TRANSVERSE COUPLING IMPEDANCES FROM FIELD MATCHING IN A SMOOTH RESISTIVE CYLINDRICAL PIPE FOR ARBITRARY BEAM ENERGIES[0]

A. Al-Khateeb[1,2], R.W. Hasse[2], O. Boine-Frankenheim[2], Wafa M. Daqa[1], I. Hofmann[2]
[1]Yarmouk University, Irbid, Jordan, [2]GSI Darmstadt, Darmstadt, Germany

Abstract

Using exact field matching techniques, the transverse resistive–wall impedance is investigated analytically for a cylindrical pipe of arbitrary wall thickness. The six components of the electromagnetic field excited by a dipolar beam mode are obtained in closed analytic form. Both transverse magnetic and transverse electric modes are excited by the beam and are coupled at the inner and outer surfaces of the resistive wall. This is used to obtain analytic expressions for the corresponding transverse coupling impedance and for the transmission coefficient. The results are applied to SIS18 and to the projected HESR and SIS100 of GSI–Darmstadt as well as to the LHC collimator and the PS Booster of CERN. Some approximate simple formulas for three important regions corresponding to small, intermediate and large frequencies in the ultrarelativistic limit were also obtained analytically.

INTRODUCTION

The resistive beam pipe usually represents the main contribution to the real part of the ring impedance at low frequencies in storage rings or synchrotrons. Recently, the resistive wall has been identified as the main driving source for the observed transverse instabilities in the FNAL Recycler [1]. As part of the FAIR accelerator project at GSI Darmstadt the HESR for anti-protons is presently being designed. This ring should operate in the energy range between 1 GeV and 15 GeV. One of the major challenges in the design of this machine is to provide very high luminosities ($> 10^{32}$ cm^{-2} s^{-1}) together with low momentum spreads for experiments with internal hydrogen targets and, hence, the control of coherent beam instabilities induced by the resistive pipe wall will be crucial.

For the design of storage rings accurate expressions for the resistive wall impedance at arbitrary beam energies are needed in order to estimate the growth rates of instabilities and to define the required feedback systems. In the conventional treatment the resistive wall impedance is calculated for ultra-relativistic beams [2, 3]. A general formalism for computing impedances of non-relativistic beams, including resistive-wall boundaries, was given by Gluckstern [4]. In [5] the resistive wall impedance for non-relativistic beams is considered using a direct solution of Maxwell's equations together with an expression for the resulting impedance. For the transverse impedance the obtained approximation agrees with the result obtained in [4].

In the spirit of our previous work on the longitudinal impedances [6] we extend these calculations to the transverse case. In the present work we consider a beam pipe of arbitrary thickness d and with finite, maybe frequency dependent, conductivity S. Expressions for the resulting transverse impedance can be found in the literature, see e.g. [3 – 8] with recent extensions in [9, 10]. These expression were usually obtained by considering only the TM mode excited by the off-centered beam. For a dipolar perturbation –as is the case for transverse oscillations– both TM and TE modes will be excited. Here we will use a linear combination of the excited TM and TE modes in order to obtain a analytic expression for the transverse coupling impedance. The boundary conditions at the three interfaces are satisfied exactly, thereby disregarding the previously employed approximate Leontovich boundary condition [11].

TRANSVERSE EXCITATION

Assume a particle beam in the form of a circular lamina of radius a that moves in the z-direction off axis with a constant longitudinal velocity $\upsilon = \beta c$ in a cylindrical pipe of radius b. If the beam center is displaced by d in the vertical direction giving rise to an electric dipole moment P, then

$$\delta\rho(r,\theta,z,t) = \frac{P}{\pi a^2}\,\cos\theta\;\delta(a-r)\;\delta(z-\beta ct)$$

accounts for the small movement of the beam center in the direction of d and it represents a correction or perturbation to the beam volume charge density. Due to the azimuthal dependence in $\delta\rho$, this term is a dipole term and is the source of the transverse impedance resulting from the off–axis motion of the beam center. Without loss of generality, we assume the beam to be displaced along the x–axis. In the frequency domain this is the source term used by Gluckstern to calculate the transverse impedance for a particle beam displaced in the x direction [4].

ELECTROMAGNETIC FIELDS

The basic wave equations satisfied by the magnetic and electric fields $\mathbf{B}, \mathbf{E}$ in a conducting medium of conductivity S are obtained from Maxwells equations and the equation of continuity with source terms of external (free) charge density ρ_c and of current density $\mathbf{j}$. Upon rewriting the Maxwell equations in cylindrical coordinates, thereby assuming a field variation with z such that $e^{ik_z z}$, one obtains six scalar equations. Solving for the transverse electromagnetic field components in terms of $E_z(r,\theta,z,\omega)$ and

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$B_z(r, \theta, z, \omega)$, we obtain the solutions in terms of modified Bessel functions I_ℓ and K_ℓ. Further, the field components will be written in each region as $E_{\ell,z}(r, z, \omega)\cos\ell\theta$ and $B_{\ell,z}(r, z, \omega)\sin\ell\theta$. Accordingly, the following equations will be used to determine the electric and magnetic fields in each region.

