# TRANSVERSE SPACE CHARGE STUDIES FOR THE ISIS SYNCHROTRON

C. M. Warsop, Rutherford Appleton Laboratory, Oxfordshire, UK.


*Abstract*

The ISIS Facility at the Rutherford Appleton Laboratory in the UK produces intense neutron and muon beams for condensed matter research. It is based on a 50 Hz proton synchrotron which, once the commissioning of a new dual harmonic RF system is complete, will accelerate about 3.5E13 protons per pulse (ppp) from 70 to 800 MeV, corresponding to mean beam powers of 0.2 MW. Following this upgrade, transverse space charge is expected to be one of the main intensity limitations, and is also a key factor for further machine upgrades. A programme of R&D on transverse space charge is now under way, aiming not only to improve the ISIS ring but also to exploit it as an experimental tool for testing theory and codes. This paper summarises the work so far, outlining calculations for coherent envelope modes on ISIS and using numerical solutions of the envelope equation to show the expected behaviour near half integer resonance. Progress on linking these predictions with more realistic beam models in space charge codes and experiments is summarised.


## BACKGROUND

The ISIS ring [1] has a mean radius of 26 m and accelerates about 2.5E13 ppp from 70 to 800 MeV in 10 ms. The beam is "painted" over the ring transverse acceptances (~ 400 π mm mr) via charge exchange injection over 150 turns. Beam is initially unbunched, and most loss occurs (5 %) during the longitudinal trapping process (~ 80 MeV). The addition of an $h$=4 RF system [2], to the existing $h$=2 will improve efficiency and allow higher intensities. The careful control of transverse dynamics has also been critical in achieving high intensity. Transverse space charge forces dominate at low energy, particularly during the bunching process where peak incoherent shifts are ~ -0.4 in both planes. The nominal betatron tunes are $Q_x$=4.31, $Q_y$=3.83, but these are adjusted during the cycle using programmable trim quadrupoles. Optimising the working point during the cycle is essential for minimising loss, e.g. pushing $Q$'s up to minimise the effects of space charge depression. In order to optimise and upgrade the machine it is important to understand more about what actually causes emittance growth and beam loss. Studies will eventually cover all key topics (3D space charge, 3D dynamics and instabilities), but here work concentrates on 2D space charge, and the effect of the important half integer resonances.

## ENVELOPE EQUATION RESULTS

Following [3,4,5], properties of the half integer resonance are studied using the envelope equation, with implicit assumptions of uniform charge density (a KV distribution for the 2D case). Behaviour is understood via standard analytical results and from numerical solutions (using [6]), in the regimes relevant to ISIS.

### *One Dimensional Beam Behaviour*

Solutions of the driven, 1D envelope equation (1) for a planar beam, with space charge [3] show key features of the more general 2D case. Parameters are: $x$ and $\varphi$ the normalised envelope and phase; $Q_0$ the zero intensity tune; $\Delta Q_{sc}$ the incoherent shift of the *unperturbed* beam; and $\Delta Q_e$ the driving error strength of harmonic $n$.

$$\frac{d^2x}{d\varphi^2} + (Q_o^2 + 2Q_o\Delta Q_e Cos[n\varphi])x - \frac{Q_o^2}{x^3} - 2Q_o\Delta Q_{sc} = 0 \qquad (1)$$

One essential result is that resonance occurs at the coherent frequency defined by the condition [3,4]:

$$Q_0 - C\cdot\Delta Q_{sc} = \frac{n}{2} \qquad (2)$$

where $C$=3/4, i.e. at an intensity 4/3 times that expected from the incoherent resonance condition ($C$=1). Space charge forces generated by envelope oscillations oppose the effects of the driving term. This condition for onset of resonance, derived for small oscillations, does not allow for the more complicated non-linear behaviour, where amplitude dependent frequency acts to limit growth [3]. The form of the envelope oscillations also depends on whether the beam is above or below resonance (2). When beam is being depressed toward the resonance by space charge, e.g. trapping on ISIS, the envelope oscillates about the equilibrium value, as indicated by numerical solutions in Figure 1 (a). As resonance is approached and exceeded the envelope oscillations become more complicated, with the appearance of additional fixed points, Figure 1 (b), (c). A number of growth mechanisms can result [3].

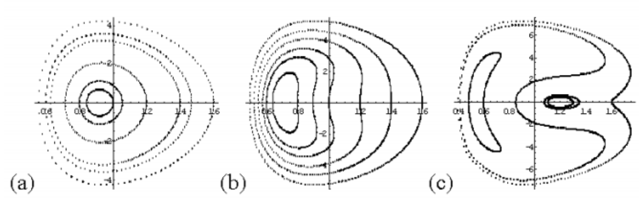

Figure 1: Normalised envelope ($x$, $x'$) sampled at a given machine azimuth (Poincaré map); (a) above, (b) approaching and (c) "depressed" below resonance.

