SPACE CHARGE INDUCED RESONANCE TRAPPING IN HIGH-INTENSITY SYNCHROTRONS

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Abstract

With the recent development of high-intensity circular accelerators, the simultaneous presence of space charge and lattice nonlinearities has gained special attention as possible source of beam loss. In this paper we present our understanding of the role of space charge and synchrotron motion as well as chromaticity for trapping of particles into the islands of nonlinear resonances. We show that the three effects combined can lead to significant beam loss, where each individual effect leads to small or negligible loss. We apply our findings to the CERN-PS experiment.

INTRODUCTION

The problem of the long term storage for high intensity bunches in a nonlinear lattice is very important for high-intensity circular accelerators. In the SIS100 [1, 2], for instance, storage for 1 second of a high intensity bunched beam ($\Delta Q_x = -0.3$) in a nonlinear lattice with a loss level not exceeding 1% is requested. This constraint stems from the large ionization cross section of a U^{+28} ion beam with residual gas atoms. A too large beam loss triggers a progressive vacuum degradation which reduces considerably the beam lifetime. The standard value of 1 W/m beam loss and protection of cold superconducting part of magnets imposes also beam loss at the % level. In order to describe the basic beam degradation mechanism deriving from the space charge in bunches, we consider a high intensity bunch stored in a ring having only one vertical lattice resonance. This assumption is done for the sake of simplicity without losing generality. First we discuss the resonance trapping in absence of the chromaticity. The main features of this dynamical system are:

- The space charge couples transverse and longitudinal planes: the instantaneous transverse Coulomb force depends on where in the longitudinal plane a particle is located;
- The self consistent effects in the absence of synchrotron motion do not cause emittance blow-up for non KV-distribution [3]: coherent resonances are not excited by transverse Gaussian distributions;
- The longitudinal motion induces, via space charge, a slow variation of transverse tunes. This condition is common in synchrotrons, for instance in the SIS100, $Q_x \sim 20$ while the longitudinal tune can be of the order of $Q_z = 10^{-3}$ so that the $Q_z/Q_x \sim 10^{-5}$; for the LHC we find $Q_z/Q_x \sim 10^{-4}$;
- The presence of a relatively small tune shift ($\Delta Q_x/Q_{x0} \sim 1.5\%$ for SIS100), does not destroy the standard transverse nonlinear dynamics, but rather induces a slow modulation of transverse tunes according to the synchrotron frequency;
- The transverse-longitudinal space charge coupling, influence, via the depression of tunes, the transverse position where the resonance condition is met.

Resonances in phase space

The main consequence is that the position of instantaneous transverse islands (resonances) in phase space are depending on the longitudinal position of particles within the bunch. According to the position of the bare tune Q_{x0} with respect to the resonance $Q_{x,res}$ and the maximum tunes shift ΔQ_x, we can distinguish the following cases:

1. $Q_{x0} - Q_{x,res} > |\Delta Q_x|$. With this condition, the tune-spread never intercepts the resonance and no island can appear in the transverse phase space;
2. $Q_{x0} - Q_{x,res} = |\Delta Q_x|$. Here the particles with maximum tune depression touch the resonance, but no islands are formed as the space charge detuning prevents the resonance condition to be met;
3. $|\Delta Q_x| > Q_{x0} - Q_{x,res} > 0$. When this condition occurs, particles with maximum tune depression (at $z = x = y = 0$) fall on the left side of the resonance (*below the resonance*), whereas particles at $z = 0$ and large transverse amplitude fall on the right side of the resonance (*above the resonance*): the islands are formed at an intermediate amplitude, which depends on the particle longitudinal position. The outer position of the fixed points occurs at $z = 0$, whereas the smaller is at $x = 0$ and it happens only in two symmetric longitudinal bunch positions. Note that the outer position of the fixed points depends on $Q_{x0} - Q_{x,res}$: for $Q_{x0} \to Q_{x,res}$ the maximum position of the fixed points is (virtually) infinite;
4. $Q_{x0} - Q_{x,res} \leq 0$. In this case the tune-spread cannot cross the resonance. When $Q_{x0} = Q_{x,res}$ practically no particle of the bunch can be resonant (only at very large amplitude).

For a particle at small transverse amplitude the depressed tune for a matched 3D Gaussian distribution can be expressed by the formula $Q_x = Q_{x0} - \Delta Q_x \exp[-0.5(z/\sigma_z)^2]$, where z is the longitudinal amplitude of the particle and σ_z the longitudinal rms size. If Q_{x0} satisfies the condition 3) we can

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define a transition longitudinal emittance ϵ_{z_t} such that if a particle has longitudinal emittance satisfying $\epsilon_z < \epsilon_{z_t}$ then it never crosses the resonance during synchrotron oscillations, whereas if $\epsilon_z > \epsilon_{z_t}$ it does it 4 times per synchrotron oscillation. The total number of particles, which can be on both sides of the resonances according to the longitudinal position is

$$\frac{\Delta N}{N} = \alpha \frac{Q_{x0} - Q_{x,res}}{|\Delta Q_x|}. \quad (1)$$

Here α depends on the topology of the islands; its lower limit is obtained by a direct integration over the distribution for particles satisfying $\epsilon_z > \epsilon_{z_t}$ and we find $\alpha > 1$. Note that Eq. 1 is valid for $Q_{x,res} < Q_{x0} < Q_{x,res} + |\Delta Q_x|/\alpha$.

