TUNE SHIFT INDUCED BY NONLINEAR RESISTIVE WALL WAKE FIELD OF FLAT COLLIMATOR

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Abstract

We present formulae for the coherent and incoherent tune shifts due to the nonlinear resistive wall wake field for a single beam traveling between two parallel plates. In particular, we demonstrate that the nonlinear terms of the resistive-wall wake field become important if the gap between the plates is comparable to the transverse rms beam size. We also compare the theoretically predicted tune shift as a function of gap size with measurements for an LHC prototype graphite collimator in the CERN SPS and with simulations.

INTRODUCTION

A common procedure for measuring collimator impedances is to detect the coherent betatron tune shift as a function of collimator-gap size. If the collimator and bunch are long, the impedance is dominated by the resistive-wall effect of the collimator jaw. However, comparing the measurement with the classical resistive-wall theory, such as described in [1], does not necessarily give the correct answer, especially if the jaw conductivity is low and/or the beam size comparable to the gap size.

In this paper, we present a generalized formula, which takes into account both the slow diffusion of the electromagnetic field through the collimator jaw, and the nonlinear components of the resistive-wall wake field. The former modifies the low-frequency impedance, while the latter introduces a dependence of the tune shift on the transverse emittance.

We benchmark our generalized theory against beam measurements with an LHC prototype collimator performed at the CERN SPS in 2004 [2].

Throughout this paper we consider Gaussian longitudinal and transverse beam distributions. In the vertical plane the distribution is truncated at the collimator aperture.

COHERENT TUNE SHIFT

Classical Theory

The classical theory of the thick-wall resistive impedance [1] is valid for a wall thickness d much larger than the skin depth δ, $d \gg \delta$, and for $\chi c/b \ll |\omega| \ll c/(b\chi^{1/3})$ with $\chi \equiv \frac{1}{Z_0 \sigma b}$, b the chamber radius, c the speed of light, $Z_0 \approx 377\ \Omega$ the vacuum impedance, and σ the wall conductivity. Typically, χ is small. For example, we find $\chi \sim 2 \times 10^{-5}$ for carbon of $\sigma \approx 10^5\ \Omega^{-1}\ \mathrm{m}^{-1}$ with half gap $b \approx 1.5$ mm, and the approximation is good over a large frequency range, from about 1 MHz to 1 THz.

Introducing $\lambda_0(\omega) = (i + \mathrm{sgn}(\omega))\sqrt{\mu_0 \sigma |\omega|/2}$ the classical impedance is

$$Z_{\mathrm{class}}(\omega) = -\frac{i}{b^3}\frac{4\pi\lambda_0(\omega)}{Z_0 \sigma c}\frac{c}{\omega}\ , \qquad (1)$$

and, for a flat chamber with half height b, the classical coherent tune shift becomes

$$\Delta Q^{\mathrm{flat}}_{\mathrm{class}} \approx \frac{\beta}{4\pi}\frac{N_b r_p L}{2\pi\gamma}\int_{-\infty}^{\infty}\frac{\pi^2}{8} Z_{\mathrm{class}}(\omega)\ e^{-\omega^2\sigma_z^2/c^2}\ d\omega\ , \qquad (2)$$

where N_b denotes the bunch population, r_p the classical proton radius, γ the Lorentz factor, and L the length of the collimator. The factor $(\pi^2/8)$ describes the difference between a flat and a round chamber [3].

Burov-Lebedev Theory

The resistive-wall impedance derived by Burov and Lebedev [4, 5] includes the effect of the finite chamber thickness as well as the correct low-frequency behaviour. It assumes $c/\omega \gg b$ and also $\beta\gamma \gg 1$ (relativistic limit; for an estimate of non-relativistic corrections see [6]). In the Burov-Lebedev theory for a vertically flat chamber of thickness d with inner radius b and outer radius $a = (b+d)$, surrounded by vacuum, the impedance can be approximated, with an accuracy better than 13%, by [4]

$$Z^{\mathrm{flat}}_{\mathrm{BL},y} \approx -i\frac{\pi^2}{12}\frac{Z_0}{2\pi b^2}\frac{1}{1+\tau/2}\ , \qquad (3)$$

where $\tau \equiv \kappa b \tanh(\kappa d)$, $\kappa \equiv \sqrt{-4\pi i \sigma\mu\omega/c^2}$, and $|\kappa| b \gg 1$ is assumed.

