# MAGNETIC FIELD MULTIPOLE MEASUREMENT WITH HALL PROBE

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*Abstract*

When assembling an insertion device, sorting algorithms are used to deduce the magnet configuration which reduces to a minimum the phase error and the field integrals before shimming. In order to carry out the sorting, magnets to be placed in the array are measured with a Helmholtz coil. This yields the magnetic dipolar moment, because Helmholtz coil measurements assume a dipolar field for each block.

In order to take into account inhomogeneities, a hypothetical new sorting method could be used, based on the measurement of multipoles corresponding to each block. In this process, the first challenge is to find a method for the fast measurement of the multipoles generated by an arbitrary block. In this paper we face this challenge and propose a method that can be implemented using a Hall probe scanning along a set of straight lines.

## INTRODUCTION

The development of narrow gap insertion devices yields a growing interest in the effect of magnetic inhomogeneities. Moreover, the FEL applications require fast techniques for characterizing the magnetic blocks and reduce the time needed to manufacture an insertion device. Magnetic inhomogeneities introduce multipolar terms that are added to those corresponding to the multipolar development of an ideal magnetic source. However, magnetic inhomogeneities are not measured with the Helmholtz coil, because it evaluates the magnetic field far from the magnet, and the multipolar terms decay faster than the dipolar with distance. Therefore, it is of great interest to develop a method for measuring the multipolar terms, feasible in terms of time consumption.

Two main principles are used when characterizing a magnetic block by its magnetic field multipoles.

First, outside magnetic sources, magnetic field $B$ can be written as a gradient of a given potential $\phi$, this potential accepts a multipolar decomposition in spherical coordinates $(r, \theta, \varphi)$:[1]

$$\phi = \sum_{l\geq 1}\sum_{m=-l}^{l}\alpha_{l,m}\frac{Y_{l,m}(\theta,\varphi)}{r^{l+1}} \quad (1)$$

Where $Y_{l,m}(\theta,\varphi)$ is the well known spherical harmonic function. Hence magnetic field can be written as follows:

$$B_i = -\sum_{l\geq 1}\sum_{m=-l}^{l}\alpha_{l,m}\nabla_i\left(\frac{Y_{l,m}(\theta,\varphi)}{r^{l+1}}\right) \quad (2)$$

Where $i$ goes from *1* to 3 and holds for the x, y, and z components. $\alpha_{l,m}$ stands for multipolar terms. In case of dipolar field, $l$=1 and $(\alpha_{1,-1},\alpha_{1,0},\alpha_{1,1})$ is the dipolar moment vector.

Second, according to Maxwell equations, all the information of the magnetic field in vacuum is contained in a 2D plane.

The previous two statements can be combined in one: known the multipolar coefficients of the field in a 2D plane, one can deduce the field in all whole 3D space.

With these ingredients, we use expression (2) to fit the magnetic field of a magnetic block measured in a planar grid. To this end, some simple mathematical treatment is needed. Implementing a change to Cartesian coordinates and the derivative in equation (2), we can write:

$$B_i(x_p) = \sum_{l\geq 0}\sum_{m=-l}^{l}\alpha_{l,m}U_{i,l,m}(x_p) \quad (3)$$

$x_p$ is the p-th Cartesian point where we measure the magnetic field, and $U_{i,l,m}(x_p)$ is a real function defined from equation (2) in the following way:

$$U_{i,l,m}(x_p) = \begin{cases} m=-l\ldots-1: -\nabla_i\left(\frac{\mathrm{Re}[Y_{l,-m}(\theta,\varphi)]}{r^{l+1}}\right) \\ m=0: -\nabla_i\left(\frac{Y_{l,0}(\theta,\varphi)}{r^{l+1}}\right) \\ m=1\ldots l: -\nabla_i\left(\frac{\mathrm{Im}[Y_{l,m}(\theta,\varphi)]}{r^{l+1}}\right) \end{cases} \quad (4)$$

The conversion to Cartesian coordinates, in addition to facilitate the previous derivation, helps when obtaining the analytical expressions for the field integrals.

To characterize a magnetic block, we will use a set of multipolar terms. This will allow us to know the field and the integral field in the space close to the block.

To obtain the values of the set of multipolar terms, we will measure the magnetic field in some points in the space and then fit the measured field with the expression (3). In this process, the coefficients to be fitted are the multipolar terms $\alpha_{l,m}$.

Note that the expression to be fitted is linear, thus the fitting will be fast. The function to be minimized is:

$$\psi^2 = \sum_p\sum_{i=1}^{3}\left(\beta_i(x_p) - B_i(x_p)\right)^2 \quad (5)$$

Where $\beta_i(x_p)$ is the measured magnetic field *i* component at $x_p$. In this way, for each block we obtain the set of multipole terms $\alpha^{block}{}_{l,m}$ that fully characterizes it.

