

# COMPARISON OF BETATRON FUNCTION MEASUREMENT METHODS AND CONSIDERATION OF HYSTERESIS EFFECTS

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## Abstract

Two methods for determining the betatron functions in a storage ring were used to survey the linear optics at Delta. The fast orbit response analysis is used to gain betatron functions at the beam position monitors (BPMs) and dipole correctors. These are compared to betatron functions measured by the tune scan method which gives the beta functions in the quadrupoles. To improve the accuracy of the betatron functions obtained by the tune scan method a measuring procedure is introduced which considers the hysteresis effects in the quadrupole magnets. Systematic deviations in the beta functions measured between the two methods have been observed. The calibration errors of the BPMs can explain the observed deviations. With the orbit response analysis also the betatron phase advances between the measurement points can be calculated. Because these do not depend on the calibration errors, unlike the betatron functions, the differences between measurement and model can be determined more precise. A comparison of both methods with the optics model will be presented.

## MEASUREMENTS

### Tune scan

The tune scan method was used in order to measure the beta functions in the 76 quadrupoles at Delta [1]. For this purpose an additional current is supplied for the individual quadrupoles. Because at Delta up to four quadrupoles are supplied by one power supply this is done with an additional power supply. The current is distributed to the magnets by a remote controlled relay cascade. Considering the effective length  $l_e$  of the quadrupole the mean beta function  $\langle\beta\rangle$  inside the quadrupole is measured. Following [2] the beta function  $\beta_c$  in the center of the quadrupole is then calculated from the measured  $\langle\beta\rangle$  and the theoretical model for the optics.

The maximum additional quadrupole current that can be set is limited by the maximum tune to the closest tune resonance either horizontal or vertical. Because of the coupling between the planes at locations with large differences between the beta functions in both planes a large current that is set to measure the small beta function can result in a too large tune shift in the other plane. At these locations the small beta functions cannot be measured with high accuracy because the tune shifts are in the order of or below the resolution of the tune measurement system. This results in

errors of the individual measurement points in the area of 1 to more than 100 %. The errors in both planes are about 20 % on average.

### Orbit response analysis

A further method that was used to determine the local beta functions  $\beta_i^b$  and  $\beta_j^c$  as well as the betatron phases  $\Psi_i^b$  and  $\Psi_j^c$  at all locations of the BPMs  $s_i$  and the correctors  $s_j$  was introduced in [3]. In Delta this are 84 locations horizontally and 80 locations vertically. Starting with model values for the beta functions and betatron phases this iterative method calculates the beta functions and the betatron phases at the same time from the elements  $R_{ij}$  of a measured orbit response matrix (ORM)  $\mathbf{R}$  [3]. The ORM contains the BPM readings for the successive variation of the corrector currents.

The errors of the elements of the measured ORMs consist of the average errors of the correctors (estimated to about 5 % for Delta) and the total position error of all BPMs (calculated to 3 %). This systematic errors do not have an effect on the calculated betatron phases [3]. The independence of the betatron phases of these errors could be verified with model calculations based on simulated errors. With systematic errors between 1 and 30 % the calculated absolute deviation of the betatron phases from the error free values is  $< 10^{-4}$  while the beta functions scale with the assumed errors [4].

## HYSTERESIS EFFECTS

To use the tune scan method at Delta the magnet current was increased with an additional power supply. With this classical method tune changes caused by hysteresis effects were observed [5]. This small effect on individual quadrupoles accumulates around the storage ring leading to a tune shift of approximately  $\pm 0.01$  horizontally and  $\pm 0.03$  vertically. This tune shift leads to a deviation of the optic. The measured hysteresis curves are extremely narrow. The course of the curves can therefore only be outlined in figure 1. Because not all magnets can be operated in saturation the magnet currents are set from zero current to their operating current (point 1). By setting an additional positive current outgoing from this starting point the magnetic field follows the initial curve (bold) up to the reaching of the predefined value (point 2). When the additional current is then taken back to the initial value this yields in a hysteresis curve deviating from the initial curve. It remains an additional remanence (point 3) which is dependent on

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the initial current of the quadrupole as well as on the amplitude of the additional current. This remanence is up to 1 % of the original field. Then from this point (3) a negative additional current can be set (point 4) which has the same amount as the positive additional current. When this negative current is then taken back again a further remanence value is reached (point 5). Increasing the additional current by the same amount once more (point 2) then completes the additional hysteresis curve (dashed). When the above procedure is repeated outgoing from the last point (5) this curve is reproduced. This results in an indefinite higher magnetic field (point 1) at the quadrupole after a tune scan measurement. A returning to the initial value of the magnetic field with the additional current is not attainable with this procedure.