Including the incoherent contribution for a flat chamber via a multiplicative factor $3/2$ [7], the corresponding coherent tune shift follows from

$$\Delta Q^{\mathrm{flat}}_{\mathrm{BL},y} \approx \frac{\beta}{4\pi}\frac{N_b r_p L}{2\pi\gamma}\int_{-\infty}^{\infty}\frac{3}{2}\ Z^{\mathrm{flat}}_{\mathrm{BL},y}(\omega)\ e^{-\omega^2\sigma_z^2/c^2}\ d\omega\ . \qquad (4)$$

General Nonlinear Theory

The nonlinear wake potential, up to infinite order in the transverse positions of both drive and probe particles, for the resistive-wall wake of a Gaussian bunch passing between two parallel plates was derived by Piwinski [8] and re-written by Bane, Irwin and Raubenheimer [9].

In the Piwinski calculation, the time dependence and the dependence on the transverse coordinates factorize. We can therefore replace the time- (or frequency-) dependent part with the more precise expression of Burov-Lebedev, while keeping Piwinski's nonlinear transverse dependence, which represents a purely geometric effect.

In the Piwinski formalism, the vertical deflection $\Delta y'$ of single particle is obtained by the negative derivative of the potential with respect to the y coordinate of the probe particle. Now considering a coherent oscillation, where the center of both drive and test-particle distribution is displaced by a small offset y_c, the centroid deflection due to this offset, related to the coherent tune shift, is obtained by differentiating $\Delta y'$ with respect to y_c, then setting y_c to zero, and finally integrating over the horizontal and vertical distributions of test and source particles. Two of the four integrations can be performed analytically after a change of variables; a double integral is left for numerical evaluation. Executing the sketched procedure [10], we obtain the general result

$$\Delta Q_{\mathrm{nl},y}^{\mathrm{flat}} \approx \frac{\beta}{4\pi}\frac{N_b r_p L}{2\pi\gamma} f_{\mathrm{nl}} \int_{-\infty}^{\infty} \frac{3}{2}\, Z_{\mathrm{BL},y}^{\mathrm{flat}}(\omega)\, e^{-\omega^2\sigma_z^2/c^2}\, d\omega \,. \tag{5}$$

This looks similar to (4), but the complexity is hidden in the factor $f_{\mathrm{nl}}(b,\sigma_x,\sigma_y)$, which is defined as

$$f_{\mathrm{nl}} \equiv \left(\frac{2b^2}{\pi^2}\right) \frac{\mathrm{erf}\left(\frac{b}{\sigma_y}\right)}{\left(\mathrm{erf}\left(\frac{b}{\sqrt{2}\sigma_y}\right)\right)^2} \int_{-\infty}^{\infty}\int_{-2\pi}^{2\pi} G(X,Y)\, \frac{e^{-\frac{Y^2b^2}{4\pi^2\sigma_y^2}-\frac{X^2b^2}{4\pi^2\sigma_x^2}}}{4\pi\sigma_x\sigma_y}\, dY\, dX \,, \tag{6}$$

where the error functions arise due to the distribution cut off at an amplitude equal to the half gap, and the function $G(X,Y)$ is

$$\begin{aligned} G(X,Y) = &-\frac{1}{8}\left(\cos\left(\frac{Y}{2}\right)+\cosh\left(\frac{X}{2}\right)\right)^{-3} \\ &\left[\left\{2Y\sin\left(\frac{Y}{2}\right)-8\cos\left(\frac{Y}{2}\right)\right\}\cosh\left(\frac{X}{2}\right)^2\right. \\ &-\{4\cos Y+12+Y\sin Y\}\cosh\left(\frac{X}{2}\right) \\ &-4Y\sin\left(\frac{Y}{2}\right)-8\cos\left(\frac{Y}{2}\right) \\ &\left.-X(\cos Y-3)\sinh\left(\frac{X}{2}\right)+X\cos\left(\frac{Y}{2}\right)\sinh X\right] \,. \end{aligned} \tag{7}$$

