TRAJECTORY STABILITY MODELING AND TOLERANCES IN THE LCLS*

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Abstract

To maintain stable performance of the Linac Coherent Light Source (LCLS) x-ray free-electron laser, one must control the electron trajectory stability through the undulator to a small fraction of the beam size. BPM-based feedback loops running at 120 Hz will be effective in controlling jitter at low frequencies less than a few Hz. On the other hand, linac and injector stability tolerances must be chosen to limit jitter at higher frequencies. In this paper we study possible sources of high frequency jitter, including: 1) steering coil current regulation; 2) quadrupole magnet transverse vibrations; 3) quadrupole current regulation with transverse misalignments; 4) charge variations coupled to jitter through transverse wakefields of misaligned RF structures; and 5) bunch length variations coupled through coherent synchrotron radiation in the bunch compressor chicanes. Based on this study, we set component tolerances and estimate expected trajectory stability in the LCLS.

INTRODUCTION AND METHOD

The electron transverse beam size in the LCLS undulator is $\sigma_{x,y} \approx 37$ μm rms. It is estimated that a trajectory with centroid offset of $|x|, |y| = \sigma_{x,y}/10$ (≈ 4 μm) will reduce the FEL power by about 2%. Some x-ray experiments, however, may wish to control the x-ray pointing to this level or better, so we set a goal here of trajectory rms stability in each plane of $|x|, |y| \leq \sigma_{x,y}/10$.

Rather than continue with this simple parameterization, we introduce the dimensionless, normalized trajectory amplitude, $A_{x,y}$, which includes both spatial (x, y) and angular (x', y') betatron oscillation components.

$$A_x \equiv \sqrt{\frac{x^2 + (x\alpha + x'\beta)^2}{\varepsilon\beta}} \tag{1}$$

Here x and x' are the (horizontal) trajectory centroid offsets at any point along the accelerator, and β, α, and ε are the Twiss parameters and geometric transverse emittance at that same point. (The vertical amplitude is similarly defined as A_y). For a free betatron oscillation, this amplitude is constant along the accelerator and independent of energy. At the source point where the trajectory receives purely an angular kick of x' (*i.e.*, $x = 0$), the amplitude reduces to:

$$A_x = x'\sqrt{\beta/\varepsilon} \tag{2}$$

For N uncorrelated kicks along the accelerator, the final rms trajectory amplitude (squared) in the undulator is:

$$A_x^2 = \sum_{i=1}^{N} x_i'^2 \frac{\beta_i}{\varepsilon_i}, \tag{3}$$

where ε_i is the local energy-dependent emittance ($\varepsilon_i = \varepsilon_N/\gamma_i$) at each kick, and we have the goal $A_{x,y} \leq 1/10$.

A total jitter tolerance budget can be formed by summing in quadrature each device's chosen kick tolerance, x_t', divided by its calculated sensitivity, x_s',

$$A_x^2 = \frac{1}{10^2}\sum_{i=1}^{N}\left(\frac{x_t'}{x_s'}\right)_i^2, \tag{4}$$

where the sensitivity is defined for the full kick of $A_x = 1/10$ ($x_s' = \sqrt{\varepsilon/\beta}/10$), but the tolerance, $x_t' \ll x_s'$, is chosen much smaller than the sensitivity in order to set the total amplitude over all kicks to $A_x \leq 1/10$. With this arrangement in Eq. (4), it is clear that choosing all tolerances at $x_t' \leq x_s'/\sqrt{N}$ produces the desired result $A_x \leq 1/10$.

This uniform treatment, however, does not leave freedom to weight tolerances based on technical difficulty, leaving some too tight and others trivially loose. A more realistic budget is formed by grouping tolerances into a few discrete levels, loosening challenging tolerances but holding tight on more standard ones. Such a budget, with three groupings ($N_1 + N_2 + N_3 = N$), is expressed by:

$$A_x^2 = \frac{1}{10^2}\left[\sum_{i=1}^{N_1}\left(\frac{x_{t1}'}{x_{s1}'}\right)_i^2 + \sum_{j=1}^{N_2}\left(\frac{x_{t2}'}{x_{s2}'}\right)_j^2 + \sum_{k=1}^{N_3}\left(\frac{x_{t3}'}{x_{s3}'}\right)_k^2\right].$$

STEERING COILS

We now apply this tolerance budgeting method to our first component of jitter: dipole steering coil current regulation. The sensitivity of each steering coil is $x_s' = \sqrt{\varepsilon/\beta}/10 = e\int|B|dl/p$, where e is the electron charge, p is the beam momentum at this magnet, and $\int|B|dl$ is the length-integrated magnetic field required to produce this kick. The full LCLS design optics listing provides the values of p, β, and ε (with $\varepsilon_N = 1.2$ μm) at each location, allowing the calculation of the sensitivity of each corrector. Figure 1 shows the sensitivity for each of 242 corrector coils along the LCLS accelerator in both planes (121 per plane), each expressed in terms of the integrated field which produces $A_{x,y} = 1/10$.

