ON THE FEASIBILITY OF A SPIN DECOHERENCE MEASUREMENT*

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Abstract

In this paper, I study the feasibility of making a turn-by-turn spin measurement to extract the spin tune from a polarized beam injected perpendicular to the stable spin direction. For the ideal case of a zero-emittance beam with no spin-tune spread, there would be no spin decoherence, and a measurement of the spin tune could easily be made by collecting turn-indexed polarization data over several million turns. However, in a real beam there is a momentum spread which provides a spin-tune spread. With a coasting beam the tune spread can cause decoherence of the spins resulting in a fast depolarization of the beam in as few as a thousand turns. With synchrotron oscillations the decoherence time can be greatly increased, so that a measurement becomes feasible with summation of the turn-by-turn data from a reasonable number of bunches ($\lesssim 100$). Three cases of 1, 2, and $2\frac{1}{2}$ Siberian snakes are considered. By using spin tune measurements for both the single and double snake cases, we could vastly improve the calibration of the optimum settings for the RHIC snakes.

INTRODUCTION

A pair Siberian snakes are used in each of the RHIC rings[1] to lock the spin tune at $\frac{1}{2}$ and to maintain a vertical stable spin direction of the closed orbit. In order to study the decoherence of spins in RHIC, it is proposed to inject polarized protons into RHIC with polarization perpendicular to the stable spin direction or at least with a large component transverse to the stable direction. In principal, if the polarization could be measured from turn to turn, then for spin tunes near $\frac{1}{2}$ we would see a spin flip every turn. Such oscillations around the stable spin direction have been seen previously[2]. Since the polarization asymmetries are small, an accurate measurement of polarization requires of order 10^7 events in the CNI polarimeter[1]. For a single bunch in RHIC the event rate at injection is usually less than one event every turn, so the observation of the spin rotation will not be seen from turn to turn. However in some cases, it should be observable in the FFT of the signal over thousands of turns. It only requires enough events to observe a definite signal in the FFT, rather than a precise value of polarization. The signal can be enhanced by adding data from multiple injections, since the injected polarization direction will always be the same on the first turn. To measure decoherence, it will be advantageous to measure the polarization with more accuracy, so that we can see the decay of the oscillating component perpendicular to the stable spin direction. In this paper I present preliminary simulations with a simple Monte Carlo event generator for the polarimeter and simple spin tracking in order to understand how to analyze such turn-by-turn polarization data.

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SIMULATION OF POLARIZATION OSCILLATIONS

The polarization of a bunch of N protons is defined as

$$\vec{P} = \frac{1}{N}\sum_{j=1}^{N} \hat{S}_j \tag{1}$$

where $\hat{S}_j$ is the normalized unit vector pointing along the $j^{\rm th}$-proton's spin. The unit vectors $\hat{x}$, $\hat{y}$, and $\hat{z}$ are respectively in the radial, vertical, and beam directions. The RHIC polarimeters are capable of measuring only components of polarization in the transverse plane with respective vertical and radial projections: $P_y = \hat{y}\cdot\vec{P}$ and $P_x = \hat{x}\cdot\vec{P}$.

Decoherence of polarization was simulated by generating a Gaussian distribution in longitudinal phase space of a number of particles (typically $N = 300$). The distribution was matched to the rf bucket with parameters: rf voltage of $V_{\rm rf} = 90$ kV, harmonic number $h = 360$, transition gamma $\gamma_t = 22.98$, and ring circumference $L = 3833.845$ m. All particles were assumed to have the same polarization at injection; however, a reduction factor to 50% polarization was later applied before generating events in the CNI polarimeter.

At fixed energy for the synchronous particle, the proton spins were tracked with energy oscillations, but no betatron oscillations; the distribution of particles was assumed to have zero transverse emittances. An average polarization for the distribution was calculated every turn. This average polarization was then projected onto the transverse plane, and random events were generated and binned in azimuth into the six detectors and written to a file every turn. Typically, it was assumed that on average there would be about one recoil carbon detected every two bunch crossings. The binned results for each turn were summed over multiple bunches with identical polarizations to increase statistics.

