THE NON-LINEAR SPACE CHARGE FIELD COMPENSATION OF THE ELECTRON BEAM IN THE HIGH ENERGY STORAGE RING OF FAIR

A. Chechenin, R. Maier and Yu. Senichev, Forschungszentrum Juelich, Germany

Abstract

In the High Energy Storage Ring, a part of the FAIR project at GSI in Darmstadt, the internal target is used [1]. To compensate the interaction of the antiproton beam with the target the electron beam cooling is needed. However, together with the cooling the non-linear space charge field of electron beam modifies the dynamic aperture for the antiproton beam due to the structural resonance influence. We investigate the possible schemes of this effect compensation using the multi-pole correctors on the HESR.

FIELD OF ELECTRON BEAM

The electron beam used to cool the anti-proton beam has two components of the interaction: the particle-particle interaction and the particle-collective field interaction. The first one is the main component allowing the anti-proton beam cooling and the second one is the effect acting on the anti-proton beam as the non-linear defocusing element. In this work we investigate the latter. From our point of view the mechanism of the heating due to e-beam has two origins. First of all, it arises, when there is the angle between two beams. It is understandable, because we have the coherent transverse kicks from the e-beam, and the e-beam plays the role of heater. The second reason for the heating can be the tune shift of antiproton beam due to the space charge field of e-beam, when the anti-proton beam cross the structural resonances, and the structural resonances heat the anti-proton beam. Besides, even without the structural resonance crossing the non-linearity causes the effective emittance growth. Here we investigate the mechanism of interaction of antiproton beam with the space charge field of e-beam in case of the zero angle between the electron and antiproton beams. The numerical simulations have been done for the HESR lattice [2].

For the e-cooler effective work the electron beam has to have the relative velocity β_e maximum closed to the antiproton beam velocity. In the HESR design its average current is expected to be equal to 1 A and the electron column should have the radius ~5 mm, which is assumed to exceed the antiproton beam size.

Now let us obtain the motion equations of the antiproton beam in the field of the electron column. Since the cooler is installed on the straight section the equations are written in the rectangular coordinate system $\{\mathrm{x}, \mathrm{y}, \tau\}$. Following the MAD presentation [3], let us write the Hamiltonian for the system with the optic elements having the mid plane symmetry and add the space charge potential of the e-beam φ_e :

$$H(p_x, p_y, x, y) = -\left[1 - \frac{2e\varphi_e(x,y)}{m_0\gamma c^2\beta_e^2} - \left(\frac{p_x}{P_0}\right)^2 - \left(\frac{p_y}{P_0}\right)^2\right]^{1/2} - \frac{e}{cP_0}A_\tau(x,y) \quad (1)$$

where $P_0 = \sqrt{\frac{E^2}{c^2} - m_0^2c^2}$ is the momentum of antiproton beam. The vector potential A_τ has the external (magneto-optical elements) and the internal (space charge e-beam field) components, that is $A_\tau = A_\tau^{ext} + A_\tau^{\mathrm{int}}$. The scalar potential φ_e is determined by the space charge field.

To find the space charge field we pass to the system moving together with the beam. The common Lorentz transformation has a view [4]:

$$\varphi_e = \frac{\varphi_e' + \beta_e A_\tau'}{\sqrt{1-\beta_e^2}};\; A_\tau = \frac{A_\tau' + \beta_e \varphi_e'}{\sqrt{1-\beta_e^2}};\; A_x = A_x';\; A_y = A_y' \quad (2)$$

where the prime means the transformation to the system moving together with the beam with the velocity $\mathrm{v} = c\beta_e$. Since in the new system the current is absent the vector potential equals to zero, and we have following relations:

$$\varphi_e = \gamma_e\varphi_e'; \qquad A_\tau = \gamma_e\beta_e\varphi_e'. \quad (3)$$

Substituting the expression for A_s in equation (1) and taking into account $\frac{e\mathrm{v}}{pc^2} = \frac{e}{m_0c^2\gamma\ \ \beta^2}\beta^2$, the final equations system is:

$$\begin{aligned}
&\frac{dx}{d\tau} = p_x; \\
&\frac{dp_x}{d\tau} = -\frac{e}{m_0c^2\gamma_e^3\ \beta_e^2}E_x^{sc} + \frac{e}{pc}\frac{\partial\ A_\tau^{ext}}{\partial\ x}; \\
&\frac{dy}{d\tau} = p_y; \\
&\frac{dp_y}{d\tau} = -\frac{e}{m_0c^2\gamma_e^3\ \beta_e^2}E_y^{sc} + \frac{e}{pc}\frac{\partial\ A_\tau^{ext}}{\partial\ y}; \\
&E_x^{sc} = \frac{\partial\ \varphi_e}{\partial\ x}; \qquad E_y^{sc} = \frac{\partial\ \varphi_e}{\partial\ y}.
\end{aligned} \quad (4)$$

