COUPLED MAPS FOR ELECTRON AND ION CLOUDS

U. Iriso† and S. Peggs‡
†CELLS, PO Box 68, 08193 - Bellaterra (Spain)
‡ BNL, Upton, NY 11973 (USA)

Abstract

Contemporary electron cloud models and simulations reproduce second order phase transitions, in which electron clouds grow smoothly beyond a threshold from "off" to "on". In contrast, some locations in the Relativistic Heavy Ion Collider (RHIC) exhibit first order phase transition behaviour, in which electron cloud related outgassing rates turn "on" or "off" precipitously. This paper presents a global framework with a high level of abstraction in which additional physics can be introduced in order to reproduce first (and second) order phase transitions. It does so by introducing maps that model the bunch-to-bunch evolution of coupled electron and ion clouds. Coupled maps reproduce then first order phase transitions, hysteresis effects, and suggest that additional dynamical phases (like period doubling, or chaos) could be observed.

INTRODUCTION

Electron cloud evolution is modeled with some significant success using complex simulation codes, typically tracking individual electrons or macro-particles, and sometimes employing 3-dimensional finite-element methods to calculate self-consistent forces and fields [1]. For the Relativistic Heavy Ion Collider (RHIC), it is found that the simulated evolution from the passage of bunch m to $m+1$ is empirically well represented by a cubic map [2, 3]

$$\rho_{m+1} \;=\; a\rho_m + b\rho_m^2 + c\rho_m^3 \,, \qquad (1)$$

where ρ [nC/m] is the linear electron cloud density. Equilibrium is obtained when $\rho_{m+1} = \rho_m \equiv \rho^*$. For example, if the cubic term in c is negligible, then the equilibrium electron density is

$$\rho^* = \begin{cases} 0 & ; \quad \text{when } a < 1 \\ \dfrac{a-1}{-b} & ; \quad \text{when } a > 1 \,. \end{cases} \qquad (2)$$

Values for the electron cloud map coefficients are typically obtained from the empirical fits after running CPU intensive simulation codes [2, 3]. An analytical expression for a is derived in [2].

PHASE TRANSITIONS

After electron cloud formation, what happens as the bunch population slowly decays? Do the electron clouds collapse suddenly, or do they slowly fade away? For a fixed set of beam pipe parameters, the coefficient a increases monotonically with the bunch population N, so that the stable electron cloud density $\rho^*(a(N))$ is also a function of bunch population [2]. Equation 2 then predicts that the phase transition from electron cloud "off" to "on" is second order – $\rho^*(N)$ increases smoothly from zero above a critical threshold population, when a becomes larger than unity. Complex simulation codes reproduce only second order phase transitions [2, 4].

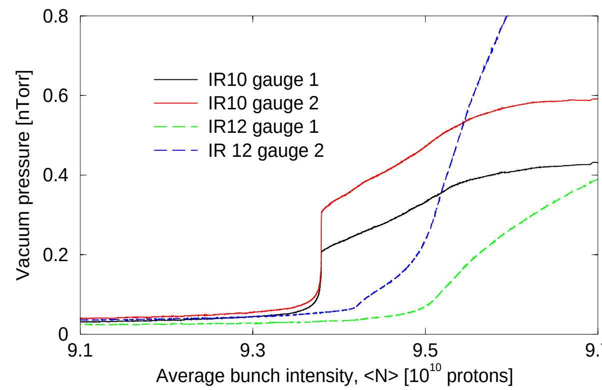

Figure 1: First and second order electron cloud phase transitions observed in the interaction regions IR10 and 12 of RHIC. The data were taken as the bunch population slowly decays.

However, experimental data shown in Fig. 1 illustrate how both first and second order phase transitions are seen in RHIC, as a threshold bunch population is crossed. While the pressure in IR12 smoothly decreases as the bunch population slowly drops, an abrupt transition is seen in IR10. This catastrophic collapse of the pressure is unexpected, especially since the surface parameters show a smooth dependence on the impact electron energy at the wall [5, 6]. The failure of simulations to reproduce these first order phase transitions, and of theory to predict them, indicates that there is missing physics in the modeling.

COUPLED MAPS AND FIXED POINTS STABILITY

A candidate for additional physics is the interplay between electron clouds and positive ion clouds, first introduced in Ref. [7], and recently discussed for RHIC [8], and the LHC [9]. Models of this interplay face two main challenges: a significant number of uncertain surface physics parameters for both electron and ions, and extremely different time scales for electron and ion cloud dynamics. Long ion lifetimes imply very long CPU times for simulations.