ONE SNAKE

The spin tune for a flat ring with no snakes is $\nu_{\rm s} = G\gamma$. Here $G = (g-2)/2 = 1.7928$ and γ is the Lorentz factor. A pair snakes on opposite sides of each RHIC ring have rotation axes relative to the beam of $45°$ and $135°$ degrees to lock $\nu_{\rm s}$ at $\frac{1}{2}$ with a vertical stable spin direction. With both snakes the stable spin direction is vertical giving spin-up in one half of the ring and spin-down in the other half.

When only one snake is turned on, the stable spin direction will lie in the horizontal plane. Normally bunches from the AGS have been injected into RHIC with essentially vertical polarization[3] at either $G\gamma = 45.5$ or 46.5. So with a single snake, a bunch with spin-up on the first pass by the polarimeter would be spin-down on the next turn and continue alternating between up and down every turn.

With a single snake generating a spin rotation of μ about an axis in the horizontal plane, stop bands in the spin tune open up around integer values of $G\gamma$. For a full snake $\mu = \pi$, and $\nu_{\rm s} = N + {}^1\!/\!_2$ where N is an integer close to $G\gamma$. For small deviations of $G\gamma$ away from $N + {}^1\!/\!_2$ and μ from $180°$, the shift of spin tune from ${}^1\!/\!_2$ is

$$\delta\nu_{\rm s} \simeq \pm\tfrac{1}{2}\,\delta(G\gamma)\,\delta\mu. \qquad (2)$$

We should be able to set the extraction energy to within 0.05 of the desired value in units of $G\gamma$. I expect that the snake rotation angles (μ) in RHIC are within $0.5°$ of $180°$.

By analogy with betatron oscillations, a spin chromaticity may be defined as

$$\xi_{\rm sp} = p\,\frac{d\nu_{\rm s}}{dp} = \beta^2 G\gamma\,\frac{d\nu_{\rm s}}{d(G\gamma)}. \qquad (3)$$

At the half integer, $\xi_{\rm sp}$ is largest, so we should expect the fastest decoherence of spins perpendicular to the stable spin direction to appear when $G\gamma$ is halfway between integers.

At integer values of $G\gamma$, the chromaticity is zero. Having $\xi_{\rm sp} = 0$ at the imperfection resonances allows complete spin flipping of all the spins with a partial snake at integer imperfection resonances as in the AGS. The top plot of Fig. 1 shows decoherence of the spins without synchrotron oscillations for several values of $G\gamma$; note the shortest decoherence time is at 46.5 with increasing decoherence times as $G\gamma$ is increased towards 47.

With a momentum spread, the spins of different energies precess at different rates and tend to move apart, but as the energies oscillate, the energies of the lower and higher momentum particles become reversed so that the spins which have moved apart then move back towards each other. In the lower plot of Fig. 1 synchrotron oscillations are included so the spin coherence oscillates for $G\gamma$ away from the integer, and at 46.5 the average decoherence time is much longer than without synchrotron oscillations. To make a quick estimate of the decoherence time in the number of turns, we can ask: how many turns does it take for a $+1\sigma$ particle in the momentum distribution to advance $180°$ ahead of the synchronous particle? This gives the rough estimate of

$$N_{\rm decoh} \simeq \frac{1}{\pi\xi_{\rm sp}(\sigma_p/p)} = \frac{1}{\pi\times 0.20\times 0.00117} \simeq 1360 \text{ turns} \qquad (4)$$