Most of the corrector coils in the SLAC linac regulate at 0.003% rms [1] of their maximum strength ($(\int Bdl)_{max} \approx$ 60 G-m), producing an rms field stability of 0.002 G-m over one minute or less. Normalizing these tolerance levels to the sensitivities in Fig. 1, and including the various

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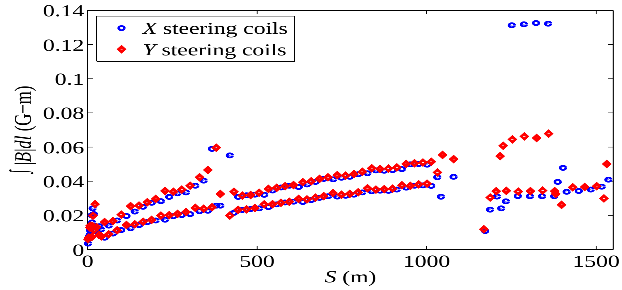

Figure 1: Sensitivity per steering coil along LCLS, expressed as fields which each produce $A_{x,y} = 1/10$.

other corrector types of the LCLS in the full sum, produces an expected rms jitter amplitude for all corrector coils per plane of $A_{x,y} \approx 6\%$, with rms regulation tolerances ranging from 0.003% to 0.01% of maximum field.

The main dipole magnets used in the chicanes and dogleg systems are grouped in series so that power supply variations do not affect the undulator trajectory. Some have independently powered weak trim coils, but at 0.01% rms regulation, these 12 trims add only 2% x-trajectory jitter.

QUADRUPOLE VIBRATIONS

Applying the budget to transverse vibrations of focusing magnets (quadrupoles and two solenoids), the sensitivity of each magnet is $x'_s = \sqrt{\varepsilon/\beta}/10 = \sigma_{\Delta x}/|f|$, where $\sigma_{\Delta x}$ is the rms transverse vibration amplitude of the magnet and f is the magnet's focal length. Figure 2 shows the vibration sensitivity for each of 148 focusing magnets along the LCLS accelerator in both planes, in terms of x and y vibration amplitudes ($\sigma_{\Delta x}, \sigma_{\Delta y}$) which produce $A_{x,y} = 1/10$.

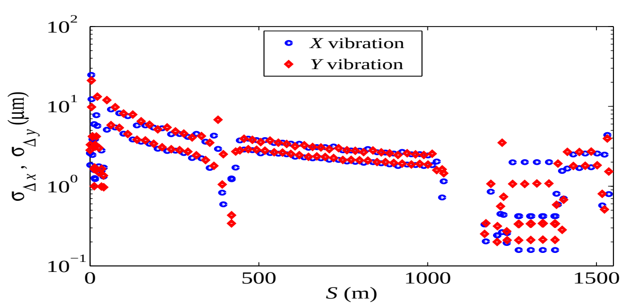

Figure 2: Vibration sensitivity per focusing magnet along LCLS, expressed as x and y vibration amplitudes ($\sigma_{\Delta x}, \sigma_{\Delta y}$) which each produce $A_{x,y} = 1/10$.

To generate the budget for vibration, we choose three tolerance groups of 500 nm, 100 nm, and 50 nm rms. Assigning each magnet to a group based on its sensitivity we find 12 magnets within the 500-nm group, 101 magnets in the 100-nm group, and 35 in the 50-nm group. This vibration component of the budget produces a jitter level of $A_{x,y} \approx 10\%$. Although this level is rather high, the small set of vibration measurements available on existing linac quadrupoles [2] suggests it will be difficult to expect a lower level. Note that most of the 100-nm magnets are existing with a few confirmed at this level, whereas all of the 50-nm magnets are new for the LCLS, so supports can be designed with this demanding goal in mind.