It is the approach from above that is most relevant on ISIS; approach from below is avoided by the programming of tunes. Numerical solutions for envelope oscillations over 20 turns, as coherent tune is depressed towards resonance, are shown in Figure 2. These show "ISIS-*like*" conditions, as coherent tune approaches the driven $n$=7 harmonic. Near resonance amplitudes increase substantially, and "beating" between natural and driven frequencies is clear. Repeating scans of Figure 2 over many values of $Q_0$ (focusing) and $\Delta Q_{sc}$ (intensity), plotting *peak* amplitudes, yields Figure 3. This shows the important "coherent advantage": incoherent resonance would occur with $C$=1 in (2), the line indicated in Figure 3. The Figure also indicates that significant envelope growth can occur as the beam *approaches* resonance, depending on $\Delta Q_e$. The triangles in Figure 3 (a) correspond to Figures 1, and the squares to Figures 2.

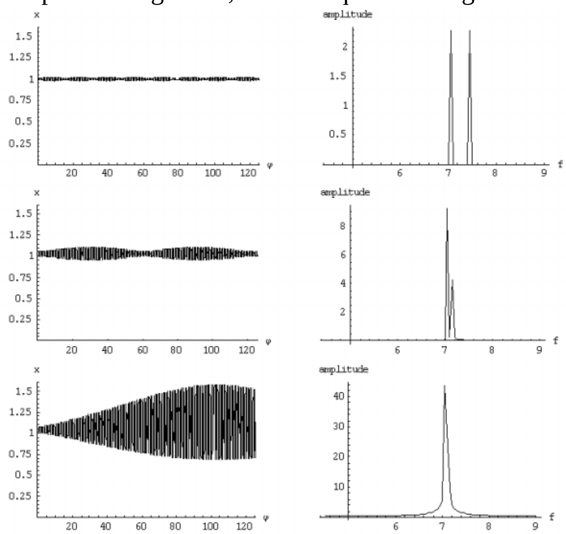

Figure 2: 1D Envelope Oscillations ($x$,$\varphi$), 20 turns with frequency spectra, approaching resonance. Corresponding parameters are shown in red on Figure 3, $\Delta Q_e$=0.01.

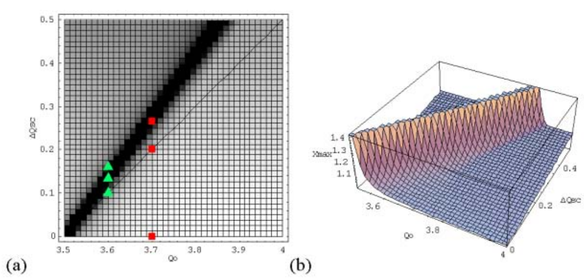

Figure 3: Max. Envelope Amplitude ($x_{max}$) as a function of Set ($Q_0$) and Depressed ($\Delta Q_{sc}$) Tune.

### *Two Dimensional Beam Behaviour*

The coupled equations for a 2D KV beam are of the form (3), where $a$ and $b$ are unperturbed semi-axes, and other symbols are standard [3].

$$\frac{d^2x}{d\varphi^2}+(Q_{xo}{}^2+2Q_{xo}\Delta Q_{xe}Cos[n\varphi])x-\frac{Q_{xo}{}^2}{x^3}-2Q_{xo}\Delta Q_{xsc}\left(\frac{a+b}{ax+by}\right)=0$$
$$\frac{d^2y}{d\varphi^2}+(Q_{yo}{}^2+2Q_{yo}\Delta Q_{ye}Cos[n\varphi])y-\frac{Q_{yo}{}^2}{y^3}-2Q_{yo}\Delta Q_{ysc}\left(\frac{a+b}{ax+by}\right)=0 \qquad (3)$$

Analytical solutions [3,4] indicate different behaviour depending on the relative tunes and emittances in the two transverse planes. On ISIS, the large tune split approximation for a round beam gives a reasonable description, yielding roughly independent and uncoupled 1D motion in each plane. Calculations of coherent frequencies using formulae for the more general cases of intermediate tune splits and non-round beams are in Figure 4, and show the results are close to the simpler case. This indicates [4] the coherent resonance condition should be approximated by (2) but with $C$=5/8.