In order to find the six integration constants at the three boundaries, we need six boundary conditions. Usually here the so-called Leontovich boundary condition [11] has been employed. This approximation consists in treating the electromagnetic field as a plane plane wave impinging on a well conducting surface. We, however, integrate exactly the differential equation for E_z across $r = a$ to get the following boundary condition concerning the discontinuity of $\partial E_z/\partial r$ at $r = a$, namely,

$$\frac{\partial E_{1,z}^{r\geq a}}{\partial r} - \frac{\partial E_{1,z}^{r\leq a}}{\partial r} = i\frac{k_z}{\epsilon_0\gamma_0^2\beta c}\,\frac{P}{\pi a^2}.$$

Here γ_0 is the relativistic factor. We also use the continuity of $E_{1,z}$ at $r = a$, the continuity of $E_{1,z}$, $B_{1,z}$, $E_{1,\theta}$, and $D_{1,r}$ at $r = b$, where $D_{1,r}$ is the radial electric displacement current of the dipolar source.

TRANSVERSE COUPLING IMPEDANCE AND TRANSMISSION

The Panofsky–Wenzel (PW) theorem states the transverse momentum imparted to charged particles moving parallel to the axis of a cavity [13]. This theorem gives a relationship between the integrated longitudinal and transverse momentum kicks a particle receives as it traverses –with constant-velocity and in paraxial limit– an isolated medium or device with an electromagnetic excitation [2, 3, 12]. The PW theorem estimates the transverse impedance from the longitudinal one at the same azimuthal number ℓ, namely,

$$Z_{\ell,\perp}^{(\mathrm{total})}(\omega) = \frac{v}{\omega}\,Z_{\ell,\parallel}^{(\mathrm{total})}(\omega) = \frac{1}{k_z}\,Z_{\ell,\parallel}^{(\mathrm{total})}(\omega)\ , \quad \ell > 0,$$

where $Z_{\ell,\parallel}^{(\mathrm{total})}(\omega)$ and $Z_{\ell,\perp}^{(\mathrm{total})}(\omega)$ are measured in $\Omega\mathrm{L}^{-2\ell}$ and $\Omega\mathrm{L}^{-2\ell+1}$, respectively. The case $\ell = 0$ has been excluded since it corresponds to an on axis beam motion with zero transverse displacement, see the last paragraph of this Section. The transverse Lorentz force will vanish identically for the rotational symmetric mode $\ell = 0$ and therefore, particles will not be deflected by the zero azimuthal electromagnetic fields.

We introduce below a generalized expression for the calculation of the longitudinal coupling impedance of an arbitrary azimuthal ℓ as a volume integral over the corresponding longitudinal electric field within the beam $E_z^{(r\leq a)}(\mathbf{r}, \omega)$ and the source electric current density $j_z(\mathbf{r}, \omega)$. Introducing the ℓ^{th} electric moment M_ℓ such that $M_\ell = Qd^\ell$ with d being the offset from the axis of the beam–pipe, we write the following,

$$Z_{\ell,\parallel}(\omega) = \frac{-1}{M_\ell^2}\int_0^a\int_0^{2\pi}\int_0^L r dr d\theta\, dz \times E_{\ell,z}^{(r\leq a)}(r, \omega)\cos\ell\theta\, e^{ik_z z}\, j_z^*(r, \theta, z, \omega)\ . \qquad (1)$$

The transverse coupling impedance $Z_{\ell,\perp}(\omega)$ is simply obtained from the PW theorem. For $\ell = 0$, Eq. (1) reduces into the well known expression of the longitudinal coupling impedance of the zero azimuthal measured in Ω [2, 4, 6]. Using equation (1) for $\ell = 1$, we obtain the following dipole longitudinal coupling impedance $Z_{1,\parallel}(\omega)$ measured in $\Omega\mathrm{L}^{-2}$ and the transverse coupling impedance $Z_{1,\perp}(\omega)$ in $\Omega\mathrm{L}^{-1}$. The resistive wall impedance then is given by the total coupling impedance minus the space charge impedance

$$Z_\perp^{\mathrm{sc}}(\omega) = \frac{iZ_0 L I_1^2(\sigma_0 a)}{\pi a^2\gamma_0^2\beta}\left[\frac{K_1(\sigma_0 a)}{I_1(\sigma_0 a)} - \frac{K_1(\sigma_0 b)}{I_1(\sigma_0 b)}\right]\ ,$$

where $Z_0 = 1/c\epsilon_0$ is the vacuum impedance, and $\sigma_0 = k_z/\gamma_0$.

In addition to the coupling impedance we also calculate the electric and magnetic transmission coefficients. As defined previously in [6] the electric transmission coefficient is defined as the ratio of the electric field behind the pipe to the field entering the pipe,

$$\tau = \frac{\textsf{transmitted electric field}}{\textsf{impinging electric field}}\ .$$

A similar definition holds for the magnetic one.

