# SIMULATION STUDY OF TRANSVERSE COUPLED-BUNCH INSTABILITIES DUE TO ELECTRON CLOUD IN KEKB LER

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*Abstract*

In this paper we report simulation results on the transverse coupled-bunch instabilities (TCBI) due to electron cloud at the KEKB Low Energy Ring (LER). The formation of electron cloud and related TCBI is investigated based on realistic solenoid field model. Studies on electron cloud in Quadrupole which could induce TCBI are also presented in this paper.

## INTRODUCTION

Electron cloud induced TCBI have been observed in the KEKB LER and well suppressed by solenoids and bunch by bunch feedback system. However, the TCBI may still be the limitation in high current operation with shorter bunch spacing for achieving high luminosity at KEKB. The main objective of this study is to obtain the information on the causes of TCBI using simulation method. The parameters used in simulation are tabulated in Table 1.

Table 1: Basic parameters of the KEKB LER

| variable | symbol | KEKB-LER |
|---|---|---|
| circumference | $L$ | 3016m |
| beam energy | $E$ | 3.5GeV |
| bunch population | $N_b$ | $3.3\times10^{10}$ |
| bunch spacing | $t_{sp} = L_{sp}/c$ | 8 ns |
| rms beam size | $\sigma_x$ | 0.42 mm |
| | $\sigma_y$ | 0.06 mm |
| bunch length | $\sigma_z$ | 4 mm |
| synchrotron turn | $\nu_s$ | 0.024 |
| betatron tune | $\nu_{x(y)}$ | 45.51/43.57 |
| chamber radius | $R$ | 0.05 m |

## SIMULATION PROGRAM

A particle in cell simulation code [1] has been developed to study the dipole coupled-bunch electron cloud effect.

Coulomb force in two-dimensional space from bunches is given by the Bassetti-Erskine formula [2] in circular chamber with assumption that the profiles of positron bunch are Gaussian with standard deviation determined by the emittance and average beta function. In order to calculate beam force in antechamber or even arbitrary shaped chamber, or displaced bunches in circular chamber, two-dimensional Finite Element Method (FEM) Poisson solver [3] was implemented in the simulation code. Figure 1(a) shows a conformal mesh representing antechamber with 2000 triangular elements. The number of elements is a tradeoff between computational overhead and accuracy. Charge is deposited into mesh elements and a charge density vector $\rho$ is determined. Potential on every mesh nodes is calculated by solving equation:

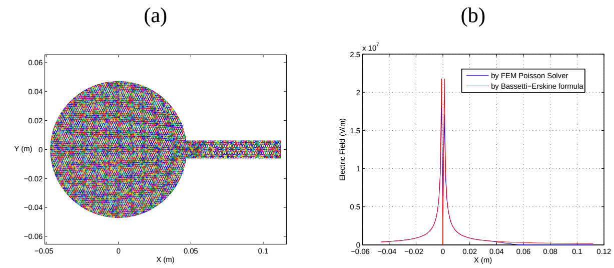


Figure 1: Triangular mesh based on KEKB antechamber geometry (a) and comparison of beam field calculation by different method (b)

$$\phi = M^{-1}\rho \quad (1)$$

where $M$ is stiffness matrix determined only by mesh geometry. Second order generalized least-squares fits is then employed to calculate electric field on desired point from potential on neighboring nodes. Figure 1(b) shows electric field of bunch along axis $x$. FEM Poisson solver works out correct electric field in slot part while Bassetti-Erskine formula is much more accurate at the vicinity of bunch.

Motion of bunches and electrons can be fully depicted by equations [4]:

$$\frac{d^2\boldsymbol{x}_p}{ds^2} + \boldsymbol{K}(s)\boldsymbol{x}_p = \frac{r_e}{\gamma}\sum_{e=1}^{N_e}\boldsymbol{F}(\boldsymbol{x}_p - \boldsymbol{x}_e)\delta_p(s - s_e) \quad (2)$$

$$\frac{d^2\boldsymbol{x}_e}{dt^2} = 2r_e c^2\sum_{p=1}^{N_b}\boldsymbol{F}(\boldsymbol{x}_e - \boldsymbol{x}_p)\delta_p[t - t_p(s_e)] + \frac{e}{m_e}\frac{d\boldsymbol{x}_e}{dt}\times\boldsymbol{B} - 2r_e c^2\frac{\partial\phi}{\partial\boldsymbol{x}} \quad (3)$$

where subscripts $p$ and $e$ of $\boldsymbol{x}$ denote positron and electron, respectively, $r_e$ is the classical electron radius, $m_e$ is the electron mass, $c$ is the speed of light, $e$ is the electron charge, $\phi$ is the electric potential due to electrons, $\delta_P$ is the periodic delta function for the circumference, and $\boldsymbol{F}$ is the Coulomb force in two-dimensional space given by the combination of the Bassetti-Erskine formula and 2D FEM Poisson sovler.