INCOHERENT TUNE SHIFT

Since the deflection depends on the transverse and longitudinal coordinate of the test particle, the collimator induces also an incoherent tune shift. From Piwinski's classical nonlinear wake potential, we can derive the following expression for the single-particle tune [10]

$$\Delta Q_{y,\mathrm{inc}}(x,y,\tau) = -\frac{1}{\left(\mathrm{erf}\left(\frac{b}{\sqrt{2}\sigma_y}\right)\right)^2}\frac{\beta}{4\pi}\,\kappa f_R(\tau) \int_{-\infty}^{\infty}\int_{-b}^{b}\tilde{G}(x,y,x_0,y_0)\frac{e^{-\frac{y_0^2}{2\sigma_y^2}-\frac{x_0^2}{2\sigma_x^2}}}{(2\pi)\sigma_x\sigma_y}\,dy_0\,dx_0 \,, \tag{8}$$

where $\kappa \equiv (1/2)N_b r_p/(\gamma\sigma_z)(L/b)\sqrt{\lambda\sigma_z}$, $f_R(\tau) = \frac{\sqrt{2}}{\pi}\int_0^\infty d\tau'/\sqrt{\tau'}e^{-(\tau+\tau')^2/2}$, and

$$\begin{aligned} \tilde{G}(x,y,x_0,y_0) = \frac{\pi^2}{4b^2}&\left[-\frac{2\sin y_-\,(y_-\cos y_- + \sin y_-)}{(\cos y_- - \cosh x_-)^2}\right. \\ &+\frac{-2\cos y_- + y_-\sin y_-}{(\cos y_- - \cosh x_-)} + \frac{2\sin y_+\,(y_+\cos y_+ + \sin y_+)}{(\cos y_+ + \cosh x_-)^2} \\ &+\frac{2\cos y_+ - y_+\sin y_+}{(\cos y_+ + \cosh x_-)} - \frac{\cos y_-\,(y_-\sin y_- + x_-\sinh x_-)}{(\cos y_- - \cosh x_-)^2} \\ &-\frac{2\sin^2 y_-\,(y_-\sin y_- + x_-\sinh x_-)}{(\cos y_- - \cosh x_-)^2} \\ &+\frac{\cos y_+\,(y_+\sin y_+ - x_-\sinh x_-)}{(\cos y_+ + \cosh x_-)^2} \\ &\left.+\frac{2\sin^2 y_+\,(y_+\sin y_+ - x_-\sinh x_-)}{(\cos y_+ + \cosh x_-)^2}\right] \,, \end{aligned}$$

with $y_+ \equiv \pi/(2b)(y+y_0)$, $y_- \equiv \pi/(2b)(y-y_0)$, and $x_- \equiv \pi/(2b)(x-x_0)$. Strictly speaking, we should also here replace the classical time dependence of the wake for a Gaussian bunch, incorporated in $f_R(\tau)$, by the equivalent function corresponding to the Burov-Lebedev theory, as we have done when computing the coherent tune shift. This would give a few percent correction (though no precision data are available) and lead to more tedious expressions, but it would not affect the transverse dependence which is at the origin of the large tune spread for small gaps.

EXAMPLES

We first consider the parameters in Table 1, which are close to those of the SPS experiment (except that vertical and horizontal planes are exchanged). Figure 1 illustrates the dependence of the coherent tune shift on the transverse emittance. Figure 2 shows the incoherent tune spread computed from (8) for collimator half gaps b of 1.5 and 1 mm. The actual experiment in the SPS exhibited some indirect evidence for a change in the tune spread. Namely, the natural oscillation amplitude was reduced when the collimator was closed [2, 11, 12], which could be explained by the enhanced Landau damping due to the larger tune spread.

In Fig. 3, the coherent tune shift observed in the SPS experiment is compared with the predictions from Eqs. (2), (4) and (5). The latter were calculated using the actual optical functions at the collimator, and, for each data point, the measured bunch intensity, bunch length, and emittances, including error propagation [10]. The nonlinear formula (5) agrees well with the experimental data, while the other two expressions, (4) and (2), deviate by factors 2 or 2.5, respectively, at the smallest gaps.