Periodic resonance crossing

When synchrotron motion is included the longitudinal motion induces a periodic migration of the islands in the phase space. If the condition 3) is satisfied, particles in the bunch may undergo a periodic crossing of the resonance. In this regime particles have a finite probability of being trapped into transverse islands. When trapping does not occur, the particle orbit makes a jump (scattering of the invariant). In Fig. 1 are shown an example of trapping a) and scattering regime b) for SIS18 with standard parameters $Q_{x0} = 4.35$ and $Q_y = 3.2$: a sextupole is added to a linear constant focusing lattice to excite the resonance $3Q_{x0} = 13$, with $\Delta Q_x = -0.1$; we used $Q_z = 5 \times 10^{-5}$ for showing the full trapping in Fig. 1a, and $Q_z = 10^{-3}$ for the scattering regime (Fig. 1b). The emittance evolution is shown in terms of the same initial emittance, in both pictures the initial particle coordinate are $x = 1.5\sigma_x, p_x = y = p_y = p_z = 0$, and $z = 3\sigma_z$.

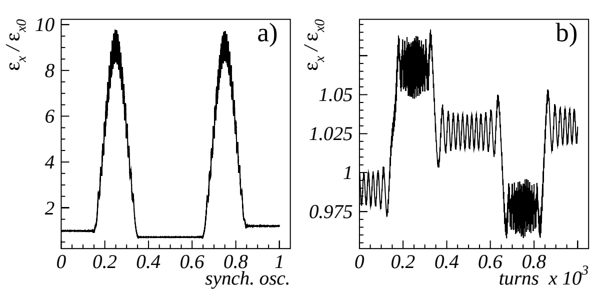

Figure 1: Full trapping a) and scattering b) of the single particle invariant.

The trapping into a resonance has been studied in [4, 5, 6]. Due to this basic mechanism all particles, which periodically cross the resonance, will slowly diffuse out to form a halo. Its density and extension depends on the number of particles that cross the resonance (Eq. 1), and on the outer position of the island (see [7]). If the outer position of islands intercepts the beam pipe or reaches the dynamic aperture beam loss occurs according to the distance from the resonance. As lost particles are characterized by large ϵ_z, the beam loss is accompained by a bunch shortening (see [8]). The long term lost particles are estimated by Eq. 1. We benchmarked the asymptotic lost particles by using the SIS18 parameters and include in the lattice a scraper placed at 3σ of the beam. We also used $Q_z = 10^{-3}$ and $\Delta Q_x = -0.1$. The beam loss is counted after 2.5×10^5 storage turns. In Fig. 2a we show the results of beam loss when the scraper is used and without it.

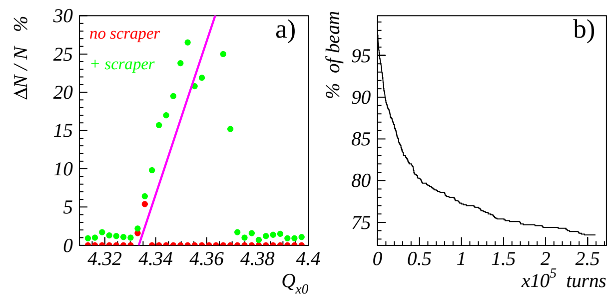

Figure 2: a) Beam loss induced by trapping/scattering of particles (green markers), in red markers beam loss by shrinking of the DA; b) Beam intensity evolution vs. storage time for $Q_{x0} = 4.3525$.

We see that a tiny beam loss occurs in the absence of the scraper because of the shrinking of the dynamic aperture. When the scraper is activated beam loss occurs (red markers): the estimate of the beam loss given by Eq. 1 with $\alpha = 1$ (pink straight line) slightly underestimates the beam loss. In Fig. 2b we show the typical long term beam loss pattern computed for $Q_{x0} = 4.3525$. Note that the loss in the region $4.352 < Q_{x0} < 4.37$ is below the analytic estimate as the asymptotic beam loss has not been reached. This is due to the small area of phase space which intercepts the scraper: in fact, slightly below $Q_{x0} = 4.37$ the beam halo barely touches the scraper and therefore the probability that a particle reaches - by the trapping/scattering process - that tiny area in phase space is very small. Therefore the characteristic time for losing all particles which periodically cross the resonance is drastically reduced.

An experimental campaign to investigate these phenomena took place at the CERN-PS in 2002-2003. A bunched beam with $\Delta Q_x = 0.075$ was stored for 5×10^5 turns and $Q_{y0} = 6.12$. The experimental finding is that the beam undergoes an emittance growth regime for $6.28 < Q_{x,0} < 6.32$ with a maximum emittance growth of 42% at $Q_{x0} = 6.265$. For tunes $6.25 < Q_{x0} < 6.28$ a beam loss regime was found with a maximum beam loss of 32% at $Q_{x0} = 6.265$. The results and simulations on this experiment are documented in Ref. [7, 8, 9]. The maximum beam loss obtained in the simulation is 8% at $Q_{x0} = 6.26$, ignoring chromaticity.