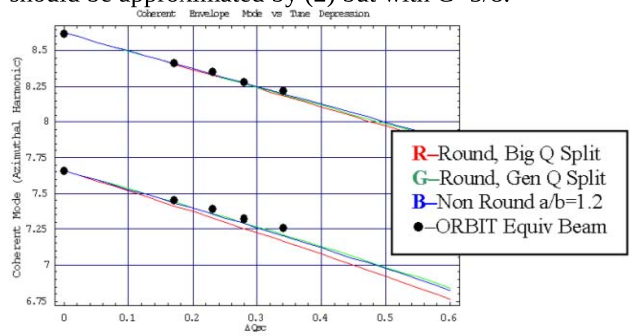


Figure 4: Coherent Frequencies vs $\Delta Q_{sc}$

Numerical solutions of the envelope equations (3) are shown in Figure 5, for nominal ISIS values, with the beam approaching vertical ($y$) resonance driven by (only) a $2Q_y$=7 gradient error ($\Delta Q_{sc}$=0.53, $\Delta Q_e$=0.01). The numerical solutions show that there is some weak interaction between planes, but that the dominant effect is a 1D process in the driven, resonant plane.

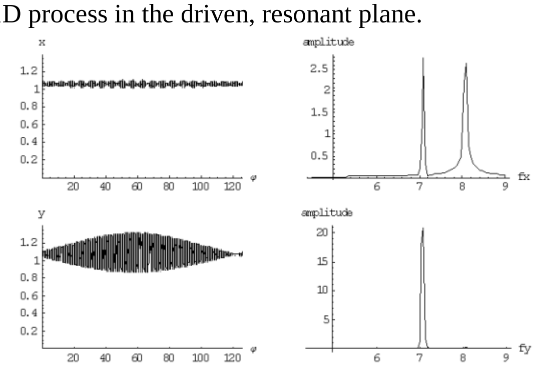

Figure 5: 2D Envelope Oscillations, top ($x$,$\varphi$), bottom ($y$,$\varphi$), 20 turns with frequency spectra.

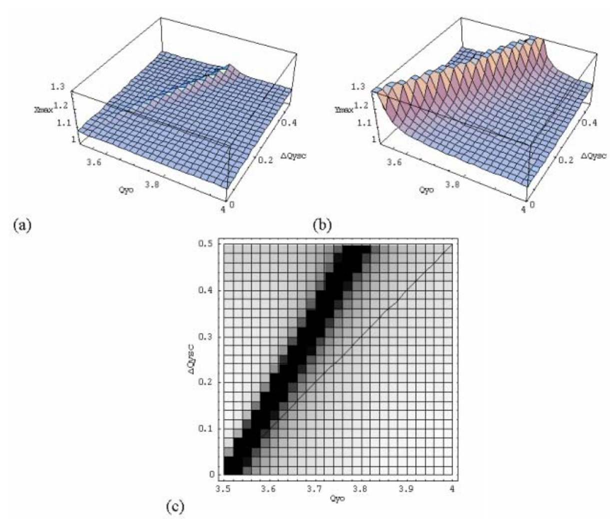

Figure 6: Max. Envelope Amplitudes as a Function of Vertical Set ($Q_{y0}$) and Depressed ($\Delta Q_{ysc}$) Tune, for driven $2Q_y$=7. (a) is Horizontal ($x_{max}$), (b) & (c) Vertical ($y_{max}$).

Figure 6 shows similar plots to Figure 3, but now peak amplitudes in the horizontal (a) and vertical (b) are shown as a function of *vertical* set tune and tune shift. The 8/5 coherent advantage is clear (c), as is the weak modulation of envelope in the non-resonant plane (a). The smooth focusing approximation is implicit in all the results above. Numerical solutions of the envelope equation using ISIS AG focusing indicate the smooth approximation is valid.

### *Summary*

The envelope equation gives some useful indications of beam behaviour. The coherent resonance (intercept $2Q_y$=7, Figure 4) of an ISIS 70 MeV coasting *equivalent beam* corresponds to an intensity of ~9E13 ppp. However, this neglects loss due to envelope oscillations, and other important effects, as shown below.

## ORBIT SIMULATIONS

To extend ideas beyond the KV beam model, and towards more realistic ISIS beams, the ORBIT code [7] is being used. For the present work, just 2D space charge is included, with no bunching, RF or momentum spread. The model has been checked by observing the preservation of a KV distribution over 100's of turns.