In a map model, the interplay between electron clouds and ion clouds is generally expressed by

$$\rho_{m+1} = f(\rho_m, R_m) \quad (3)$$
$$R_{m+1} = g(\rho_m, R_m)\ , \quad (4)$$

where R_m [nC/m] is the ion cloud density after the passage of the m'th bunch. (Both ρ and R are defined to be positive.) In the following, we use the vector $\vec{r}$ for the electron and ion densities

$$\vec{r}_m = \begin{pmatrix} \rho_m \\ R_m \end{pmatrix} . \quad (5)$$

A fixed point is found when $\vec{r}_{\mathrm{m+1}} = \vec{r}_m \equiv \vec{r*}$. Furthermore, we need the fixed point to be *stable*. That is, small perturbations around the fixed point $\vec{r*}$ must result in an evolution that converges towards the fixed point.

Close to a fixed point $\vec{r*}$, the linear motion in one time step from bunch passage m to $m+1$ is

$$\vec{r}_{\mathrm{m+1}} = J\vec{r}_m\ , \quad (6)$$

where J is the 2x2 Jacobian matrix from Eqs. 3 and 4. Reference [11] shows that a fixed point is stable if one of the two following pairs of conditions is fulfilled:

$$i)\quad t^2 < d^2 \quad ;\ \text{and} \quad d^2 < 1 \quad (7)$$
$$ii)\quad t^2 > d^2 \quad ;\ \text{and} \quad |t| + \sqrt{t^2 - d^2} < 1\ , \quad (8)$$

where the convenient definitions $t \equiv \mathrm{Tr}(J^2)/2$, and $d \equiv \det(J)$ have been introduced.

A SIMPLE COUPLED MAPS MODEL

In order to visualize the phenomena that these conditions can generate, we use an example for the functions f and g based on the cubic map for the electron density (Eq. 1). Consider the "proof-of-principle" coupled maps

$$\rho_{m+1} = (a + yR_m)\rho_m + b\rho_m^2 + c\rho_m^3 \quad (9)$$
$$R_{m+1} = AR_m + Y\rho_m \quad (10)$$

If the coupling coefficients are turned off ($y = Y = 0$), then the electron cloud map Eq. 1 is recovered, along with the uncoupled ion map

$$R_{m+1} = AR_m\ . \quad (11)$$

Due to the inertia of the massive ion cloud, we expect $A \approx 1$. There are two coupling mechanisms in Eqs. 9 and 10:

1. Electrons generate a positive ion cloud by colliding with the rest gas in the vacuum chamber. This is represented by the term $Y\rho_m$ in Eq. 10. Y is positive, but its order of magnitude is not trivially apparent.

2. The slow moving positive ions enhance the probability of electron survival between one bunch passage and the next. This is represented by the term yR_m. Moreover, the presence of an ion cloud also tends to neutralize the negative electron space charge of the accumulated electron cloud. For these reasons, physical values of y are positive, typically smaller than a, and of the same magnitude as b, so $y < a$ and $y \sim |b|$.

Numerical application

Next, we assume that all the coupled map coefficients are constants except for the bunch to bunch electron cloud gain, a. We presume that a depends linearly on the bunch population according to [2, 3]

$$a = 0.4 + 0.1\,(N/10^{10})\ . \quad (12)$$

The coupled map coefficient values used throughout below and quoted in Table 1 are illustrative – they are not intended to quantitatively reproduce RHIC results.

Table 1: Map parameters used in the following examples.

a	b	c	y	A	Y
Eq. 12	-0.1	-0.08	0.4	0.96	0.03

FIRST ORDER PHASE TRANSITIONS, HYSTERESIS, AND CHAOS

Figure 2 shows the results of a dynamical simulation in which the coupled maps are applied directly, first as the bunch population is slowly decreased, and then as it is slowly increased. The solid line shows that the stable electron cloud density decreases as the bunch population is reduced, until at $N \approx 4.7 \times 10^{10}$ the electron cloud collapses catastrophically. When the bunch population is then slowly increased, no electron (or ion) cloud forms up to a population of $N = 6.0 \times 10^{10}$, when the cloud grows rapidly to a stable stationary value. Figure 3 shows the flow in (ρ, R)

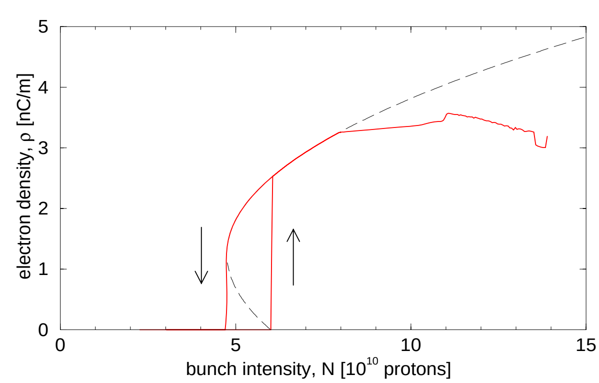

Figure 2: Evolution of the electron cloud density as the bunch population N is first slowly decreased, and then slowly increased. The precipitous and hysteretic behavior is characteristic of first order phase transitions.