Now we have the equations system with the space charge term and the external term determined by the expressions (1) and (2). The scalar potential is calculated by solving the Poisson equation in the moving coordinate system:

$$\nabla^2 \varphi(x, y) = -\frac{1}{\varepsilon_0} \rho(x, y) \, . \qquad (5)$$

NON-LINEAR TUNE SHIFT FOR UNIFORM AND GAUSSIAN DISTRIBUTIONS

Let us consider two cases, when the electron beam has the uniform and the Gaussian distributions. For the uniform distribution it is the electron column with the current I_e, the radius r_e and the space density is:

$$\rho(r) = \begin{cases} \dfrac{I_e}{\pi r_e^2 \beta_e c}, & \text{for } r < r_e \\ 0, & \text{for } r > r_e. \end{cases} \qquad (6)$$

For the Gaussian distribution it is:

$$\rho(r) = \frac{I_e}{2\pi \sigma_e^2 \beta_e c} \exp\left(-\frac{r^2}{2\sigma_e^2}\right), \qquad (7)$$

where σ_e is the dispersion of distribution.

For the electron beam with the axial symmetry the equation (5) is:

$$\frac{1}{r} \cdot \frac{\partial}{\partial r}(r \cdot E_r) = -\frac{\rho}{\varepsilon_0}, \qquad (8)$$

Then the electrical field for the uniform distribution is:

$$E_r = \begin{cases} \dfrac{I_e}{2\pi \varepsilon_0 c r_e^2} \cdot r, & \text{for } r < r_e \\ \dfrac{I_e}{2\pi \varepsilon_0 c} \cdot \dfrac{1}{r}, & \text{for } r > r_e, \end{cases} \qquad (9)$$

and for the Gaussian distribution is:

$$E_r = \frac{I_e}{2\pi \varepsilon_0 c} \cdot \frac{1}{r} \left(1 - e^{-\frac{r^2}{2\sigma_e^2}} \right) \qquad (10)$$

Figures 1 and 2 (red curves) show the field of electron beam calculated by (9) and (10). We can see for both distributions, if the antiprotons exceed the e-beam size, they experience the non-linear action from the e-beam space charge field.

Since we suspect the tune shift as the main reason for the structural resonance crossing, we calculate the tune shift of the antiproton beam with and without electron beam. The particles are tracked in the lattice with the e-beam field, and the total tune versus the amplitude of oscillation is determined. The results are shown on figures 3 and 4.

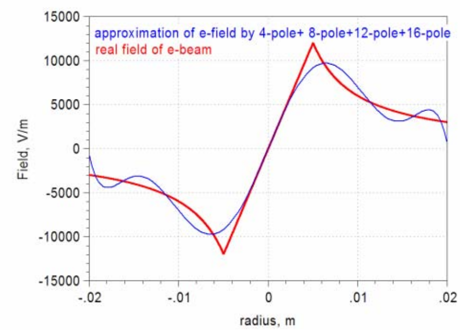

Figure 1: The space charge field of the e-beam with the uniform distribution and its approximation by multipoles.

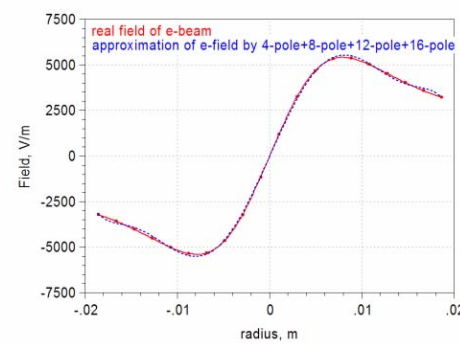

Figure 2: The space charge field of the e-beam with the Gaussian distribution and its approximation by multipoles.

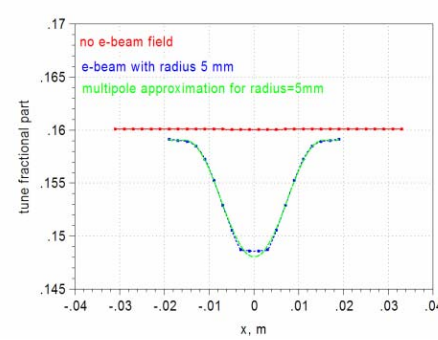

Figure 3: The tune fractional part of anti-proton beam at the uniform e-beam distribution with I_e=1 A and r_e=5 mm.

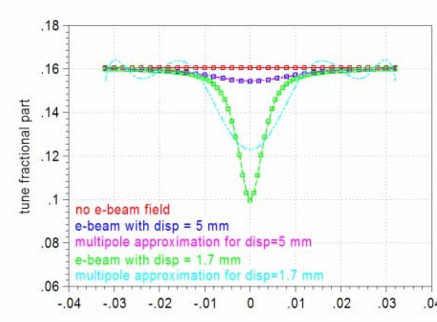

Figure 4: The tune fractional part of anti-proton beam at Gaussian e-beam distribution with I_e=1 A and σ_e=5 mm.