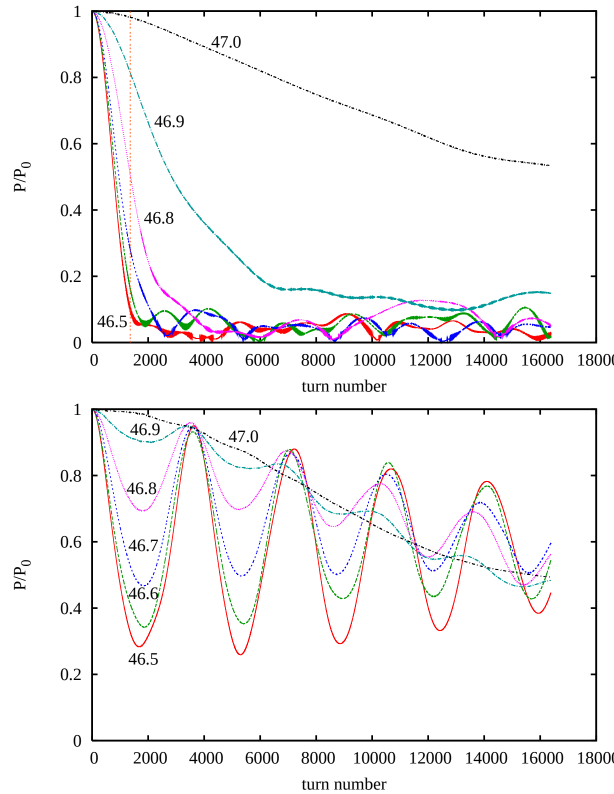

Figure 1: Simulated spin decoherence of a bunch of 300 particles with longitudinal emittance 0.3 eVs. Curves of decreasing negative slopes from $G\gamma = 46.5$ to 47 in steps of 0.1 are shown. The upper plot without synchrotron oscillations shows a rapid decrease in polarization, while the lower plot with synchrotron oscillations ($V_{\rm rf} = 90$ kV, $Q_{\rm sy} = 3\times10^{-4}$) shows longer coherences of an oscillatory nature for $G\gamma$ away from integral values. In both cases, spins were tracked with the snake detuned to $\mu = 179.5°$. The light vertical line in the upper plot indicates the decoherence estimate of $N_{\rm decoh} = 1360$ as described in the text.

when there are no synchrotron oscillations. With synchrotron oscillations the spins recohere after approximately one synchrotron period:

$$\frac{1}{Q_{\rm sy}} \simeq \frac{1}{0.000299} = 3344 \text{ turns}. \qquad (5)$$

MORE SNAKES

With both full snakes in a RHIC ring, the stable spin direction at injection is vertical, so to measure decoherence, we would want to inject the bunches with horizontal polarization. Spins of different energies which decohere in one half of the ring are flipped over in the other half, so they recohere in the other half of the ring. This should lead to much longer decoherence times as shown in Fig. 2.

In order to inject horizontally polarized protons into RHIC, we would either need to add a 50% snake in the

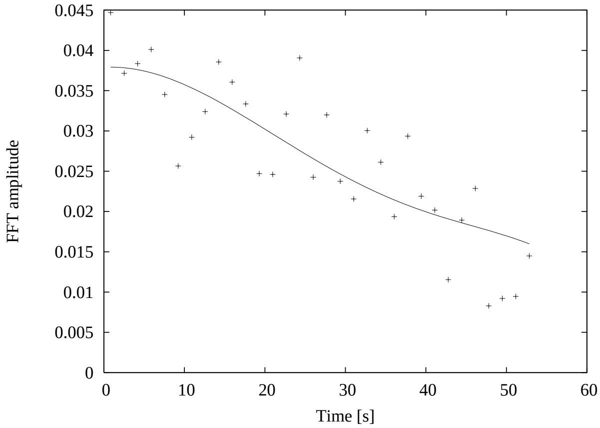

Figure 2: The data points are the FFT amplitudes simulated for a case with two snakes divided into 32 segments of 2^{17} turns. The solid curve is the average polarization scaled to fit the FFT data. The scaling factor was fit to be 0.0380 ± 0.0013. The snakes were detuned to have $\mu = 179.5^\circ$ and 90.5° between the snake axes.