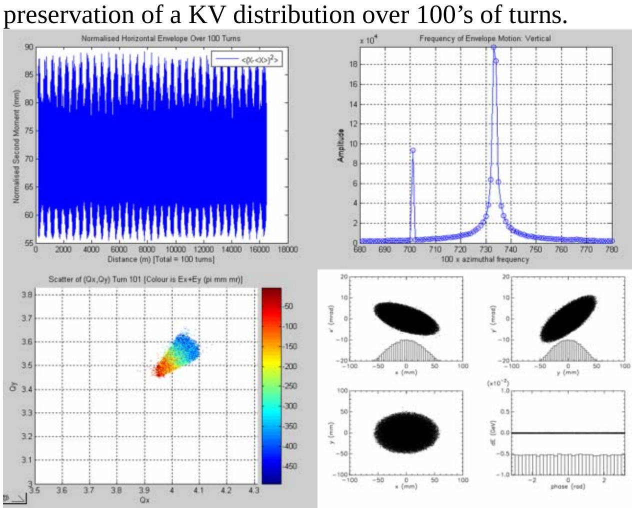


Figure 7: ORBIT Results – Coasting 5E13 ppp, 70 MeV

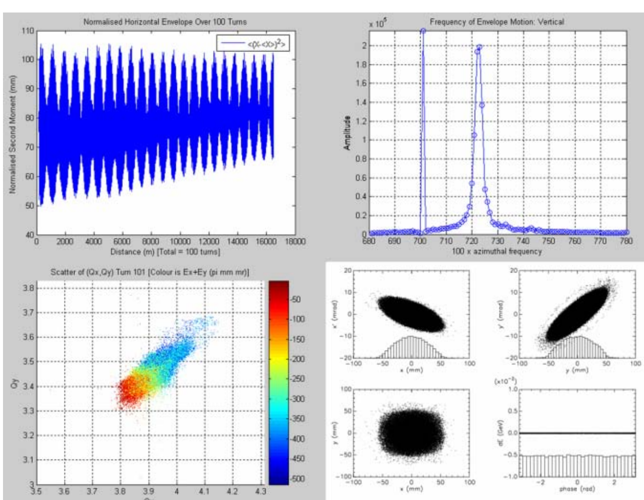


Figure 8: ORBIT Results – Coasting 7E13 ppp, 70 MeV

A detailed model of the nominal ISIS lattice is used to study a coasting 70 MeV beam, with harmonic quadrupole errors included to excite the $2Q_y$=7 resonance. A reasonably representative (slightly mismatched) 2D waterbag distribution ($\varepsilon_{rms}$=66 π mm mr) is tracked for ~ 100 turns. The beam intensity is increased so space charge shifts the beam towards resonance. ORBIT functions are used to find normalised envelopes, coherent frequencies, the incoherent tune footprint and phase space distributions after 100 turns.

Results are shown in Figures 7 (5E13 ppp) and 8 (7E13 ppp). In Figure 7 it can be seen that the beam sits on the incoherent resonance ($2Q_y$=7) for 100 turns with negligible growth, suitably accompanied by regular envelope oscillations, consistent with the coherent resonance model. As space charge increases, the coherent resonance gets stronger, Figure 8, the envelope is seen to grow, and halo is generated. A scan of simulations from 0 to 6E13 ppp result in a conserved $\varepsilon_{rms}$, and the coherent frequencies plotted in Figure 4, which follow the theoretical predictions. Above this $\varepsilon_{rms}$ grows, as in Figure 8. The source of halo is of some interest, with likely mechanisms being parametric resonance and redistribution of the non-stationary waterbag beam.

## SUMMARY AND PLANS

Half integer coherent resonance theory has been applied to ISIS, and the main features have been observed in simulations as expected. However, more complicated non-core model behaviour is also seen, with the generation of halo. This, along with extensions to include momentum spread, longitudinal motion, coupling and higher order effects, will be the subject of future work. To allow related experimental work, upgrades and studies of diagnostics are under way [8]. Experiments with the machine in storage ring mode are also planned.

## ACKNOWLEDGEMENTS

Valuable input from D Adams, S Brookes, R Brodie, C Prior and K Tilley is gratefully acknowledged.

## REFERENCES

[1] B. Boardman *et al*, "SNS: Description of Accelerator And Target" [ISIS], RAL Report RL-82-006
[2] A. Seville *et al*, "Progress on Dual Harmonic Acceleration on the ISIS Synchrotron", EPAC '06
[3] F. Sacherer, PhD Thesis, University of California 1968, UCRL-18454
[4] R. Baartman, "Betatron Resonances with Space Charge", AIP Conf. Proc. CP448, 1998
[5] A. V. Fedotov, I. Hofmann, "Half-integer Resonance Crossing in High Intensity Rings", Phys. Rev. ST-AB Vol. 5, 024202 (2002)
[6] *Mathematica*®: See www.wolfram.com (**NDSolve**: Runge-Kutta and various adaptive routines)
[7] J. A. Holmes *et al*, ORBIT User Manual, ORNL Technical Note SNS/ORNL/AP/011
[8] B. G. Pine *et al*, "Modelling of Diagnostics for Space Charge Studies on the ISIS Synchrotron", EPAC'06