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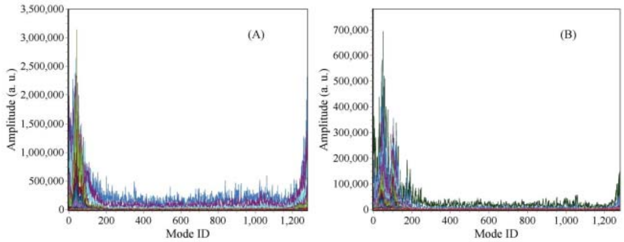


Figure 2: Observed unstable modes with 45 Gauss solenoid field for the horizontal plane (a) and the vertical plane (b)

Electrons' movement in simulation is calculated using Runge-Kutta method with adaptive stepsize control. It causes heavy computational overhead especially in tracking simulation. Simulation program was parallelized with the OpenMP since electrons move independently to each other between calculations of electron cloud potential. Overall performance is therefore greatly enhanced in shared-memory multiprocessor system.

## TRACKING OF COUPLED BUNCH INSTABILITY IN 45 GAUSS REALISTIC SOLENOID FIELD

In tracking simulation, bunches in a ring interact with cloud by solving Eq. 1–3 numerically for hundreds or more revolutions. The transverse amplitude of each bunch is obtained as a function of time. Fourier transformation of the amplitude of all bunches gives a spectrum of unstable modes.

At KEKB LER, about 75% of circumference are covered by solenoid with 45 Gauss magnetic field at the center. Experiments were carried out using fast position monitor [5]. Mode spectra deriving from experimental data as shown in Figure 2 can be compared with that from the simulation, as shown in Figure 5.

Here we focus on equal polarity configured solenoids which magnetic field can be exactly expressed as an expansion of series [6] as

$$B_z = B_0 \frac{2ka}{\pi} \sum_{n=1}^{\infty} \sin nhk K_1(nka) I_0(nkr) \cos nkz + B_0 \frac{2h}{\lambda}$$

$$B_r = B_0 \frac{2ka}{\pi} \sum_{n=1}^{\infty} \sin nhk K_1(nka) I_1(nkr) \sin nkz \quad (4)$$

where the $K_n$ and $I_n$ are modified Bessel function of order n, $a$ is the solenoid radius, $2h$ is the solenoid length, $\lambda$ is the distance between adjacent solenoids.

Figure 3 shows the magnetic field within 2 equal polarity configured solenoids with $B_0 = 45$ Gauss, $a = 70$mm. $\lambda = 1$ meter, $h = 0.4$m in (a) and 0.25m in (b), which implies gap size between solenoids is 20cm (small gap) or 50cm (large gap). The number of terms in series used in simulation is 20.

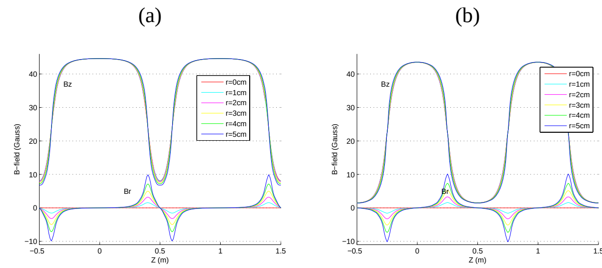


Figure 3: Realistic solenoid field within 2 equal polarity configured solenoids of $B_0 = 45$ Gauss, 20cm gap is assumed in (a) and 50cm in (b).

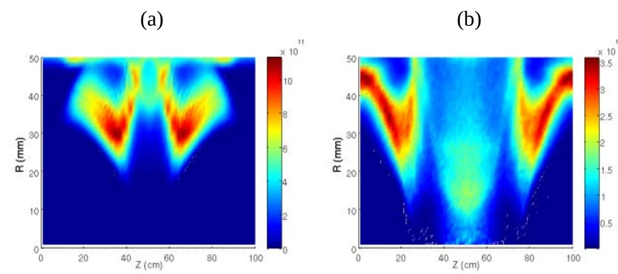


Figure 4: Electron cloud distribution in 45 Gauss EP configured solenoid field, (a) for small gap case and (b) for large gap case.

On contrary to in uniform solenoid field, electrons in the magnetic field given by Figure 3 are not confined along the chamber surface. On the assumption of uniform photoemission on circular chamber surface, electrons distribution is of circular symmetry. Therefore normalized volume density plot calculated on concentric rings with 1mm$\times$1cm cross section could be helpful in understanding spacial distribution of electron cloud. As shown in Figure 4(a), large amount of electrons are trapped around the end of solenoid where we can see a electron "donut" with radius of about 3cm.