RESULTS

For the calculation of the impedances and transmission coefficients we provide different examples for the existing SIS 18, the projected SIS 100, the antiproton machine HESR as well as for the collimator part of LHC. For the different machines we employ the following parameters:

Table I: Machine parameters

	SIS18	SIS100	HESR	LHC
Circumference [km] L	0.216	1.080	0.570	27
DC conductivity $[\Omega\ \mathrm{m}]^{-1} S$	10^6	10^6	10^6	10^5
Aperture radius [cm] b	10	4	5	0.2
Beam radius [mm] a	5	20	1	0.2
Wall thickness [mm] d	0.3	0.2	10	0.2
Reference γ_0	1.0122	1.12	2	7500

Here the conductivity of stainless steel, $S = 10^6[\Omega\ \mathrm{m}]^{-1}$, is used throughout or, according to [14], for the LHC collimator, the frequency dependent conductivity $S = S_{\mathrm{DC}}(1 + i\omega\tau_0)$ with $\tau_0 = 0.8\mathrm{ps}$.

Fig. 1 shows a comparison between the exact result and the previously known thin wall approximation textbook formula [4]

$$Z_{\perp,\mathrm{textbook}} = \frac{L\beta}{\pi b^3}\sqrt{\frac{Z_0}{2S\omega}}. \qquad (2)$$

As can be seen, for this case only results for irrelevant very small frequencies differ from the exact result. The

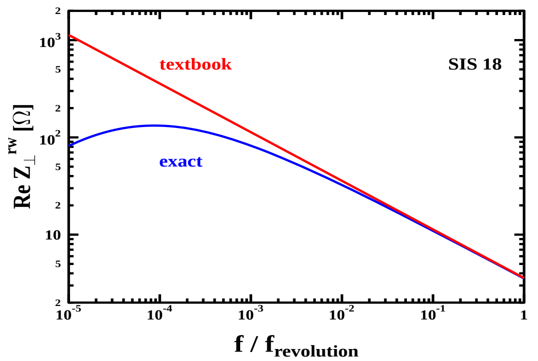

Figure 1: Comparison between exact (blue) and textbook result eq.(2), of the real part of the transverse resistive wall impedance (SIS18)

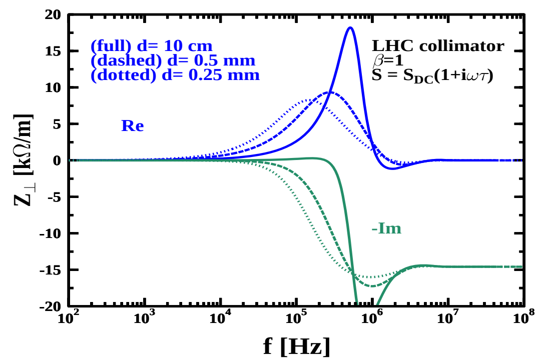

Figure 2: Transverse impedances for different wall thicknesses LHC collimator)

asymptotic static limit, however, is correctly reproduced by the exact calculation. The next example, Fig. 2, shows the transverse impedance for different wall thicknesses, exhibiting nicely the transition from thick to thin wall. Both previous features also manifest themselves in Fig. 3. Here for wall thicknesses larger than 0.1 mm the thick wall approximation suffices. Finally we show an example of the transverse transmission coefficient and compare it with the longitidinal one derived in [6]. One sees that the longitudinal transmitted field is shielded much better than the transverse one. In case of SIS 100 for a wall thickness of the order of a tenth of a millimeter still a large fraction of the field strays into the ouside vacuum. The magnetic transmission

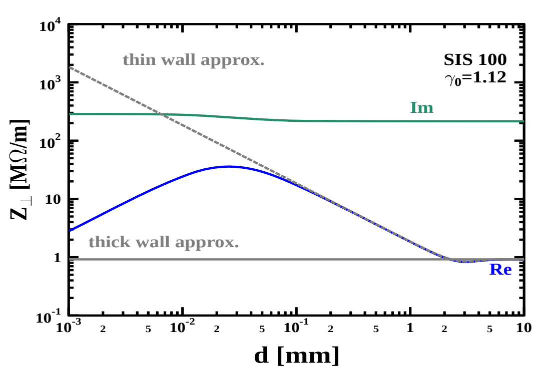

Figure 3: Transverse exact resistive wall impedances (blue, green) together with the thin wall approximation result for SIS 100

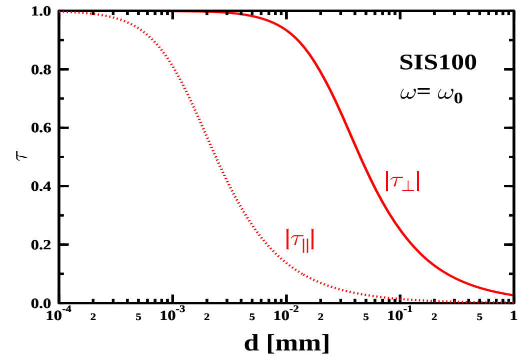

Figure 4: Comparison of transverse and longitudinal transmission coefficients for SIS 100 at revolution frequency

coefficient also has been calculated but is negligibly small as compared to the electric one.

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