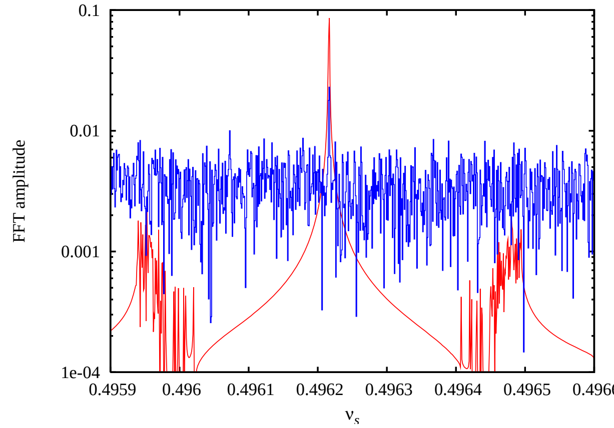

Figure 3: FFT amplitude for the case of two and a half snakes over the first 2^{18} turns. The histogram (blue) shows FFT amplitude of simulated polarimeter data with an average of 50 detected events per turn, and the curve (red) shows the FFT amplitude of the P_x average from 300 protons. Note the synchrotron sidebands in the P_x signal. This data was simulated with $G\gamma = 45.55$, $\mu = 179.5^\circ$, $\Delta\phi = 0.5^\circ$ for the two full snakes and a 50% partial snake with rotation axis 0.5° out of the horizontal plane.

transfer line or extract from the AGS at $G\gamma$ of an integer. Since radial polarization has not been demonstrated at an integer, I am not sure whether we can inject a large horizontal polarization into RHIC.

In addition to two Siberian snakes in each RHIC ring, we have a number of spin rotators around two of the interaction regions. It is possible to power any one of these rotators as a 50% partial snake having a rotation axis along the radial direction (a Type-II partial snake). It turns out that for a ring with two full snakes on opposite sides of the ring with rotation axes in the horizontal plane at 90° to each other, we can add an additional partial snake of any strength with its rotation axis in the horizontal plane and still keep the spin tune $\nu_s = ½$. A 50% partial snake will tip the stable spin direction 45° away from the vertical even with the other two full snakes. In this case a vertically injected spin at $\nu_s = ½$ will alternate between vertical and horizontal every other turn. By using the silicon detectors which are located on the 45° diagonals in the CNI polarimeters we can measure both vertical and radial polarizations. By picking a rotator near the STAR experiment the stable spin direction will have both large vertical and radial components. Fig. 3 demonstrates that a spin tune could be extracted from a turn-tagged measurement of polarization with 2½ snakes..

CONCLUSIONS

Simulations of spin decoherence from energy spread combined with a Monte Carlo generator of the polarimeter signals show that the spin tune may be extracted from turn-tagged polarimeter data by using a standard FFT algorithm. Given that the signal does not decohere too quickly, it should be possible to extract the spin tune with extremely high precision using a modest number of bunches.

Since these simple simulations only include synchrotron oscillations, future simulations will extend this to a more complete model of RHIC with betatron oscillations, orbit bumps, and solenoids.

Measurements of the spin tune with a single RHIC snake at injection with $G\gamma = 45.5$ is sensitive to the snake rotation angle (μ). With two snakes at $G\gamma = 46$, the spin tune at injection is sensitive to the rotation axes (ϕ) of the two snakes, and rather insensitive to the snake rotation angles (μ). By combining measurements of the two cases, a much better calibration of the RHIC snakes might be possible.

A case with 2½ snakes has been chosen to first verify that the spin tune can be extracted by this method.

ACKNOWLEDGMENTS

The idea for this study originated in a discussion with Gerry Bunce one evening in the Main Control Room for the AGS and RHIC. The idea of turn-by-turn polarimeter measurements came from Mei Bai and Thomas Roser in connection with the possibility of spin-tune measurements by tilting the spin coherently with an ac dipole.

REFERENCES

[1] I. Alekseev et al., "Polarized proton collider at RHIC", *Nucl. Inst. and Meth.*, A 499, 392 (2003).

[2] N. Akchurin et al., Phys. Rev. Lett. **69**, 1753 (1992).

[3] W. W. MacKay, et al., 'Spin Transport from AGS to RHIC with Two Partial Snakes in AGS", these proceedings.