DEPENDENCE OF THE ELECTRON BEAM POLARIZATION EFFECT IN THE INTRA-BEAM SCATTERING RATE ON THE VERTICAL BEAM EMITTANCE*

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Abstract

We have calculated the magnitude of depolarization jump in Intra-Beam-Scattering rate using one- and two-dimensional approximations for the function of distribution over transverse particle momentum in electron/positron storage rins. According to the 2D approximation, the jump magnitude essentially depends upon a ratio between the vertical and radial beam emittances. We present the calculation results in comparison with our recent experiments on resonant depolarization at VEPP-4M.

INTRODUCTION

Measurement of Intra-Beam-Scattering (IBS) rate is applied in the resonant depolarization technique to detect the beam polarization in the electron/positron storage rings [1, 2]. Depolarization jump in the counting rate of scattering particles (Touschek electrons or positrons) occurs at the instant when the beam becomes unpolarized due to a fulfilment of the external spin resonance condition. Magnitude of the jump depends on a square of polarization extent, some other beam parameters as well as the position of counters relative to the beam orbit. The larger jump provides the higher accuracy in absolute calibration of the particle energy which is proportional to the spin precession frequency [3]. According to our calculations based on the ordinary one-dimensional approach for consideration of an elementary scattering act, the jump magnitude at VEPP-4M may achive about 7% at the beam energy $E = 1840$ MeV (about 8% at $E = 1550$ MeV) and the polarization extent $\zeta = 80\%$. Typically, in the experiments we observe the jumps not exceeding 3%. On the estimations, such large a discrepancy can be hardly explained by a decrease in the polarization extent at the distance from the depolarizing spin resonances in the booster VEPP-3, the source of polarized beams for VEPP-4M, as well as in the VEPP-4M ring. To understand this fact we have considered the 2D interaction approachment which allows for a vertical component of relative velocity in particle collisions. Below we give the calculation of jump magnitude in both approximations. We also present results of our recent experiments at VEPP-4M on study of the dependence of jump magnitude upon the vertical beam size.

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CALCULATION OF THE DEPOLARIZATION JUMP IN THE INTRA-BEAM-SCATTERING RATE

One-dimensional interaction approach

A scintillation counter is moved inward the aperture of vacuum chamber at the distance A from a closed orbit position. In accordance with the theories of IBS in storage rings [4] and polarization effects in electron-electron scattering [5], the counting rate of Touschek electrons encountering at non-relativistic relative velocity (ν) in the center-of-mass system (CMS) can be expressed as

$$w = \frac{\sqrt{\pi} r_0^2 c N^2}{\gamma^5 V_b (\sigma_p c/E)^3} \left(I_1 + \zeta^2 I_2\right). \qquad (1)$$

where N is a beam population; $V_b = 8\pi^{3/2}\sigma_X \sigma_Y \sigma_Z$ is a beam volume in the lab system, $\sigma_{X,Y,Z}$ are the radial, vertical and longitudinal beam sizes respectively; $\gamma = E/mc^2$ is a relativistic factor; r_0 is the electron radius; $\sigma_p = mc\gamma\sigma_{X'}$ is a spread of transverse momentum $p = m\nu/2 \approx m\nu_X/2$ in CMS, $\sigma_{X'}$ is a horizontal angular spread. This equation is obtained when neglecting the vertical component of particle motion ($\nu_Y \to 0$). Quantities I_1 and I_2 are presented through the integrals

$$I_1 = \int_{\varepsilon_1}^{\varepsilon_2} \frac{1}{\chi^2} \left\{ \frac{\chi}{\varepsilon_1} - 1 + \frac{1}{2}\ln\frac{\varepsilon_1}{\chi} \right\} e^{-\chi} d\chi +$$

$$+ \int_{\varepsilon_2}^{\infty} \frac{1}{\chi^2} \left\{ \chi \left(\frac{1}{\varepsilon_1} - \frac{1}{\varepsilon_2} \right) + \frac{1}{2}\ln\frac{\varepsilon_1}{\varepsilon_2} \right\} e^{-\chi} d\chi,$$

$$I_2 = \int_{\varepsilon_1}^{\varepsilon_2} \frac{1}{2\chi^2} \ln\frac{\varepsilon_1}{\chi} e^{-\chi} d\chi + \int_{\varepsilon_2}^{\infty} \frac{1}{2\chi^2} \ln\frac{\varepsilon_1}{\varepsilon_2} e^{-\chi} d\chi,$$

$$\varepsilon_{1,2} = [\Delta p_{1,2}/(\gamma\sigma_p)]^2. \qquad (2)$$

Δp_1, and Δp_2 in (2) are respectively the upper and lower limits of the deviation of total particle momentum in the laboratory system from an equilibrium value. Δp_1 is a minimal momentum deviation, needed for particles to fall on the counter. For simplicity, we consider the possibility to detect the particle during only a first turn after the scattering. A contribution of particles detected at subsequent turns is not so significant since the magnitude $\Delta p/p > 1\%$ being of interest exceeds the RF separatrix size $\sim 0.6\%$.