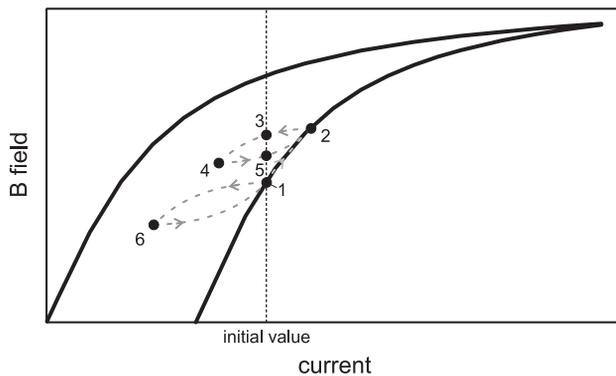


Figure 1: Part of a hysteresis curve (bold) of a quadrupole magnet (schematic). Two additional hysteresis curves are marked (dashed) which arise from increasing and reducing the quadrupole current from the initial value (see text).

If the additional current is set in negative direction first another hysteresis curve results (point 6). When switching the additional current off the initial value (point 1) is reached again and no additional remanence remains. In the examined additional current interval down to - 3 A the starting field is reproducible in the accuracy of the field measurement ( $< 0.01$  mT). When a positive additional current on the left side of the hysteresis curve in figure 1 is set the same effect occurs as when setting a negative additional current on the right side of the curve.

By use of this new method no more tune changes caused by hysteresis effects could be observed during the measurement. The horizontal and vertical tunes remained constant within the absolute measurement precision of  $\pm 0.001$  ( $Q_x = 9.163$  and  $Q_z = 3.281$ ). The inaccuracy appearing through the hysteresis effect during the current reduction (point 6 in figure 1) leads to an error in the magnetic field variation up to 1 %. Because of this the beta functions calculated from the tune scan method are systematically too small. This error is negligible compared with the accumulated disturbance by using the classical method since it appears only at one quadrupole during the variation of the magnetic field.

## RESULTS

### Betatron functions

The ORM analysis was compared with the tune scan method. For that purpose in figure 2 only those points are displayed where both methods allow to measure the beta function at the same place. The standard deviation of the measurements of both methods amounts horizontally to  $\sigma_x = 3.166$  m and vertically to  $\sigma_z = 4.911$  m. The beta functions determined from the ORM are almost always horizontally smaller than those of the tune scan measurement. Vertically they are often larger, particularly in the locations of maximum beta functions. This in both planes opposite scaling of the beta functions can be explained with a systematic error of the BPM and corrector calibrations used for the ORM analysis. The systematic error of the tune scan method was kept small by performing shunt measurements for the additional power supply.