QUADRUPOLE REGULATION

Quadrupole and solenoid magnets with imperfect current regulation and transverse misalignments also add trajectory jitter. The sensitivity of each magnet is $x'_s = \sqrt{\varepsilon/\beta}/10 = (\sigma_I/I)|\Delta x|/|f|$, where Δx is the static misalignment, σ_I/I is the rms relative current regulation of the magnet, and f is the magnet's focal length. Figure 3 shows the regulation sensitivity for each of 156 focusing magnets, each with 200-μm misalignment, expressed in terms of the relative current change to produce $A_{x,y} = 1/10$.

Again, to generate the budget for misaligned quadrupole regulation, we choose three tolerance groups of 0.1%, 0.05%, and 0.025% rms. Assigning each magnet to a group based on its sensitivity we find 14 magnets in the 0.1% group, 104 magnets in the 0.05% group, and 38 in the 0.025% group. This component of the budget produces a jitter level of $A_{x,y} \approx 5\%$, based on expected regulation levels of 0.01 to 0.02% over a few seconds. Note that even though a few sets of magnets are powered in series on a single power supply, their misalignments are uncorrelated and random with an estimated 200-μm rms, so the resulting kicks at each magnets are also uncorrelated.

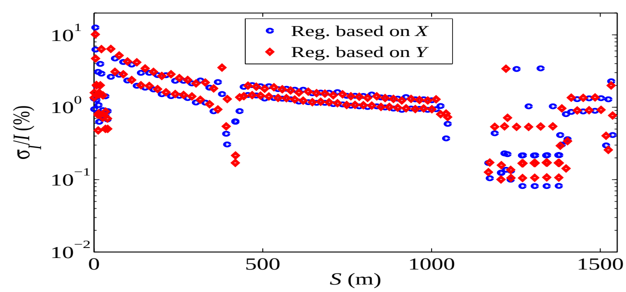

Figure 3: Regulation sensitivity per focusing magnet, each with 200-μm misalignment, along LCLS, expressed as relative current change which each produces $A_{x,y} = 1/10$.

WAKEFIELDS AND CHARGE JITTER

Trajectory jitter can also be caused by pulse-to-pulse bunch charge variations coupled with the transverse wakefields of misaligned RF accelerating structures. The point-charge wake function for the SLAC S-band structure (~300 3-m structures in the LCLS) can be approximated by [3]

$$W_x(z) \approx \mathcal{A}\frac{4Z_0 s_\perp c}{\pi a^4}\left[1 - \left(1 + \sqrt{\frac{z}{s_\perp}}\right)e^{-\sqrt{z/s_\perp}}\right], \quad (5)$$

where, Z_0 is the free-space impedance (377 Ω), $s_\perp$ is the characteristic length (0.56 mm), a is the iris radius (11.6

mm), and $\mathcal{A} \approx 1.1$. For X-band (one 60-cm structure in LCLS), based on Ref. [4] and fit to the same form as Eq. (5), we have $s_\perp \approx 0.40$ mm, $a \approx 4.72$ mm, and $\mathcal{A} \approx 1.00$.

For a transverse wakefield, the kick angle along the bunch-length coordinate, z, per RF structure is

$$x'(z) = \frac{N_e e^2 L \Delta x}{\gamma m c^2 l_b} \int_{-l_b/2}^{z} W_x(z - z')dz', \qquad (6)$$

where N_e is the bunch population, L the accelerating structure length, Δx the structure misalignment, and for simplicity we use a uniform distribution with full-width bunch length l_b. Since the structure is short compared to the betatron wavelength, we neglect both the betatron phase variation and the acceleration over the structure. Then taking the centroid as $x'(z=0) = x'_0$, the mean kick angle is

$$\begin{aligned} x'_0 &= \mathcal{B} N_e \Delta x \equiv \mathcal{A}\frac{4 Z_0 s_\perp e^2 L}{\pi \gamma m c a^4 l_b} N_e \Delta x \\ &\times \left[\frac{l_b}{2} - 6 s_\perp + \left(3\sqrt{2 l_b s_\perp} + l_b + 6 s_\perp\right) e^{-\sqrt{\frac{l_b}{2 s_\perp}}}\right] \end{aligned}$$

