LIFETIME LIMIT FROM NUCLEAR INTRA-BUNCH SCATTERING FOR HIGH-ENERGY HADRON BEAMS

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Abstract

We discuss the possibility and importance of nuclear scattering processes inside a bunched hadron beam. Estimates are presented for the LHC.

INTRODUCTION

The electromagnetic scattering of charged particles inside a bunch off each other gives rise to two important phenomena frequently affecting storage-ring operation, namely emittance growth due to *intrabeam scattering* and beam lifetime limitation by the *Touschek effect*. The former is related to multiple small-angle scattering, and the latter to single large-angle scattering.

In this paper we explore whether nuclear scattering processes could also become important as the brightness and intensity of new accelerators is pushed ever further. At the center-of-mass momenta of interest, protons scatter elastically. For ions with their internal degrees of freedom more complicated scattering processes are possible, for example fusion or nuclear excitation. By contrast, mixed electromagnetic-nuclear processes, like pair creation followed by electron capture or electromagnetic dissociation, which dominate in beam-beam collisions, are not thought to be important at the non-relativistic energies in the hadron-beam rest frame.

The numerical estimates we present refer to the LHC in either proton or ion operation at top energy and injection.

PROTON NUCLEAR CROSS SECTION

At 7 TeV, the rms transverse momentum spread of protons in an LHC bunch in both beam and laboratory frame is of order

$$p_{\perp,\mathrm{rms}} \approx \sqrt{\frac{\epsilon_{x,y}}{\beta_{x,y}}}\, p_{\|,\mathrm{lab,av}} \approx 14\ \mathrm{MeV}/c\ , \qquad (1)$$

where $p_{\|,\mathrm{lab,av}}$ denotes the average longitudinal momentum in the laboratory frame, $\beta_{x,y} \approx 100$ m the average beta function, and $\epsilon_{x,y}$ the transverse emittance ($\gamma\epsilon_{x,y} = 3.5\ \mu$m). The longitudinal momentum in the beam frame is smaller, namely

$$p_{\|,\mathrm{rms}} \approx \frac{1}{\gamma}\, p_{\|,\mathrm{lab,rms}} \approx 1\ \mathrm{MeV}/c\ . \qquad (2)$$

As is common in problems of Touschek or intrabeam scattering, we are mainly concerned with scattering of the transverse plane into the longitudinal.

Scattering can be characterized by an invariant scattering amplitude M which is related to the scattering cross section via Fermi's golden rule

$$\sigma = \frac{1}{j_{\mathrm{inc}}} \frac{2\pi}{\hbar} |M|^2 \rho_f\ , \qquad (3)$$

where j_{inc} denotes the incident flux and ρ_f the density of final states.

The invariant amplitude for Coulomb scattering is

$$M_{\mathrm{em}} = \frac{4\pi\alpha}{q^2}\ , \qquad (4)$$

with $\alpha \approx 1/137$ the fine-structure constant, and q the momentum transfer during the scattering.

For nuclear scattering, we approximate the scattering amplitude as

$$M_{\mathrm{nucl}} = \frac{4\pi g}{q^2 + m_\pi^2 c^2}\ , \qquad (5)$$

which corresponds to the classical Yukawa theory; see, *e.g.*, [1]. In (5), $g \approx 1$ denotes the coupling constant of the nuclear Yukawa potential, also known as πNN coupling constant and equal to the factor $(4\pi f^2)$ used in some historical papers. The g parameter was determined in low-energy scattering experiments. The second parameter, $m_\pi \approx 140$ MeV, characterizes the range of the force, and we take it to be equal to the mass of the pi meson. The theory described in [1, 2] contains a cutoff at momentum transfers equal to about 6 times the pion mass, which is of no consequence for our application. Although more advanced theoretical frameworks for nuclear scattering exist, we consider formula (5) practical and sufficiently precise for our order of magnitude estimates.

