

The features of Doppler effect for radiating or absorbing quantum system in motion at parameters close to extreme condition $(\beta n(\omega)\cos\theta)\approx 1$ are studied. It was shown that the correct use of energy and momentum conservation laws, the dependence on radiation and absorption frequencies on $(1-\beta n(\omega)\cos\theta)$ contains a discontinuity. The value of these frequencies has a jump discontinuity of (1+√2). The positions of these discontinuities depend on the value $(1-\beta n(\omega)\cos\theta)$, when normal Doppler effect transforms into abnormal.

INTRODUCTION

The classical notation of the Doppler effect

$$\omega=\Omega_0/\gamma(1-\beta n(\omega)\cos\theta) \quad (1)$$

connects the frequency of radiation ω in laboratory coordinate system with the frequency of the radiation Ω_0 in moving coordinate system, which moves with the velocity $\vec{v}=\vec{\beta}c$, c is the speed of light. Here $n(\omega)$ is the index of refraction; θ is the angle between wave vector $\vec{k}$ in the laboratory frame and velocity **v** of moving system; γ is the gamma factor. The Doppler effect was examined in the first Einstein's work on relativity theory.

For practical use the formula, which connects the frequency of radiation ω in laboratory system with the transition frequency Ω_{21} between two discontinuous energy levels ε_2 and ε_1 of the particle, which is moving with the velocity $\vec{v}=\vec{\beta}c$, is more useful.

In the most simple case of low transition frequency (when it is possible to ignore the recoil effect) it is possible to replace Ω_0 by Ω_{21} in Eq. (1). In this case from Eq. (1) follows, that in free space (at $n(\omega)=1$) the radiation frequency at $\theta=\pi$ and at $\beta\to 1$ has its minimum value $\omega=\omega_{\min}=\Omega_{21}/2\gamma$. The maximum value $\omega=\omega_{\max}=2\gamma\Omega_{21}$ corresponds to the radiation angle $\theta=0$ and $\beta\to 1$. Here $\gamma=1/\sqrt{1-\beta^2}$.

The analysis of the literature has shown that near the condition $\beta n(\omega)\cos\theta=1$, the transformation of radiation and absorption spectra generally are not studied. Unknown is the difference between radiation and absorption frequencies.

Due to strong influence of the recoil effect in the range of $\Delta(\beta,\omega,\theta)$ the radiation and absorption frequencies depend on the system parameters and medium properties.

Peculiarities of absorption and radiation processes in the range of parameters bordering with the condition $\beta n(\omega)\cos\theta=1$ are examined below on the basis of conservation laws

PECULIARITIES OF THE DOPPLER EFFECT IN THE RANGE OF CRITICAL PARAMETERS

The emission of the photon with frequency ω_{rad} and momentum $\Delta\vec{p}=\hbar\vec{k}$ by the particle, that has initial momentum $\vec{p}_{initial}=\vec{p}$, mass m and two motion energy levels - initial (upper) ε_2 and final (lower) ε_1, is characterized by the laws of energy and momentum conservation

$$\begin{aligned}
&\vec{p}_{final}=\vec{p}-\Delta\vec{p};\ \vec{k}=\vec{e}_k n\omega_{rad}/c;\ n\equiv n(\omega_{rad});\\
&E_{initial}=\sqrt{p^2c^2+(mc^2+\varepsilon_2)^2};\\
&E_{final}=\sqrt{(\vec{p}-\Delta\vec{p})^2c^2+(mc^2+\varepsilon_1)^2};\\
&\hbar\omega_{rad}=E_{initial}-E_{final};\\
&\hbar\omega_{rad}=\sqrt{\vec{p}^2c^2+(mc^2+\varepsilon_2)^2}-\\
&\sqrt{(\vec{p}-\Delta\vec{p})^2c^2+(mc^2+\varepsilon_1)^2}
\end{aligned} \quad (2)$$

Excluding from the energy conservation law $\Delta\vec{p}$ and linearizing the expressions

$$(mc^2+\varepsilon_{1,2})^2=m^2c^4+2mc^2\varepsilon_{1,2}^*, \quad (3)$$

we can receive

$$\begin{aligned}
&\hbar\omega_{rad}=\sqrt{p^2c^2+m^2c^4+2mc^2\varepsilon_2^*}-\\
&[p-\hbar(\omega_{rad}/c)n\cos\theta]^2c^2+\\
&[n\hbar(\omega_{rad}/c)\sin\theta]^2c^2+m^2c^4+2mc^2\varepsilon_1^*]^{1/2}
\end{aligned} \quad (4)$$

Here $\varepsilon_{1,2}^*=\varepsilon_{1,2}(1+\varepsilon_{1,2}/2mc^2)$ is so-called efficient energy of every level, θ is the angle between particle momentum $\vec{p}$ and wave vector $\vec{k}$ of the emitted quantum.