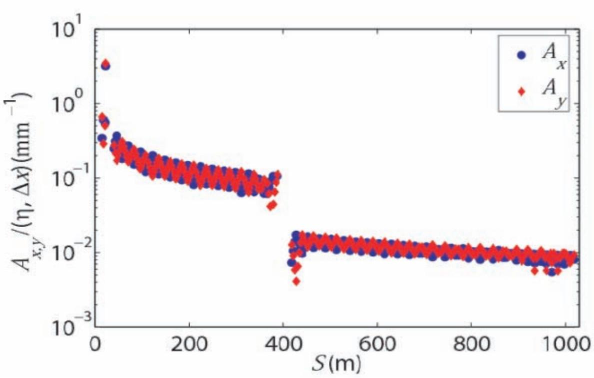

Figure 4: $A_{x,y}(\eta, \Delta x)$ normalized to charge jitter, η, and cavity alignment, Δx, along the linac.

The kick amplitude due to a charge variation is

$$A_x = x'_0(\eta)\sqrt{\beta/\varepsilon} = \Delta x \mathcal{B} N_{e0} \eta \sqrt{\gamma\beta/\varepsilon_N}, \qquad (7)$$

where $\eta \equiv \Delta N_e / N_{e0}$ is the relative charge jitter and we retain only this jitter-dependent term. Similarly, we can look at the accumulated amplitude after N uncorrelated kicks,

$$A_x^2 = \sum_{i=1}^{N} \Delta x_i^2 \mathcal{B}_i^2 N_{e0}^2 \eta^2 \frac{\gamma_i \beta_i}{\varepsilon_N} = \frac{(\sigma_X N_{e0} \eta)^2}{\varepsilon_N} \sum_{i=1}^{N} \mathcal{B}_i^2 \gamma_i \beta_i,$$

with σ_X the rms RF structure misalignments over the linac.

For the entire LCLS accelerator, after inserting values, including $eN_{e0} = 1$ nC, we have

$$\begin{aligned} A_x &\approx (3.6\ \text{mm}^{-1})|\eta|\sigma_X \\ A_y &\approx (3.9\ \text{mm}^{-1})|\eta|\sigma_Y \end{aligned} \;. \qquad (8)$$

As an example, we take $\sigma_X = \sigma_Y = 200$ μm, an rms charge jitter of 2% (LCLS specification), and find the trajectory jitter due to wakefields is only $A_{x,y} \approx 2\%$. The jitter induced by bunch length variations is smaller yet.

Examining the relation more closely, noting Eq. (7) and

$$A_x = x'_0 \eta \sqrt{\gamma\beta/\varepsilon_N}, \qquad (9)$$

we see that small γ and large β increase the kick. Figure 4 shows $A_x/(\eta\Delta x)$ for all structures along the linac. Due to the $1/a^4$-dependence of the wakefield, the largest jitter contribution is from the one 60-cm long X-band structure at 260 MeV. The total effect is, however, quite small.

CSR AND BUNCH LENGTH JITTER

Bunch length variations can also cause trajectory jitter due to coherent synchrotron radiation (CSR) energy loss in the 2nd bunch compressor chicane (BC2). Charge variations also induce jitter, but bunch length jitter dominates.

Numerical calculations are much more accurate than simplified formulas here, and for the BC2 chicane we find

$$A_x \approx 2(\Delta\sigma_z/\sigma_{z0})_{rms}. \qquad (10)$$

Note that only the x trajectory is affected by CSR in the horizontal dipole magnets. With rms bunch length jitter $(\Delta\sigma_z/\sigma_{z0})_{rms} \approx 10\%$, we can expect a fairly large CSR effect: $A_x \approx 20\%$, $A_y = 0$. This will improve if better RF phase stability is eventually achieved ($< 0.1^\circ$ rms).

CONCLUSION

Assuming uncorrelated jitter sources (given the wide variety of sources this seems to be a reasonable assumption) and adding up these six main components in x and y, plus drive laser pointing jitter on the cathode, we have the following estimated undulator trajectory stability, which is dominated in the x-plane by CSR (see Table 1). This is beyond our goal, but considered a realistic estimate.

Table 1: Summary of undulator trajectory jitter.

Mechanism	A_x **(%)**	A_y **(%)**
steering coils	6	6
main dipole trims	2	0
magnet vibration	10	10
misaligned quad. regulation	5	5
wakefields + charge jitter	2	2
BC2 CSR + σ_z jitter	20	0
Drive laser pointing jitter	3	3
Total →	**24**	**13**

REFERENCES

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[4] K.L.F. Bane, SLAC NLC-Note 9, Feburary 1995.