## SIMULATING THE MEASUREMENT OF THE BLOCKS

To study the feasibility of this method, we have used the blocks designed for the Apple II undulator HU71 to be installed at ALBA. Please note that in this paper we have carried out RADIA numerical simulations,[2] but the method can be easily implemented using Hall probe measurements. In the near future, real measurements will be carried out to cross-check its feasibility.

The main parameters of the blocks (see Figure 1) geometry are listed below. The blocks have a rectangular geometry with two notches, with some inhomogeneities artificially added.

Table 1: Block geometrical characteristics

| | |
|---|---|
| gap | 15.50 mm |
| period | 71.36 mm |
| High (z direction) | 30.00 mm |
| Length (y direction) | 17.66 mm |
| Width (x direction) | 34.00 mm |
| Notches high (z direction) | 5.00 mm |
| Notches Width (x direction) | 5.00 mm |

The starting point to characterize a magnetic block is the measurement of the three components of the magnetic field in one plane, as shown in Figure 1. Closer to the blocks, major is the influence of high order multipoles, and thus more terms will be needed in expression (3).

The plane of measurement is the undulator mid plane. To take advantage of the *on the fly* measurements with the Hall probe, the grid in which magnetic field will be measured is a set of straight lines parallel to the undulator axis. Figure 1 shows this arrangement.

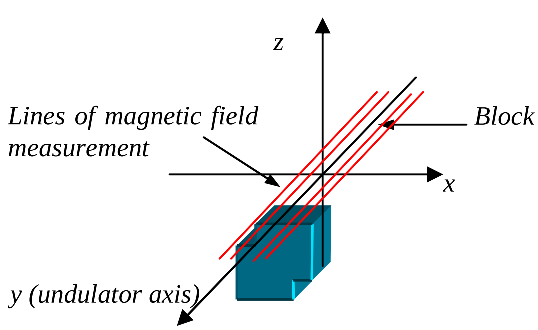


Figure 1: Scheme of the magnetic field measurement. Note the notches in two of the corners of the blocks

### *Parameters of the fitting*

By using l=10 in the expansion in equation (3), we found, according to calculations, that 9 lines of length 7 periods (500 mm) of magnetic field measurement distributed above the magnet are enough to fit the field and its integrals in the region of interest (at a distance $\Delta z \geq gap/2$ from the faces of the magnet and $x \in$ [-10 mm, 10 mm]), with errors around 1 G for the field and 1 G·cm for the field integral. The measured field for one block is typically ¼ of that existing in the final assembled undulator, i. e. $\beta_i \sim 0.2$ T. In this case, the typical values of the $\alpha_{l,m}$ ~$10^3$ for l=1, ~$10^5$ for l=2, etc.

In a hypothetical real measure of a real block, several strategies could be applied to minimize both measure and calculation times. For example, several magnets can be placed one after the other, the measurement range of the Hall prove being several times bigger than the measurement space needed for of one block.

From the calculation point of view, the fitting involves the calculation of one matrix related to the grid geometry, which has to be calculated only once for each insertion device (few tens of minutes with a Pentium IV 3GHz PC) and particular calculations for each magnet block. If 10 terms are used in the multipolar expansion, the particular calculations are solved in less than a second per block.

## RESULTS

We have implemented this algorithm in a C program running in a Pentium IV 3 GHz PC. We call the method described above Magnetic Multipolar Fitting (MMF for short) and we have compared it with the usual method. In which a Helmholtz Coil system or similar is used to measure the equivalent averaged magnetization. For this reason we call it Average Magnetization Measurement (AMM for short).

In the case of AMM, for fast prediction of the block field in any point by means of well known analytical formulas, permeability equal 1 and the block shape are used. AMM gives a quite good description of the field far from the magnets surfaces in any direction, but not close to the blocks.

In the MMF case, the field is well approximated by the fitted multipolar components from where the field is measured to far away from the block. In other words, the plane of measurement separates the region where the multipoles are well describing the field and the region where they are not. This is because, closer to the magnets, multipoles of higher order than that used in the fitting have more relevance.

A comparison between AMM and MMF is presented in Figure 2. Here we define the error as the difference between the field or integral calculated with Radia from the exact geometry of the block, and the fields or integrals derived from the application of AMM or MMF to a set of values calculated in a grid of points (in the case of MMF) or from Helmoltz geometry set up (in the case of AMM).

Independently of the capabilities of MMF and AMM in reproducing the magnetic field distribution, we have to take into account that in an undulator, as blocks have finite permeability, the neighborhood magnets will change the magnetic field of every single magnet. Then, when reproducing the field of the whole undulator one expects a periodic difference between the actual field and the simple superposition of fields produced by each magnet. The effect of this difference will mainly impact on field integrals (due to the end sections), but not in the phase error. However, the effect in the field integrals should be easily overcome by trim magnets shimming.