Table 1: Example parameters.

bunch population	N_b	10^{11}
hor. beta function	β_x	93 m
vert. beta function	β_y	25 m
dispersion function	D_y	0 cm
norm. transv. emittance	$\gamma\epsilon_{x,y}$	1.5 μm
rms hor. beam size	σ_x	0.72 mm
rms vert. beam size	σ_y	0.37 mm
rms bunch length	σ_z	0.21 m
circumference	C	6912 m
vertical tune	Q_y	26.135
beam momentum	p	270 GeV/c
collimator half gap	b	1.5 mm
collimator thickness	d	30 mm
collimator resistivity	ρ	10 $\mu\Omega$m
collimator length	L	1 m

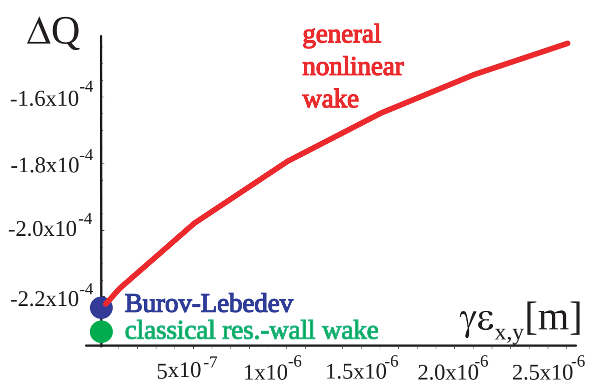

Figure 1: Coherent tune shift computed from the general nonlinear formula (5) as a function of the two transverse normalized emittances, for $\beta_x = 93$ m, $\beta_y = 25$ m, $\gamma \approx 288$, and a vertical half gap $b = 1.5$ mm. The linear estimate based on the flat-wall Burov-Lebedev formula, (4), and the one from the classical theory, (2), are also indicated. As can be seen, the latter two theories are strictly applicable only in the limit of vanishing emittance, where the nonlinear contributions disappear.

SUMMARY AND CAVEAT

We have described a new formula for the coherent betatron tune shift of a Gaussian bunch induced by a flat-chamber resistive wall impedance, including the nonlinear components of the wake field. The latter give rise to a dependence of the tune on the two transverse emittances and, in particular, to a significant tune-shift reduction for beam sizes comparable to the collimator gap. A similar expression was derived for the incoherent tune spread created by the nonlinear collimator wake. The two formulae for coherent and incoherent tune shift, respectively, may explain experimental observations with an LHC prototype collimator at the SPS from 2004.

Our treatment is not fully self-consistent, as we impose the shape of the vertical distribution to be a truncated Gaussian, which it might be on a first passage, but unlikely on later turns. The Gaussian approximation appears acceptable as long as the collimator gaps are large enough that only a small portion of the beam is scraped.

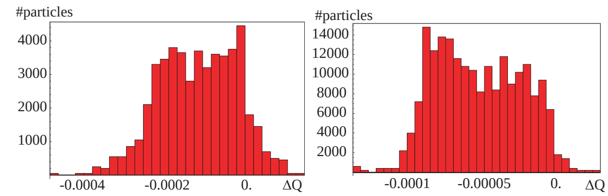

Figure 2: Vertical incoherent tune distribution calculated by a Monte-Carlo evaluation of (8), assuming a Gaussian beam distribution in all three dimensions with $\sigma_x = 0.72$ mm, $\sigma_y = 0.37$ mm, $\sigma_\tau = 1$, and considering a half gap of $b = 1.0$ mm (left) and 1.5 mm (right). The area of each histogram is normalized to 1. Note the different horizontal scale.

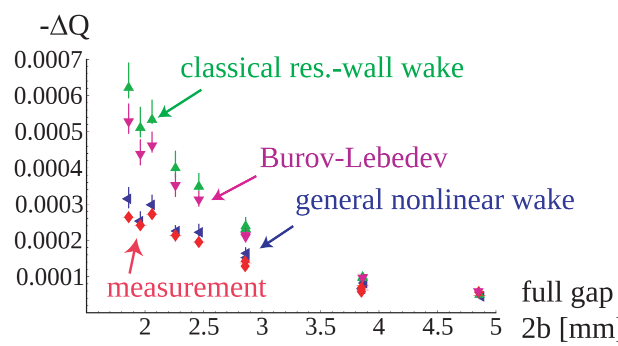

Figure 3: Measured data points from the SPS experiment [red rhombi] compared with predictions from classical theory (2) [light green upward triangles], Burov-Lebedev theory (4) [purple downward triangles] and general nonlinear theory (5) [blue sideward triangles]. The error bar on the predicted values contains both the statistical and the systematic error.

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