Obviously in case of the uniform distribution, when the electron column edge is sharper, and the total electron charge is concentrated in the smaller volume, the tune is modified stronger by a factor of almost two. For the Gaussian and the uniform distributions the tune shift is about 0.006 and 0.012 accordingly. Thus, for both distributions the tune shift is not enough strong in order to cross the integer structural resonances, and we do not expect the significant influence of the electron beam on the antiproton beam. Although some of the particles will interact with the lower order structural resonances, which have to be studied for the long term instability.

Now let us study the e-beam non-linearity influence. The calculated total tune (see fig. 3 and 4) can be represented as the function of the radius:

$$\nu_x = \nu_{x0} - \Delta\nu_x + a_1 r_x^2 + a_2 r_x^4 + \ldots, \qquad (11)$$

where $\Delta\nu_x$ is the linear detune, or using $r_x = \sqrt{\beta_x \varepsilon_x}$, the total tune can be represented as:

$$\begin{aligned} \nu_x &= \nu_{x0} - \Delta\nu_x + a_1\beta_x \cdot \varepsilon_x + a_2\beta_x^2 \cdot \varepsilon_x^2 + \ldots = \\ &\nu_{x0} - \Delta\nu_x + \left.\frac{\partial\nu_x}{\partial\varepsilon_x}\right|_{\varepsilon_x=0} \cdot \varepsilon_x + \frac{1}{2!}\left.\frac{\partial^2\nu_x}{\partial\varepsilon_x^2}\right|_{\varepsilon_x=0} \cdot \varepsilon_x^2 + \ldots, \end{aligned} \qquad (12)$$

where the non-linear tunes coefficients are:

$$\left.\frac{\partial\nu_x}{\partial\varepsilon_x}\right|_{\varepsilon_x=0} = a_1\beta_x; \quad \left.\frac{\partial^2\nu_x}{\partial\varepsilon_x^2}\right|_{\varepsilon_x=0} = 2a_2\beta_x^2 \qquad (13)$$

The table 1 shows the numerical data for the e-beam current 1 A, the beam radius 5 mm and the cooler length 30 m. We can see the linear tune shift is enough small value. However, the non-linearity is the significant value, and it is essentially more than the non-linear tune shift coming from the chromatic sextupoles.

To understand the influence of the electron beam field non-linearity on the antiproton beam we launched the antiproton beam with and without the electron beam having the initial phase distribution in the form of line. The modification of the line allows observing the non-linearity effect. Figure 5 shows the result of such tracking. You can see in the absence of electron beam the phase line of the antiproton beam remains to be almost unchanged. And on the contrary the phase line becomes twisted in the spiral for the range occupied by the electron beam due to its non-linear field. Due to this effect the effective phase area of the antiproton beam grows.

To compensate this effect we use the multi-pole correctors. The non-linear field is approximated by the polynomial composition of the quadrupole, octupole, dodecapole and dioctapole fields (see fig. 1, 2, blue curve) and the minimization of the mean-square deviation in the range $r<R_{av}$ of the space charge force averaged for the Uniform and Gaussian distributions correspondingly:

$$\mathrm{Min}_{r<R_{av}}\left\{\left[E_r(r) - \left(b_1 \cdot r + b_3 \cdot r^3 + b_5 \cdot r^5 + b_7 \cdot r^7\right)\right]^2\right\}. \qquad (14)$$

In result we get the multi-pole compensation of the non-linear electron field. Figure 5 shows the central part (r<20 mm) of the corrected phase line, and you can see this curve has the smaller effective phase area.

Table 1: The non-linear tunes for the e-beam current

Distribution	Gaussian	Uniform
a_1	4.1x10	$1.2\text{x}10^2$
a_2	$-1.3\text{x}10^5$	$-4.3\text{x}10^5$
$\Delta\nu_x$	0.0063	0.012
$\left.\frac{\partial\nu_x}{\partial\varepsilon_x}\right\|_{\varepsilon_x=0}$	4100	12000

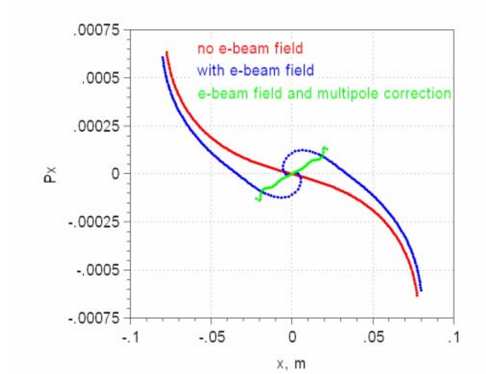

Figure 5: The phase line modification of antiproton beam at the Gaussian electron beam distribution with I_e=1 A and σ_e=5 mm.

The disadvantage of this method is the limited order of multipole. In our case it is m<8. Therefore we can not synthesize the exact copy of the electron beam. As consequence together with the correction of phase area occupied by electron beam the whole area outside of electron beam becomes unstable due to these multipoles.

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