space for $N = 5 \times 10^{10}$ protons/bunch. These plots result from tracking several cases with different initial conditions. Two different *basins of attraction* coexist: one corresponding to the fixed point $\vec{r*}_1 = (0, 0)$, the second corresponding to the fixed point $\vec{r*}_3 = (1.81, 1.36)$. This feature is the origin of the hysteresis and the first order phase transitions: depending on the initial conditions (ion density value), the system evolves towards $\vec{r*}_1$ or $\vec{r*}_2$. Note that there is a second fixed point at $\vec{r*}_2 = (0.69, 0.52)$, which becomes a *global repeller*. Following Eqs. 7 and 8, this point is unstable. The presence of a coupled ion cloud enhances the

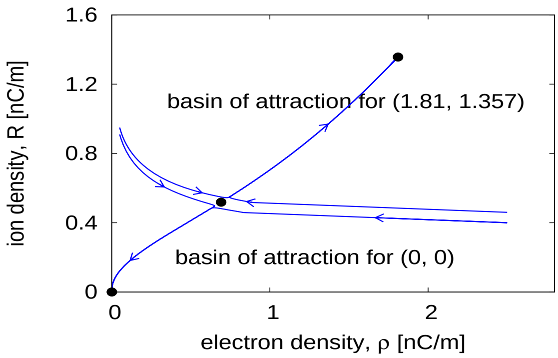

Figure 3: Coupled motion in (ρ, R) space is tracked following the coupled maps for $N = 5 \times 10^{10}$ protons/bunch. There are two *basins of attraction* for $N = 5 \times 10^{10}$: one containing the fixed point $\vec{r*}_1 = (0, 0)$, and the second containing $\vec{r*}_3 = (1.81, 1.36)$. The fixed point $\vec{r*}_2 = (0.69, 0.52)$ sits on the boundary between the two basins, acting as a global repeller. Thus, the system evolves towards $\vec{r*}_1$ or $\vec{r*}_3$, depending on the initial conditions.

electron survival, and stable and non-zero electron clouds are created even when $a < 1$. Enhanced electron survival due to the presence of an ion cloud is also considered in Ref. [12], but the ion cloud density is not allowed to evolve. The importance of the model stems from its ability to show the possibility of abrupt transitions even with a smooth dependence of the map coefficients on electron cloud parameters (such as bunch population or length). Recall that all coefficients remain constant except $a(N)$, which changes linearly with the bunch intensity.

Figure 2 shows chaotic behaviour for bunch populations above $\sim 8.0 \times 10^{10}$, above which different dynamical phases become active. Figure 4 shows the evolution of the electron and ion clouds for different bunch populations, always starting with the same (arbitrary) initial cloud densities. The clouds decay away or build to stable solutions with $N = 3 \times 10^{10}$ or 6×10^{10} protons per bunch respectively, consistent with classical expectations (see Fig. 2). However, the clouds evolve into a stable period-2 oscillation when $N = 9 \times 10^{10}$ protons/bunch. Coupled maps enable the generation of period doubling and chaos, behavior that does not occur in the smoothed world of differential equations. Such additional dynamical phases have not (yet) been observed in electron clouds in accelerators, but it is possible they occur at, or near, typical operating conditions. An understanding of coupled cloud dynamics from the map perspective may prove important in enhancing accelerator performance.

CONCLUSIONS

Bunch-by-bunch maps are more appropriate than differential equations in modeling coupled cloud dynamics, because of the rapid evolution of the electron cloud after the violent transient of a bunch passage. The "proof-of-principle" form of the coupled maps presented here

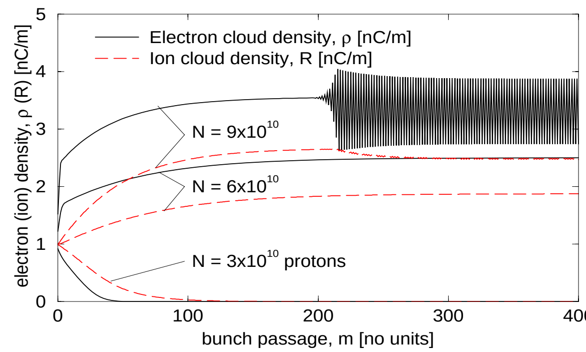

Figure 4: Dynamical evolution of the electron and ion cloud densities as a function of time (bunch passage number) for 3 different bunch intensities, $N = 3 \times 10^{10}, 6 \times 10^{10}$, and 9×10^{10} protons/bunch.

can generate electron and ion clouds that turn "on" and "off" precipitously even with smooth and continuous dependence of the map coefficients. Such first order phase transitions are sometimes seen in practice, but are not predicted by contemporary simulation codes, which model electron clouds in isolation. Other coupling mechanisms than those presented here are also plausible.

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