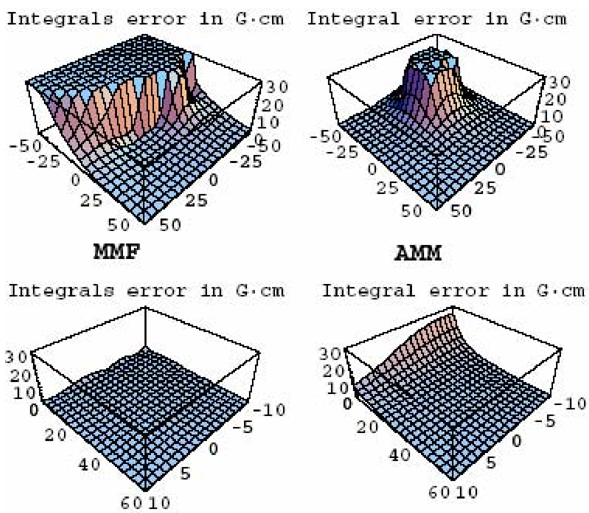


Figure 2: Transverse distribution of the field integral error (with respect to the field created by a block without neighbor magnets) for the AMM and MMF. Upper pictures show the error around the magnet and the lower in the zone of interest (mid gap). The electron trajectory corresponds to the line z=0.

In Figures 3 and 4 we present the error in the fitting of the magnetic field along the axis of the undulator for both MMF and AMM. In Figure 4, the black curve represents the difference of the fields generated by a single isolated block and the same block when it is in the undulator array. The green curve represents the error of the MMF fitting, which is very close to the black curve, indicating that this error does not depend on the magnet but is caused by the permeability effect. The red curve represents the error of the AMM, which is clearly bigger and, additionally, very dependent on the level of the magnet inhomogeneities.

## CONCLUSIONS

We have shown the feasibility of the MMF method and we have compared it with the traditional method AMM. The next step would be the study of the influence of measurement errors in both methods. This study could be done via simulations, but only the application in a real case would test the real feasibility of MMF.

Not only single blocks can be characterized in this way, also sets of blocks in modules. Sets of two vertically magnetized blocks with opposite magnetization direction (A magnets) with one horizontally magnetized block (B magnet) in between can then be characterized. In this way we take advantage of the theoretically null field integral of such a group of magnets, and the measurement will only describe the errors to be minimized in the sorting process. For such cases, the set of blocks is characterized with three sets $\alpha_{l,m}$ coefficients, each one using the centre of each block as origin to the multipolar decomposition. In an application to a sorting code, this would decrease at a half the number of elements to be permutated.

A further step would be the implementation of the method into a sorting algorism. This is now under study.

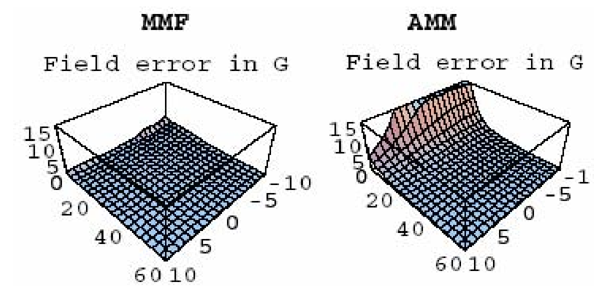


Figure 3: Transverse distribution of the squared difference in the fitted magnetic field (with respect to the field created by the block without neighbors) for the AMM and MMF in the region of interest.

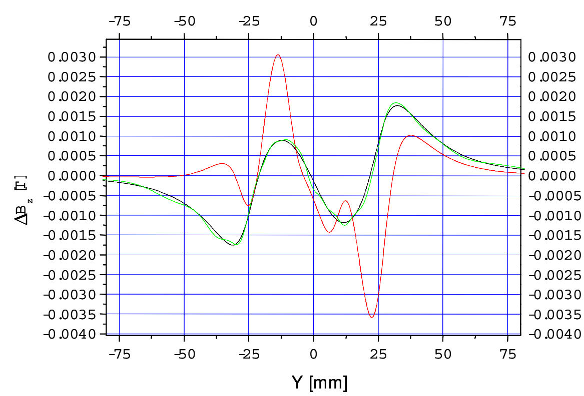


Figure 4: Difference of the vertical component of the magnetic field along the longitudinal direction with respect to the field created by a block with neighbors. Red (AMM), green (MMF) and black (magnet without neighbors).

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## REFERENCES

[1] Gray. C. G “Simplified derivation of the magnetostatic multipole expansion using scalar potential”, Am. J. Phys. 46 (1978) 582-583.

[2] Radia 4.1, April 1997. Download at www.esrf.fr/ Accelerators/Groups/InsertionDevices/Software/Radia.