Received expression has the exact solution

$$\omega_{rad} = \frac{\gamma mc^2}{\hbar(n^2-1)}[-(1-\beta n\cos\theta) \pm \sqrt{(1-\beta n\cos\theta)^2 + 2\frac{\varepsilon_2^{\ *}-\varepsilon_1^{\ *}}{\gamma^2 mc^2}(n^2-1)} \quad (5)$$

that allows to find expression for the radiation frequency in the whole interval of $(1-\beta n(\omega)\cos\theta)$, including strict equality $(1-\beta n(\omega)\cos\theta)=0$.

Note that the radiation frequency ω_{rad} and the absorption frequency ω_{abs} (it will be analyzed later) have to be positive values in Eq. 5.

Moreover, with the decreasing of the particle speed $(\beta \to 0)$, the condition $\omega_{rad}, \omega_{abs} \to |\varepsilon_2^* - \varepsilon_1^*|/\hbar$ has to be fulfilled.

Let's consider all possible variations of parameters in the problem.

In the case, when $1-\beta n(\omega)\cos\theta > 0$, equation (5) takes the form

$$\omega_{rad} = \frac{\gamma mc^2}{\hbar(n^2-1)}[-(1-\beta n\cos\theta) + \sqrt{(1-\beta n\cos\theta)^2 + 2\frac{\varepsilon_2^{\ *}-\varepsilon_1^{\ *}}{\gamma^2 mc^2}(n^2-1)}] \quad (6)$$

From equation (6) follows that the radiation with positive frequency ω_{rad} is possible only at condition

$$(\varepsilon_2^{\ *}-\varepsilon_1^{\ *})(n^2-1)/\gamma^2 mc^2 > 0 \quad (7)$$

From the equation (6), imposing the condition

$$(1-\beta n\cos\theta)^2 >> 2|\varepsilon_2^{\ *}-\varepsilon_1^{\ *}||n^2-1|/\gamma^2 mc^2, \quad (8)$$

we can find the expression

$$\omega_{rad} \approx \frac{\varepsilon_2^{\ *}-\varepsilon_1^{\ *}}{\gamma\hbar(1-\beta n\cos\theta)}, \quad (9)$$

The condition Eq. (7) corresponds to the point A on the branch AB on fig. 1.

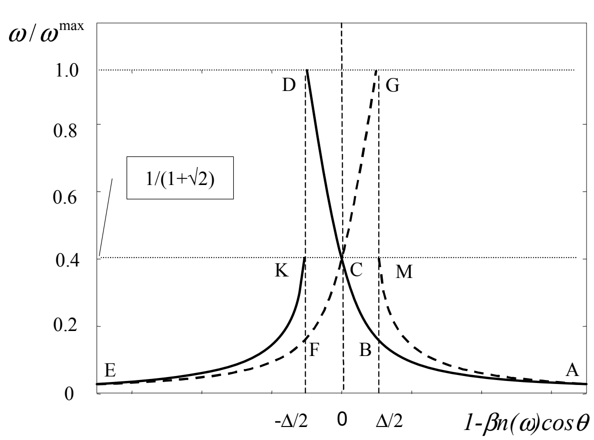

Fig.1. The dependence of the frequencies of radiation (branches AM and GE) and absorption (AD, KE) for the region of normal (AM, AD) and abnormal (GE, KE) Doppler effects. Inside the interval [-Δ/2, Δ/2] the Doppler formula is obviously incorrect. The value ω^{max} is determined by formulas (19) and (28).

In another extreme case

$$(1-\beta n\cos\theta)^2 << 2\frac{|\varepsilon_2^{\ *}-\varepsilon_1^{\ *}|}{\gamma^2 mc^2}|n^2-1| \quad (10)$$

from the same Eq. (6) follows fundamentally different and before this unknown result

$$\omega_{rad} \approx \sqrt{2\frac{mc^2(\varepsilon_2^{\ *}-\varepsilon_1^{\ *})}{(n^2-1)\hbar^2}} \quad (11)$$

This condition corresponds to the point C on the branch BC, presented on fig. 1.