EFFECT OF THE CHROMATICITY

Including chromaticity complicates the particle dynamics. The key feature of the space charge driven tune modulation stems from the symmetry of the longitudinal distribution: the tune modulation has a periodicity which is half of the synchrotron one. The tune modulation introduced by the chromaticity, instead, has the same periodicity as the synchrotron motion. When space charge maximum detuning and maximum chromaticity detuning are comparable, the resulting slow modulation of the transverse tunes is the composition of these two effects, which have different frequencies. In Fig. 3a we show the single particle invariant in one synchrotron oscillation as for Fig. 1a, but now including the effect of the chromaticity. For a particle with $\delta p/p = 2.3 \times 10^{-3}$ the natural chromaticity yields a maximum detuning of $\delta Q_c = 0.01$. In Fig. 3b we plot the islands at $z = 0$ for $\delta Q_c = 0.01$, where loss of momentum takes place (red), and for $\delta Q_c = -0.01$ where gain of momentum occurs (blue). The asymmetry of the position of the fixed points with respect to half a synchrotron oscillation is evident. The overall effect is that islands are pushed further out and increase the halo size. For comparison, in Fig. 3b we plot also the islands in absence of chromaticity (black curve).

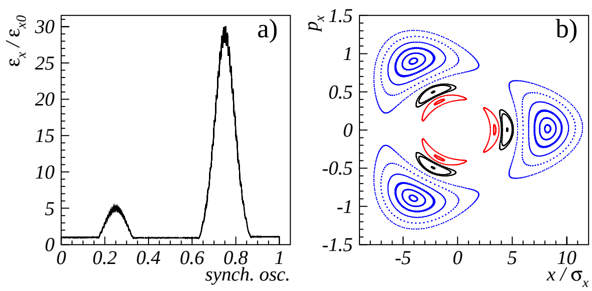

Figure 3: a) Asymmetry of the invariant in one synchrotron oscillation; b) Transverse islands for $\delta Q_c = +0.01$ (red), $\delta Q_c = -0.01$ (blue), corresponding to the loss/increase of particle momentum. For comparison we plot in black the islands when the chromaticity is absent.

Space charge - Chromaticity induced beam loss

An intuitive approach to qualitatively understand the behaviour of beams in presence of the chromaticity is to consider the chromaticity induced tune shift. If $Q_{x0} + \delta Q_c$ gets close to the resonance from above, the trapped particles will be (virtually) brought to infinity. If ΔQ_c is the tunespread induced by the chromaticity, setting the bare tune in the region $Q_{x,res} < Q_{x0} < Q_{x,res} + \Delta Q_c$ will always allow some particles in the bunch to hit the pipe in our model of SIS18. A more detailed description of this beam loss is found in [10]. In Fig. 4a we plot beam loss when chromaticity is included and the beam pipe is shifted to $100\sigma_x$. As the beam used has a chromaticity induced tunespread of $\Delta Q_c = 0.01$, we see a beam loss regime for $4.333 < Q_{x0} < 4.343$. Note also a beam loss region for $4.32 < Q_{x0} < 4.333$. Considering this argument for the CERN-PS experiment, having an rms momentum spread of $\Delta p/p = 1.5 \times 10^{-3}$, the maximum tune spread correspondent to the natural chromaticity is $\Delta Q_c = 0.028$, which corresponds to the witdh of the observed beam loss regime.

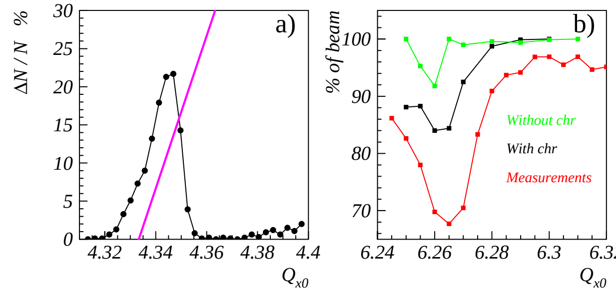

Figure 4: a) beam loss induced by chromaticity; b) New beam loss prediction for the PS-experiment: the black curve gives the simulated beam loss in presence of chromaticity.

We have then repeated the simulation made in [8] including the chromaticity (Fig. 4b). In red we show the measured beam loss, in green the beam loss computed in [8], where the effect of chromaticity was absent, and in black the simulation where the natural chromaticity of the PS synchrotron is included. The beam loss increases up to 16%, which is about 50% of the total measured beam lost.

OUTLOOK

Trapping phenomena are an important subject in high intensity machines as well as in electron cloud rings [11]. The chromaticity plays an important role in the space charge induced trapping/scattering. The CERN-PS experiment modeling has considerably improved by including the chromaticity, and a beam loss regime as broad as the experimental one has been found. The absolute number of lost particles is still 50% smaller than in the experiment. This discrepancy will be subject of suture studies, which should include fully self-consistent simulation.

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