We performed tracking simulation in the magnetic field

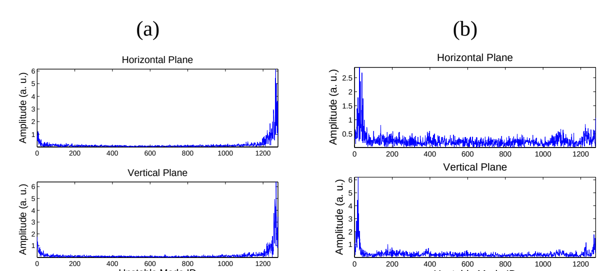


Figure 5: Unstable modes by tracking in realistic solenoid field of 45 Gauss, (a) for small gap case and (b) for large gap case.

given by Figure 3 with the same fill pattern as in experiment: a bunch train of 1154 bunches with 8ns bunch spacing was assumed. For the beam current of 600 mA, a bunch contains $3.3 \times 10^{10}$ positrons. Figure 5 shows unstable mode spectra obtained from tracking. Unstable modes are always larger than $M - \text{Int}(\nu)$ or smaller than $M/2 - \text{Int}(\nu)$ which proves the "focusing" nature of wake force due to electron cloud in solenoid field. In large gap case, unstable modes are consistent with experimental ones. Electron cloud distribution in Figure 4(b) shows that a big number of electrons are trapped in the gap region between equal polarity configured solenoids and they can approach very close to beam orbit. Electrons' gyration frequency of about 5.6 MHz can be used to explain the unstable modes when the magnetic field in gap is as weak as 2 Gauss. Mode spectra suggest unstable modes observed in experiment reflecting the movement of electrons trapped between solenoids.

## WAKE FIELD AND GROWTH RATE OF THE COUPLED-BUNCH INSTABILITY IN QUADRUPOLE MAGNET

We use simulation method to evaluate wake field. Bunches pass through the chamber center until the electron cloud get saturated. The $i$-th bunch with a slight displacement $\Delta y_i$ passes through the cloud exciting a dipole wake which affects the subsequent bunches. The subsequent $j$-th bunch experiences momentum kick $\Delta p_j$ and the dipole wake field is computed as

$$W_{1,y}(z_j - z_i) = \frac{\gamma}{N_p r_e} \frac{\Delta p_j}{\Delta y_i} \tag{5}$$

where the actual momentum kick $\Delta p_j$ is calculated in the simulation.

For a bunch train uniformly filled with $M$ bunches, the equation of motion gives a dispersion relation,

$$(\Omega_m - \omega_\beta) L/c = \frac{N_p r_e c}{2\gamma\omega_\beta} \sum_{\ell=1}^{N_w} W_1(-\ell L_{sp}) \times \exp\left(2\pi i \ell \frac{m + \nu_\beta}{M}\right) \tag{6}$$

where $N_w$ is the range of the wake field, $L_{sp}$ is bunch spacing, $\omega_\beta = \omega_0 \nu_\beta$ is the betatron angular frequency, $\nu_\beta$ is the horizontal or vertical tune, the collective mode oscillation number is given by $m = 0, 1, 2, \ldots, M-1$. The imaginary part of $\Omega_m L/c = T_0/\tau_m$ is the growth rate per revolution for the $m$th mode.

Figure 6(b) shows wake field and growth rate due to electron cloud in a quadrupole magnet. The normal value of the quadrupole field gradient in KEKB LER is 10.3T/m. In simulation, bunch 300 was displaced by 5mm in $+y$ direction after electron cloud get saturated with average volume density of $3.8 \times 10^{12}$ m$^{-3}$. Bunch population of $4.95 \times 10^{10}$, corresponding to 1A beam current, is adopted to get stronger beam force. However neither obvious oscillation nor evident unstable mode can be identified on wake field plot or growth rate spectrum. For the electron cloud distribution as shown in Figure 6(a), we can see most of electrons are trapped at region where local magnetic field is as high as 400$\sim$500 Gauss and magnetic flux perpendicular to beam force. Such a strong magnet field finally limits the change of electron cloud distribution which is essential to coupled-bunch effect.

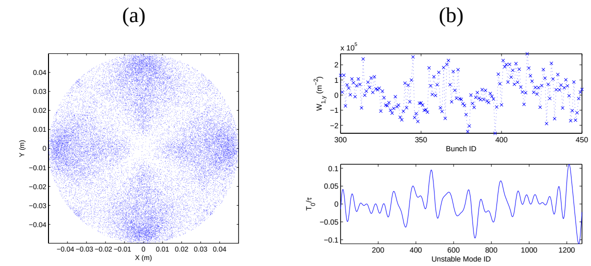


Figure 6: (a)Electron cloud distribution in the transverse x-y plane within Quadrupole magnet. (b) Wake field and growth rate of coupled-bunch instability. Initial displacement in $+y$ direction is 5mm.

## SUMMARY

A particle-in-cell simulation program has been developed to study transverse coupled-bunch instability due to electron cloud in KEK LER. Calculation method was firstly introduced.

Tracking simulation in a realistic solenoid field suggests unstable modes observed in experiment maybe due to electrons trapped in the gap region between solenoids.

Wake field calculation shows that strong magnetic field in quadrupole magnet limits the change of electron cloud distribution and no unstable mode has been found.

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