Let's consider the case $(1-\beta n(\omega)\cos\theta) < 0$. If

$$(\beta n\cos\theta - 1)^2 > 2|\frac{(\varepsilon_2^{\ *}-\varepsilon_1^{\ *})}{\gamma^2 mc^2}(n^2-1)|, \quad (12)$$

then Eq. (5) takes the form

$$\omega_{rad} = \frac{\gamma mc^2}{\hbar(n^2-1)}((\beta n\cos\theta-1) - \sqrt{(\beta n\cos\theta-1)^2 + 2\frac{\varepsilon_2^{\ *}-\varepsilon_1^{\ *}}{\gamma^2 mc^2}(n^2-1)}) \quad (13)$$

Equation (13) has a sense (frequency is positive) at the fulfillment of the next condition

$$\frac{(\varepsilon_2^{\ *}-\varepsilon_1^{\ *})}{\gamma^2 mc^2}(n^2-1) < 0 \cdot \quad (14)$$

The condition $1-\beta n(\omega)\cos\theta < 0$ is possible only when $(n^2-1) > 0$. In that case the condition (12) can be fulfilled only at $\varepsilon_2^* < \varepsilon_1^*$, that corresponds to the abnormal Doppler effect.

From (13) we can find solution

$$\omega_{rad} \approx \frac{\varepsilon_1^{\ *}-\varepsilon_2^{\ *}}{\gamma\hbar(\beta n\cos\theta-1)}[1-\frac{(\varepsilon_1-\varepsilon_2)(n^2-1)}{2\gamma^2 mc^2(\beta n\cos\theta-1)^2}], \quad (15)$$

that is different from the approximate solution (1) only by small values.

The equation (15) determines the branch EK on fig.1 and it is seen that the radiation frequency is increasing with decreasing of the absolute value of the difference $(\beta n(\omega)\cos\theta - 1)$. The maximum value of the radiation frequency, Eq. (11), corresponds to the point K.

In another case

$$(\beta n\cos\theta-1)^2 < 2|\frac{(\varepsilon_2^{\ *}-\varepsilon_1^{\ *})}{\gamma^2 mc^2}(n^2-1)| \quad (16)$$

equation (6) takes the form

$$\omega_{rad} = \frac{\gamma mc^2}{\hbar(n^2-1)}((\beta n\cos\theta-1) + \sqrt{(\beta n\cos\theta-1)^2 + 2\frac{\varepsilon_2^{\ *}-\varepsilon_1^{\ *}}{\gamma^2 mc^2}(n^2-1)}) \quad (17)$$

Analysis of equations (16) and (17) demonstrates, that the second term under the radical in (17) is positive

$$\frac{(\varepsilon_2^{\ *}-\varepsilon_1^{\ *})}{\gamma^2 mc^2}(n^2-1) > 0. \quad (18)$$

From the solution (6a) follows, that the radiation frequency decreases at the decrease of the absolute value of the difference $\beta n(\omega)\cos\theta - 1$. This solution corresponds to the branch DC on fig.1 and corresponds to the normal Doppler effect $(\varepsilon_2^* > \varepsilon_1^*)$ in the parameters range $(n^2-1) > 0$ and $(1-\beta n(\omega)\cos\theta) < 0$.

The maximum value of the radiation frequency on this branch in the point D equals

$$\omega_{rad}^{\max} \approx \sqrt{2\frac{mc^2(\varepsilon_2^* - \varepsilon_1^*)}{(n^2-1)\hbar^2}}(1+\sqrt{2}) \quad (19)$$

and in (1+√2) times exceeds the maximum value ω_{rad} in the point K on the branch EK, see Eq. (11).

It can be seen that the radiation frequency is discontinuous. The point, where ω_{rad} is discontinuous, is

$$(1-\beta n\cos\theta)_{K,D} \equiv -\Delta/2 = -\sqrt{2\frac{(\varepsilon_2^* - \varepsilon_1^*)}{\gamma^2 mc^2}(n^2-1)} \quad (20)$$

The reason of such behavior of ω_{rad} is connected with the fact that stepwise transition from normal to abnormal Doppler effect takes place when $(1-\beta n(\omega)\cos\theta) < 0$ in the points K,D, while at standard effect treatment the same transition takes place when $(1-\beta n(\omega)\cos\theta) = 0$. Similarly, it is possible to consider the process of photon absorption by the moving particle, internal state of which is characterized by two energy levels.

