NOVEL CONCEPTION OF BEAM TEMPERATURE IN ACCELERATOR AND APPLICATIONS*

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Abstract

In this paper, we will introduce a novel conception of beam temperature in accelerator, discuss the calculation method. And finally the author will show an example on the beam temperature in IHEP-made 45MW klystron.

INTRODUCTION

The beam temperature conception can be used in the studies of beam in accelerator. We consider the particle beam as ideal gas, and in accelerator case, the gas is not in balance state, so we call the beam temperature as general temperature.

BEAM TEMPERATURE

First, let us see non-relativistic case. For 1-dimensional model, suppose that particles in a beam in a laboratory frame has velocities

$$v_1, v_2, \ldots, v_i, \ldots v_N, \quad (1)$$

and momenta

$$p_1 = mv_1, p_2 = mv_2, \ldots, p_i = mv_i, \ldots, p_N = mv_N. \quad (2)$$

where N is the total number of particles in the beam. The average momentum with respect to the laboratory frame is

$$\bar{p} = \frac{1}{N}\sum_i p_i = \frac{m}{N}\sum_i v_i = m\bar{v}, \quad (3)$$

where $\bar{v}$ is the average velocity in the laboratory frame.

Next, let us turn to consider the relativistic case. Again, suppose that particles in a beam in a laboratory frame has velocities

$$v_1, v_2, \ldots, v_i, \ldots v_N, \quad (4)$$

and momenta

$$p_1 = m_1 v_1, p_2 = m_2 v_2, \ldots, p_i = m_i v_i, \ldots, p_N = m_N v_N. \quad (5)$$

Since v_i and speed of light, c, are comparable,

$$m_i = m(v_i) = \frac{m_0}{\sqrt{1-\frac{v_i^2}{c^2}}} = \frac{m_0}{\sqrt{1-\beta_i^2}} \quad (6)$$

is no longer a constant, where m_0 is the rest mass. The average momentum with respect to the laboratory frame is

$$\bar{p} = \frac{1}{N}\sum_i p_i = \frac{m_0}{N}\sum_i \frac{v_i}{\sqrt{1-\beta_i^2}} = \frac{m_0\bar{v}}{\sqrt{1-\bar{\beta}^2}} \quad (7)$$

From this equation, one can find out $\bar{v}$ in the following way. Dividing Eq.(7) by c and then taking square, we get

$$\left(\frac{1}{N}\sum_i \frac{\beta_i}{\sqrt{1-\beta_i^2}}\right)^2 = \frac{\bar{\beta}^2}{1-\bar{\beta}^2} = \frac{1}{1-\bar{\beta}^2} - 1 \quad (8)$$

It gives

$$1-\bar{\beta}^2 = \frac{1}{1+\left(\frac{1}{N}\sum_i \frac{\beta_i}{\sqrt{1-\beta_i^2}}\right)^2}, \quad (9)$$

or

$$\bar{\beta}^2 = 1 - \frac{1}{1+\left(\frac{1}{N}\sum_i \frac{\beta_i}{\sqrt{1-\beta_i^2}}\right)^2}, \quad (10)$$

Namely,

$$\bar{v} = c\left[1 - \frac{1}{1+\left(\frac{1}{N}\sum_i \frac{\beta_i}{\sqrt{1-\beta_i^2}}\right)^2}\right]^{1/2}. \quad (11)$$

This is the velocities of 'mass center'. So, the velocity of the I-th particle related to the mass center is

$$u_i' = \frac{u_i - v}{1-\frac{u_i v}{c^2}} \quad (12)$$

and the kinetic energy is

$$E_i'(u_i') = m_i(u_i')c^2 = \frac{m_0 c^2}{\sqrt{1-\beta_i'^2}} \quad (13)$$

in which $\beta_i' = \frac{u_i'}{c} = \frac{(u_i - v)c}{c^2 - u_i v}$

so

$$E_i'(u_i') = \frac{(c^2 - u_i v)m_0 c^2}{\sqrt{(c^2-u_i v)^2 - (u_i - v)^2 c^2}} \quad (14)$$

and the average moving kinetic energy of the beam is

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$$\overline{E_i'} = \frac{m_0 c^2}{N} \sum_i \frac{(c^2 - u_i v) m_0 c^2}{\sqrt{(c^2 - u_i v)^2 - (u_i - v)^2 c^2}} \qquad (15)$$

$$= kT$$

and T is the general temperature of the beam.