The relative importance of the two scattering amplitudes depends on the value of q, as is illustrated in Fig. 1. For large-angle scattering the nuclear interaction is dominant. Combining (4) and (5), we have

$$M_{\mathrm{nucl}} \approx \frac{q^2}{m_\pi^2 c^2} \frac{g}{\alpha} M_{\mathrm{em}}\ . \qquad (6)$$

For a momentum transfer $q \approx 1$ MeV/c, (6) yields $M_{\mathrm{nucl}} \approx 0.007 M_{\mathrm{em}}$, which means that the associated nuclear scattering cross section is 20,000 times smaller than the electromagnetic one. On the other hand, for a scattering momentum transfer equal to $\sqrt{2}$ times the rms transverse momentum spread in the LHC at 7 TeV, $q \approx 20$ MeV/c, (6) becomes $M_{\mathrm{nucl}} \approx 8 M_{\mathrm{em}}$.

In order to crosscheck our result, experimental data of proton-proton scattering are shown in Fig. 2. The figure demonstrates that for kinetic energies in the center-of-mass up to about 140 MeV (blue arrow) the total cross section

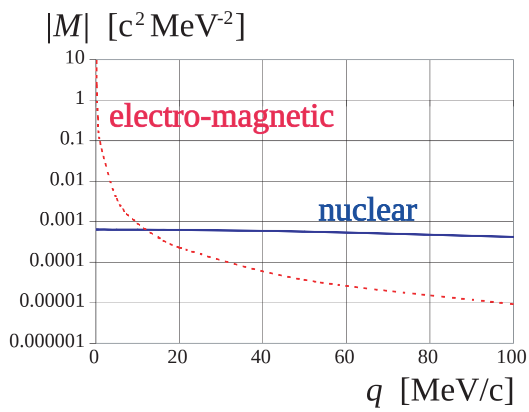

Figure 1: Electromagnetic and nuclear scattering amplitude $|M|$ as a function of momentum transfer q.

equals the elastic one, and inelastic processes become important only from pion-mass energies onwards. Even for the LHC, we are interested in the low-energy region, indicated a by a red arrow, where the cross section is of order 100–300 mbarn.

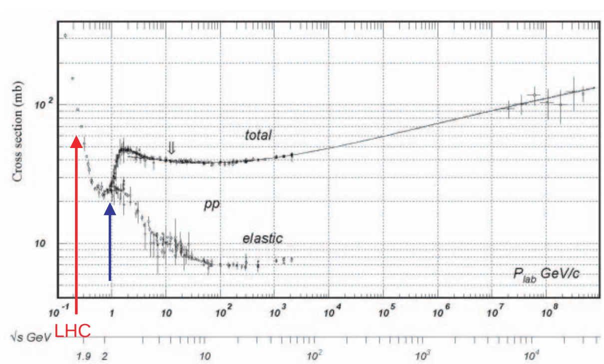

Figure 2: Total and elastic cross sections for pp collisions as a function of laboratory beam momentum and total center-of-mass energy [3]. Below the blue arrow all interactions are elastic; the red arrow indicates the energy corresponding to the LHC rms transverse momentum spread at 7 TeV.

PROTON BEAM LIFETIME

Assuming that every nuclear scattering event leads to the loss of two particles, from the corresponding total cross section $\sigma_{\rm tot} \approx 300$ mbarn, and proceeding in analogy to 1-dimensional Touschek calculations [5], we may estimate the corresponding beam lifetime $\tau_{\rm nucl}$ as

$$\frac{1}{\tau_{\rm nucl}} \approx 4\frac{p_{\|,{\rm lab,av}}\sigma_{\rm tot}}{m\gamma^2}\frac{N_b}{8\pi^{3/2}\beta_x\sigma_y\sigma_z} , \qquad (7)$$

where N_b denotes the bunch population, β_x the horizontal beta function, σ_y the vertical beam size, σ_z the rms bunch length, and $\sigma_{x'}$ the rms horizontal beam divergence. The factor γ^2 arises from two Lorentz transformations into the beam frame and back to the laboratory frame. One factor of two takes into account the difference between 1- and 2-dimensional calculations found for Touschek scattering [4].