The upper momentum limit Δp_2 is determined by a geometric aperture limit. In our calculations $\Delta p_1/p \approx 0.9\%$ at $A = 1$ cm. Polarization effect is a ratio

$$\Delta = \frac{\zeta^2 I_2}{I_1} < 0, \tag{3}$$

which appears as a "jump" in the counting rate if fast depolarization of the beam occurs The greater Δp_1 corresponds to the larger jump $|\Delta|$. But at the great p_1s (the large values of A) the counting rate w decreases. Beside, one must not make a significant decrease in beam lifetime with bringing the counter near the beam orbit (the small As). So, there exists an optimal range for the counter position which is $A \approx 1$ cm in the case of VEPP-4M. Fig.1 demonstrates the dependence of the jump magnitude on the position A calculated in the 1D approachment at $E = 1840$ MeV. In optimum, the jump achieves 7%.

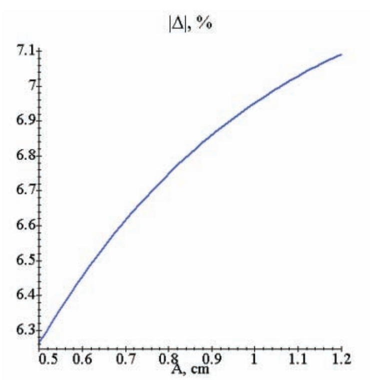

Figure 1: Jump vs. the counter position at E=1840 MeV, $\zeta = 80\%$ in the 1D approximation.

Two-dimensional interaction approach

Evidently, the first 2D relativistic consideration to describe the Touschek lifetime including the polarization effect was made in [6]. For our purpose we have developed a reductive, reasonably restricted by a non-relativistic case, approach taking into account only the particles falling into the counter of finite sizes.

In [7] one of us introduced the modified function of distribution over the transverse momentum p considering two-dimensional character of mutual particle-particle interactions. The coupling can be characterized by an additional parameter $k = \sigma_{X'}/\sigma_{Y'}$, where $\sigma_{Y'}$ is the vertical angular spread. In the so-called "round" beam $k \to 1$ and in the flat one $k \to \infty$. The distribution function of interest is

$$f(k,p)dp = \frac{2kp}{\sigma_p^2} \cdot S(u,k)dp, \quad (p > 0)$$

$$S(u,k) = \exp\left[-\frac{u}{2}(1+k^2)\right] I_0\left[\frac{u}{2}(1-k^2)\right]. \tag{4}$$

Here $p = m\nu/2$; ν is the relative velocity of colliding particles inside a beam ($\nu^2 = \nu_X^2 + \nu_Y^2$); $u = p^2/\sigma_p^2$; $\sigma_p = mc\gamma\sqrt{\sigma_{X'}^2 + \sigma_{Y'}^2}$; I_0 is the modified Bessel function. At $k \to \infty$ the distribution function approaches the form corresponding to the one-dimensional collision case [4]. At $k \to 1$ the distribution becomes the two-dimensional Maxwell one: $f(p) \propto p \cdot \exp\left(-p^2/\sigma_p\right)$. The use of the modified distribution function changes the forms of universal characteristic functions which describe the diffusion and losses rates in IBS theory [7].

Generally, in the consideration of polarization effect measured by IBS-based polarimeter, one must take into account the modified distribution function too. The counting rate of Touschek particles in the approach of 2D collisions is

$$w = \frac{N^2}{\gamma^2 V_b}\int \nu d\sigma f_p(k,p)dp = \frac{\pi r_0^2 c N^2 (J_1 + \zeta^2 J_2)}{\gamma^5 V_b (\sigma_{X'}^2 + \sigma_{Y'}^2)^{3/2}}$$

with

$$J_1 = \int_{\varepsilon_2}^{\infty} \chi^{-1/2}\left[\chi\left(\frac{1}{\varepsilon_1} - \frac{1}{\varepsilon_2}\right) + \frac{1}{2}\ln\frac{\varepsilon_1}{\varepsilon_2}\right] S(\chi,k)d\chi +$$

$$+ \int_{\varepsilon_1}^{\varepsilon_2} \chi^{-3/2}\left[\frac{\chi}{\varepsilon_1} - 1 + \frac{1}{2}\ln\frac{\varepsilon_1}{\chi}\right] S(\chi,k)d\chi,$$

$$J_2 = \frac{1}{2}\int_{\varepsilon_2}^{\infty} \chi^{-3/2}\ln\frac{\varepsilon_1}{\varepsilon_2} S(\chi,k)d\chi +$$

$$+ \frac{1}{2}\int_{\varepsilon_1}^{\varepsilon_2} \chi^{-3/2}\ln\frac{\varepsilon_1}{\chi} S(\chi,k)d\chi. \tag{5}$$

Limits in integrals () have the same origin as in 1D collision case (see()) but now with $\sigma_p = mc\gamma\sqrt{\sigma_{X'}^2 + \sigma_{Y'}^2}$. The jump magnitude is found from (3) where I_1 and I_2 are substituted by J_1 and J_2 respectively. The results of the jump