APPLICATION

As an example of the application of the beam temperature, we consider to analysis the electron beam temperature in klystron. Because we usually analysis klystron in one dimension, that's so called DISK mode, so it will be a very simple case to be analysis using general beam temperature concept.

We consider IHEP-made 45MW klystron, which klystron has 5 cavities, and the RF output power is 45MW at beam voltage 310KV, current 360A, and we obtain 33MW output power at 260KV. We obtained the distribution of velocity of electron in this klystron by using disk-mode analysis, the results shown in Figure 1.

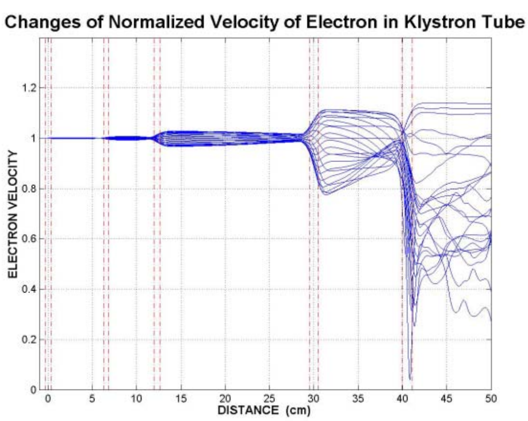

Figure 1: IHEP-made 45MW klystron

We can obtain the beam temperature changes along the klystron shown in figure 2.

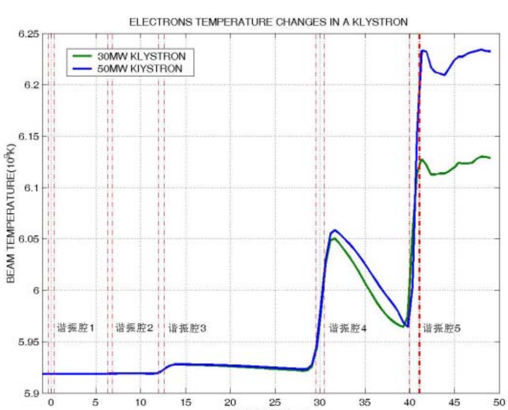

Figure 2: IHEP-made 45MW klystron

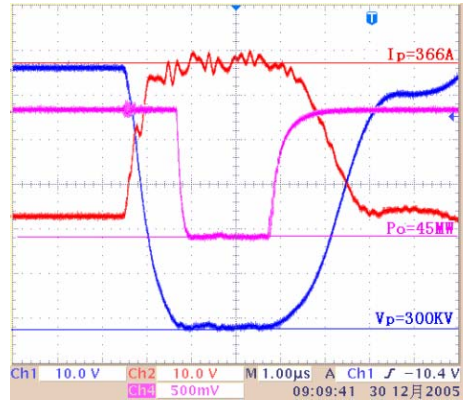

Figure 3: waveform of IHEP-made 45MW klystron

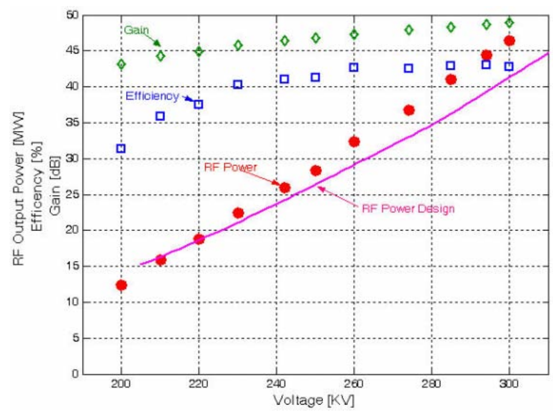

Figure 4: results of IHEP-made 45MW klystron

Figure 5: IHEP-made 45MW klystron

CONCULSION

We obtain new aspect of beam in accelerator by using general beam temperature concept, more further work should be done in the relationship between the beam temperature and beam parameter, such as energy spread and emittance etc. The author would like to thank Dr. Huang Chao-guang for helping and beneficial discussion to this job.