Inserting LHC parameters at 7 TeV, namely $\sigma_x \approx \sigma_y \approx 220$ μm, $\sigma_z \approx 7.5$ cm, $N_b \approx 1.15 \times 10^{11}$, $\gamma \approx 7461$, $\beta_x \approx 100$ m, and $\sigma_{\rm tot} \approx 300$ mbarn, we find $\tau \approx 2.6\times 10^{11}$ s or 3×10^6 days, which is enormous. For the LHC injection energy of 450 GeV, using $\sigma_x \approx \sigma_y \approx 0.88$ mm, $\sigma_z \approx 11.5$ cm, and $\gamma \approx 480$, the estimated lifetime still is a considerable 10^{11} s or 10^6 days. With the nominal number of 2808 bunches these rates amount to a total of one nuclear scattering event on every tenth or third turn, respectively, for the entire beam. At constant normalized emittance the dependence on the beam energy is weak.

COMPARISONS

For comparison the Touschek effect for round beams in LHC, *i.e.*, the effect of single electromagnetic scattering, translates into an effective LHC beam lifetime of 2×10^7 s (230 days) at injection and 4.5×10^7 s (520 days) at 7 TeV [4]. At both energies, the Touschek scattering rate is about 3000 times larger than nuclear scattering. This corresponds to the cross section ratio (6) expected at $q \approx 1.6$ MeV/c.

We can also compare the nuclear scattering cross section with the cross section relevant for classical low-angle intrabeam scattering. The latter contains a Coulomb logarithm of the ratio of maximum and mimum impact parameter. The maximum impact parameter (minimum q) is normally taken to be the minimum of the rms horizontal beam size and the Debye length $\lambda_D \approx \epsilon_{x,y}\gamma(\pi)^{1/4}\sqrt{\sigma_z/(r_pN_b)}$. For LHC at 7 TeV, $\sigma_x \approx 200$ μm, and $\lambda_D \approx 3$ mm, so that the beam size is the correct upper limit. As for the minimum impact parameter, it is the maximum of the classical and quantum-mechanical limits, which are $b_{\rm min,class} \approx r_pc^2/v_\perp^2 \approx r_p\beta_{x,y}/(\gamma^2\epsilon_{x,y}) \approx 5$ fm, and $b_{\rm min,qm} \approx \hbar/(m_pv_\perp) \approx \lambda\!\!\!^{-}/(\gamma\sqrt{\epsilon_{x,y}/\beta_{x,y}}) \approx 13$ fm. The quantum-mechanical minimum impact parameter is slightly larger.

The relation between impact parameter b and momentum transfer q is $q \approx 2c^2m_pr_p/(bv)$. The total Coulomb cross section is obtained by integrating the differential Rutherford cross section $d\sigma/d\Omega|_{\rm Ruth.} \approx 4\alpha^2E^2\hbar^2/(q^4v^2)$ over Ω, using $d\Omega = 2\pi\sin\theta d\theta$ and $q^2 = 4p^2\sin^2(\theta/2)$, with the result

$$\sigma_{\rm Ruth.} \approx \frac{\pi\alpha^2\hbar^2b_{\rm max}^2}{\beta^2r_p^2m_p^2c^4} \qquad (8)$$

where $\beta \equiv v/c$ and m_p the proton mass. Inserting for $b_{\rm max}$ the maximum impact parameter, *i.e.*, the rms beam size, and taking $\beta \approx 0.025$ for the relative velocity in the two-particle center-of-mass frame for the 7-TeV LHC, the total electromagnetic scattering cross section is estimated as 36 Mbarn, which is 10^8 times larger than the 0.3 barn of nuclear scattering. The electromagnetic cross section is dominated by contributions from low-angle scattering: The average squared scattering angle is $<\theta^2>\approx 2(2r_pmc)/b_{\rm max}v)^2/p_{\|,av,lab}^2\ln(b_{\rm max}/b_{\rm min})$ which amounts to 8.8×10^{-21}.

DISCUSSION

We know that nuclear interactions with the residual gas and in beam-beam collisions will limit the LHC beam lifetime to about 100 h or 20 h, respectively. The corresponding nuclear cross section at low energy is at least comparable in magnitude, if not higher, than the nuclear cross sections characterizing gas scattering or interactions with the other beam (Fig. 2).