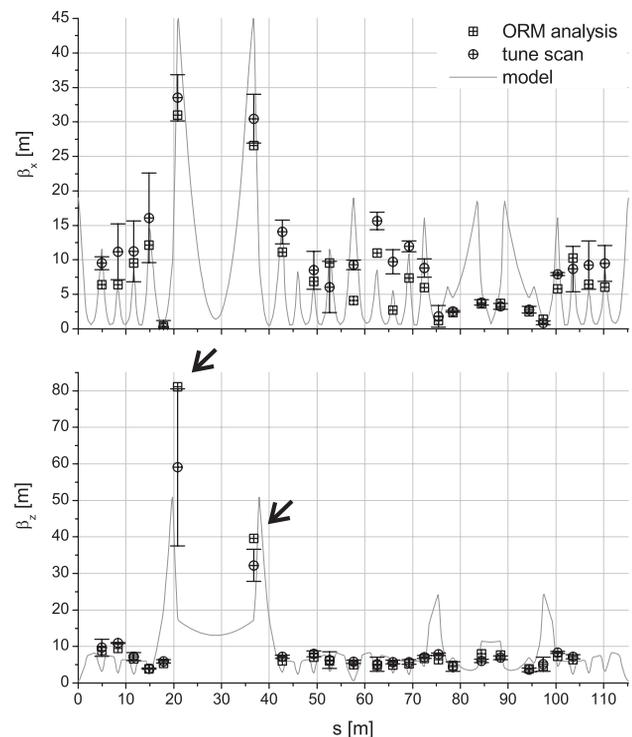


Figure 2: Beta functions in the storage ring Delta (standard optics,  $E = 1.5$  GeV, injection bump). The errors of the beta functions calculated with the ORM analysis are smaller than 5 %. Arrows are marking offsets between model and measurement.

The measured amplitudes of the beta functions show asymmetries compared to the optics model, particularly around the FEL (arrows in figure 2). Additional measurements [4] show that the asymmetry is caused by an closed orbit bump in the injection area, which is needed for the injection. The bump through sextupoles causes additional focusings not included in the optics model.

### Betatron phase advances

Since the betatron phases show a higher precision than the beta functions they are used to examine the deviations of the measured values from the model. The betatron phases are calculated from the analysis of the ORMs, too. In the following only the relative phase advances are used. Therefore the phase advances between the BPMs to the next element  $i$  respectively are calculated and the difference between model values  $t$  and measured values  $e$  are formed:

$$\Delta\Psi = (\Psi_{i+1} - \Psi_i)_t - (\Psi_{i+1} - \Psi_i)_e. \quad (1)$$

Numeric errors can occur in the calculation of the phases between nearby BPMs and correctors due to very small phase advances between these elements. Because of this only the BPMs are regarded in the analysis.

For all measurements a phase beating appears. The course seen in figure 3 always arises.

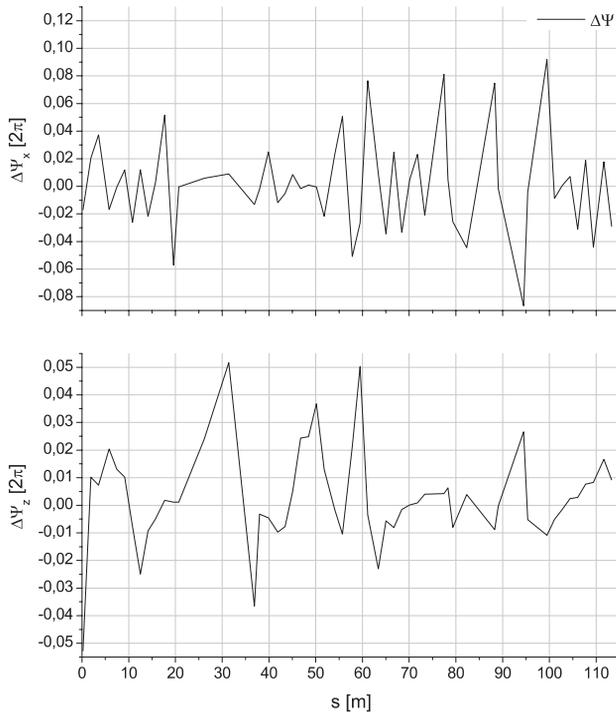


Figure 3: Horizontal (above) and vertical (below) phase beating in the storage ring Delta (standard optics,  $E = 1.5$  GeV, injection bump).

To check whether higher multipoles are responsible for the phase beating an ORM was taken at set orbit and with sextupoles turned off to reduce the nonlinear effects in the storage ring as far as possible. The phase beating does not change. Therefore sextupoles can be excluded as a source for a phase beating. Probably longitudinal deviations of the magnet and BPM locations from the model are responsible for the beating. In a next step survey data will be included in the model.

### CONCLUSION

It was shown that it is important for the tune scan method to correctly deal with the hysteresis effects of the magnets. Both methods have been applied to the Delta storage ring, leading in between the errors to the same result. Systematic deviations in the beta functions can be explained by the systematic errors of the BPMs and correctors. Asymmetries in the optical functions have been observed and could be explained by additional focussing in the injection area which was not yet included in the model.

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