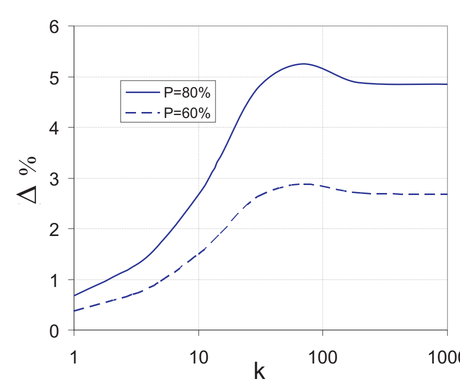

Figure 2: Jump vs. the coupling parameter (k) for two values of the polarization degree at $E = 1840$ MeV, $A = 1$ cm.

magnitude calculation based on (5) are plotted in Fig.2 for two values of the beam polarization extent at VEPP-4M as a function of the coupling parameter k at $E = 1840$ MeV, $A = 1$ cm, $\sigma_p = 1.4 \cdot 10^{-4}$. The betatron coupling in routine conditions of VEPP-4M operation is characterized by $k \sim 10$ that corresponds to the ratio between the vertical and radial emittances $\approx 10^{-2}$. According to the curves in Fig.2, the depolarization jump in counting rate of Touschek pairs can not exceed 2.5% at the polarization extent $\leq 80\%$.

EXPERIMENTAL RESULTS

In the resonant depolarization experiments we use two scintillation counters which are moved in the horizontal plane from opposite sides of the vacuum chamber inward the aperture and positioned symmetrically in respect to the beam closed orbit. Two-fold coincidence curcuit allows to pick out the IBS contribution from a total counting rate of two counters since the trajectories of Touschek pair electrons also lie symmetrically on each side of the equilibrium particle orbit. This contribution ($\sim 60\%$) dominates over the background caused by gas scattering and showers from the vacuum chamber walls. Typical jump in the counting rate of Touschek coincidences shown in Fig.3 makes up about 2.5% in a good agreement with the 2D calulation results above.

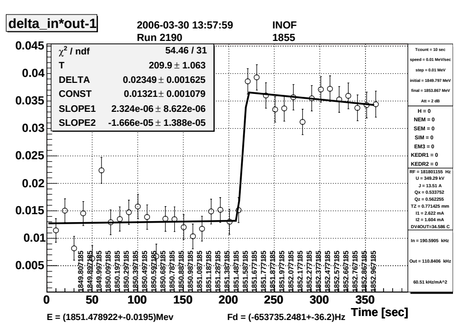

Figure 3: Typical jump in the counting rate of Touschek coincidences.

With the aim to clarify the influence of the vertical emittance on the jump we have performed the experiment on enlargement of the vertical beam size by changing a field in one of the VEPP-4M skew quads. The jump in the total counting rate (without detachment of the background) measured at three values of the vertical beam size is presented by points versus the electron bunch current in Fig.3. The dependence of jump magnitude on the vertical size in the last figure is distinct. Moderate drop of the jump magnitude with the current can be explained by an increase of the background.

The same data are used in the plot representing the jump in Touschek coincidences versus the vertical beam size. The jump grows from a level $\leq 1.5\%$ up $\approx 2.5\%$ with increase of the coupling parameter k by ≈ 1.6 times. This fact is in accordance with the calculational curve in Fig.2 at $\zeta = 80\%$.

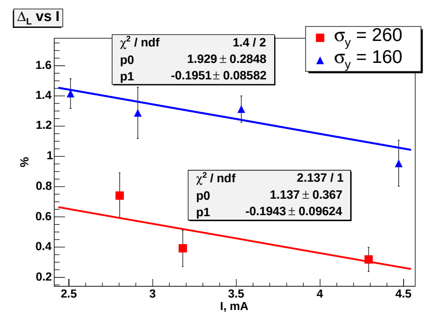

Figure 4: Dependence of the jump in a total counting rate of two counters (without detachment of background) on the bunch current at two values of the vertical beam sizes (given in a.u.).

CONCLUSION

Our theoretical consideration and experiments have revealed the significant influence of the ratio between the vertical and horizontal beam emittances on a magnitude of the depolarization jump measured by means of the IBS-based polarimeter. It's necessary to take into account this influence in determination of the polarization extent by the jump magnitude. One must provide a minimal vertical emittance to obtain larger jumps which are preferrable in the viewpoint of resonant depolarization techniqe application for a precise beam energy calibration.

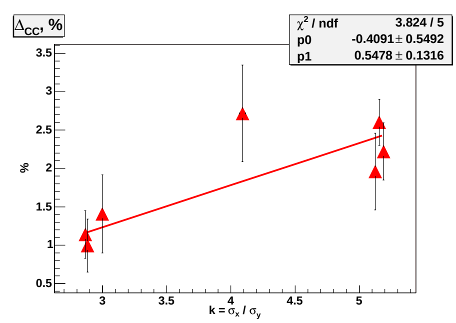

Figure 5: Measured dependence of the jump in the Touschek particle counting rate on the ratio of the horizontal and vertical beam sizes.

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