After similar derivations as for radiation case, we can find the absorption frequency

$$\omega_{abs} = \frac{\gamma mc^2}{\hbar(n^2-1)}((1-\beta n\cos\theta) \mp \mp\sqrt{(1-\beta n\cos\theta)^2 - 2\frac{\varepsilon_2^* - \varepsilon_1^*}{\gamma^2 mc^2}(n^2-1)}) \quad (21)$$

If $(1-\beta n(\omega)\cos\theta) > 0$, then equation (21) takes the form

$$\omega_{abs} = \frac{\gamma mc^2}{\hbar(n^2-1)}((1-\beta n\cos\theta) - -\sqrt{(1-\beta n\cos\theta)^2 - 2\frac{\varepsilon_2^* - \varepsilon_1^*}{\gamma^2 mc^2}(n^2-1)}) \quad (22)$$

At the condition

$$(1-\beta n\cos\theta)^2 > \left| 2\frac{(\varepsilon_2^* - \varepsilon_1^*)}{\gamma^2 mc^2}(n^2-1) \right| \quad (23)$$

necessary requirement $\omega_{abs} > 0$ can be satisfied when

$$\frac{(\varepsilon_2^* - \varepsilon_1^*)}{\gamma^2 mc^2}(n^2-1) > 0 \quad (24)$$

This corresponds to the normal Doppler effect $(\varepsilon_2^* > \varepsilon_1^*)$. In this case the branch AM on fig.1 describes solution (22). The maximum absorption frequency on this branch corresponds to the point D and equals

$$\omega_{abs} \approx \sqrt{2\frac{mc^2(\varepsilon_2^* - \varepsilon_1^*)}{(n^2-1)\hbar^2}} \quad (25)$$

On the other hand, when

$$(1-\beta n\cos\theta)^2 < \left| 2\frac{(\varepsilon_2^* - \varepsilon_1^*)}{\gamma^2 mc^2}(n^2-1) \right|, \quad (26)$$

absorption frequency is positive, when the following condition is fulfilled

$$\frac{(\varepsilon_2^* - \varepsilon_1^*)}{\gamma^2 mc^2}(n^2-1) < 0 \quad (27)$$

This corresponds to the abnormal Doppler effect when $(1-\beta n(\omega)\cos\theta) > 0$. At classical treatment only normal Doppler effect takes place at such condition. In this case equation (22) is described by the branch GC on fig.1.

The maximum absorption frequency on the point G equals

$$\omega_{abs}^{\max} \approx \sqrt{2\frac{mc^2(\varepsilon_2^* - \varepsilon_1^*)}{(n^2-1)\hbar^2}}(1+\sqrt{2}) \quad (28)$$

and (1+√2) times exceeds the maximum value on the point M (See Eq. (25)). The coordinate of this point is equal

$$(1-\beta n\cos\theta)_{G,M} \equiv \Delta/2 = \sqrt{2\frac{(\varepsilon_2^* - \varepsilon_1^*)}{\gamma^2 mc^2}(n^2-1)} \quad (29)$$

In the case, when $1-\beta n(\omega)\cos\theta < 0$, expression (21) takes the form, different from the Eq. (22)

$$\omega_{abs} = \frac{\gamma mc^2}{\hbar(n^2-1)}(-(\beta n\cos\theta - 1) + +\sqrt{(\beta n\cos\theta - 1)^2 - 2\frac{\varepsilon_2^* - \varepsilon_1^*}{\gamma^2 mc^2}(n^2-1)}) \quad (30)$$

The solution has physical meaning when the following condition is fulfilled

$$\frac{(\varepsilon_2^* - \varepsilon_1^*)}{\gamma^2 mc^2}(n^2-1) < 0, \quad (31)$$

in this case $(1-\beta n(\omega)\cos\theta) < 0$ can be fulfilled only when $\varepsilon_2^* < \varepsilon_1^*$, that corresponds to the transition from the upper level ε_1^* to the lower level ε_2^* at abnormal Doppler effect. This solution corresponds to the branch EC on fig.1.

CONCLUSION

Peculiarities of quanta radiation and absorption processes at critical and extreme conditions examined above demonstrate complicated radiation and absorption spectra structure. The structure of spectra depends on degree of proximity to the condition $1-\beta n(\omega)\cos\theta$ and make up for the deficiency in such fundamental classical phenomenon as the Doppler Effect. It was shown that the maximum frequency of radiation or absorption corresponds to symmetrical deviations from extreme condition.

It is important to notice that the main spectrum peculiarities don't correspond to the exact condition $\beta n(\omega)\cos\theta = 1$, but are removed symmetrically relative to this point in both directions.