Why the effect of intrabunch nuclear scattering on the beam lifetime is so much smaller can be understood by comparing the equivalent proton densities in the beam rest frame. For scattering of particles inside the same bunch the density of the beam itself enters in the scattering rate. Due to Lorentz expansion, the beam is γ times longer than in the laboratory frame, which translates into a particle density

$$\rho^*_{\rm beam} \approx \frac{N_b}{(2\pi)^{3/2}\sigma_x\sigma_y\sigma_z\gamma} \approx 3\times 10^{14}\ \ {\rm m}^{-3}\ , \qquad (9)$$

where the numerical value refers to LHC at 7 TeV, and the asterisk as superindex indicates that the density refers to the beam frame.

The LHC design assumes a residual volume density of hydrogen molecules, ρ_{H_2} of 10^{15} m^{-3}, which corresponds to 100 h nuclear beam lifetime. In the beam frame the density is enhanced by Lorentz contraction, and it converts into the effective proton density

$$\rho^*_{p,{\rm H}_2} \approx \gamma 2\rho_{H_2} \approx 1.4\times 10^{19}\ \ {\rm m}^{-3}\ . \qquad (10)$$

The collisions with the counter-propagating beam only occur at 2 primary interaction points. The effective density of the opposing beam is obtained by averaging these collision in time, *e.g.*, over one turn. Transforming to the rest frame of the original beam, also this density is enhanced by Lorentz contraction,

$$\rho^*_{\rm 2nd\ beam} \approx 2\frac{N_b\gamma}{(2\pi)^{3/2}\sigma^*_x\sigma^*_y C} \approx 1.6\times 10^{19}\ \ {\rm m}^{-3} \qquad (11)$$

where $\sigma^*_{x,y} \approx 16$ μm denotes the spot size at the collision point.

According to (9)—(11), in its rest frame the density of the beam is almost five orders of magnitude lower than both the density of the residual-gas protons and of the time-averaged opposing beam. This explains most of the large difference in the associated beam lifetimes. The non-relativistic relative velocities for intrabunch scattering contribute another factor of about 40 at 7 TeV, which accounts for the remaining difference.

FUSION

The only possibility for proton-proton fusion is

$$p + p \to d + e^+ + \nu_e\ , \qquad (12)$$

which involves the weak interaction and for 1-MeV protons has a cross section of 10^{-23} barn. Incidentally, this extremely low cross section explains the age of the sun which is of order 10^{10} years [6].

For ions heavier than hydrogen the situation is different. The fusion cross sections are higher, rising linearly between energies 10 and 20 MeV and then reaching a plateau at a value of about 1 barn [6], as is illustrated for oxygene in Fig. 3. However, the density of heavy ions in an LHC bunch is about 10^3 times lower than the density of LHC proton bunches. The lower particle density far outweighs the higher cross sections.

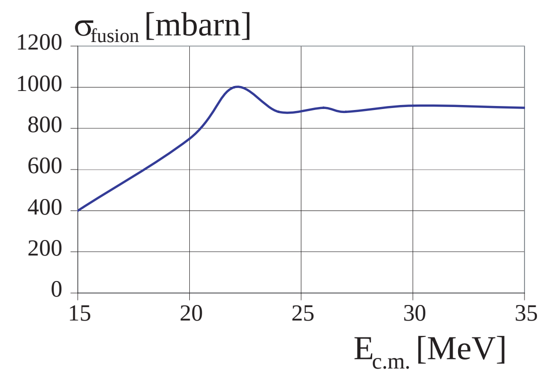

Figure 3: Fusion cross section for ^{16}O-^{16}O system as a function of c.m. energy (freely redrawn from [6]).

CONCLUSIONS

Protons inside an LHC bunch undergo elastic nuclear scattering off each other about 3000 times less frequently than classical Touschek scattering. Heavier ions can in addition experience fusion reactions with higher cross sections. Due to the reduced ion beam density in LHC, the total rate of fusion events, however, is even lower than the elastic nuclear-scattering event rate of protons. About one ion fusion event is expected every few thousand turns for the nominal ion beam. Since the reaction products, e.g., neutrons, positrons, electrons or photons, will be Lorentz boosted, in principle they could be detected, but they will not noticably contribute to heat load or radiation damage.

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