LONGITUDINAL COHERENT OSCILLATION INDUCED IN QUASI-ISOCHRONOUS RING

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Abstract

The analytical formula for the longitudinal coherent oscillation is explained. It contained a path-length oscillation, which had never been considered. The formula is useful to analyse the beam oscillation data and identify the noise source especially in a quasi-isochronous ring. Preliminary results at NewSUBARU are explained.

INTRODUCTION

It is a common understanding that a stable power supply is required for a stable operation of an accelerator. One extreme case of such operation is a quasi-isochronous (QI) operation of an electron storage ring, which means a very small momentum compaction factor (α). Under QI condition, the rf bucket size is extended along the energy displacement axis while that along the time axis remained the same. Then a QI electron storage ring can store an extremely short electron bunch, as short as pico-second. However in QI operation, a small change of a path-length for a revolution produces a big energy shift.

According to some experiences of QI operation at BESSY-II [1] and NewSUBARU [2], the longitudinal coherent fluctuation, or oscillation, can be a main practical limit for the bunch shortening. In order to reduce the beam fluctuations, it is necessary to know where it comes from. There are three possible sources of an observed beam oscillation;

(1) Phase or voltage ripple of the rf acceleration field.
(2) Magnetic field ripple.
(3) Monitor noise.

In QI operation the beam is much sensitive to the noise than a typical ripple monitor of power supplies. Then it is reasonable to use beam to identify the noise source. On the analysis of the longitudinal oscillation in an electron storage ring, there has been some theoretical [3] and experimental studies [4-9]. However their studies did not consider the externally excited fluctuation of path-length, which is introduced in this article.

Most of the analysis in this article is limited in the linear treatment. Although non-linear terms are important in the real operation, the understanding in the linear system is essential.

EQUATIONS OF LONGITUDINAL OSCILLATION

The time deviation τ and the relative energy deviation ε from the reference particle obey the well-known differential equations

$$\frac{d\tau}{dt} = -\alpha\varepsilon + \frac{\Delta L}{L_0}, \tag{1a}$$

$$\frac{d\varepsilon}{dt} = \frac{eV_{RF}\omega_{RF}\cos\phi_0}{T_{REV}E}(\tau + \Delta T) - 2\alpha_E\varepsilon. \tag{1b}$$

Here α is the momentum compaction factor, L_0 is the circumference, e is the electron charge, V_{RF} is the rf acceleration voltage, ω_{RF} is the angular rf frequency, ϕ_0 is the rf synchronous phase, T_{REV} is the revolution period, E is the energy of the reference particle, and α_E is the longitudinal damping coefficient. ΔT and ΔL are the rf noise and the path-length noise. For the calculation with one frequency component, the noises are written as

$$\Delta T = \Delta_{RF} e^{j\omega t}, \tag{2a}$$

$$\Delta L = \Delta_L e^{j\omega t}. \tag{2b}$$

Here ω is the angular frequency of the noise and Δ_{RF} and Δ_L are amplitudes of that frequency component. The equations of motion Eqs.(1) are simplified as

$$\frac{d\tau}{dt} = -\alpha\varepsilon + \Delta_L e^{j\omega t}, \tag{3a}$$

$$\frac{d\varepsilon}{dt} = \frac{{\omega_S}^2}{\alpha}(\tau + \Delta_{RF} e^{j\omega t}) - 2\alpha_E\varepsilon. \tag{3b}$$

Here ω_S is the synchrotron oscillation frequency given by

$${\omega_S}^2 = \frac{eV_{RF}\omega_{RF}\alpha\cos\phi_0}{T_{REV}E}. \tag{4}$$

The stationary solution of Eqs. (3) is given as

$$\varepsilon = \frac{({\omega_S}^2/\alpha)(\Delta_L + j\omega\Delta_{RF})}{{\omega_S}^2 - \omega^2 + 2j\alpha_E\omega} e^{j\omega t}, \tag{5a}$$

$$\tau = \frac{(j\omega + 2\alpha_E)\Delta_L - {\omega_S}^2\Delta_{RF}}{{\omega_S}^2 - \omega^2 + 2j\alpha_E\omega} e^{j\omega t}. \tag{5b}$$

PATH-LENGTH NOISE

Analytical Formulae

A slow horizontal dipole deflection, either by a magnet ripple or eddy current field, produces a closed orbit distortion (cod) and then a small change of the path-length for a revolution. This shift of the path-length is a

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possible source of a coherent longitudinal oscillation. The cod produced by a single deflection θ_S is given by a well-known equation,

$$x(s) = \frac{\sqrt{\beta_S \beta(s)}}{2\sin\pi\nu}\theta_S \cos\left[\left|\psi(s)-\psi_S\right|-\pi\nu\right]. \qquad (6)$$

Here x and s are horizontal and azimuthal coordinates. β and ψ are the horizontal beta function and betatron phase and β_S and ψ_S are those at the deflection point. ν is the horizontal tune. The shift of the path-length for one revolution ΔL is given by

$$\Delta L = \int_0^{L_0} \frac{x(s)}{\rho(s)} ds = \eta_S \theta_S. \qquad (7)$$

Here ρ is the curvature of radius, and η_S is the dispersion function at the deflection point. The closed orbit shift by the deflection is a sum of Eq. (6) and the displacement by an energy shift to cancel ΔL.

$$x(s) = \left\{\frac{\sqrt{\beta_S \beta(s)}}{2\sin\pi\nu}\cos\left[\left|\psi(s)-\psi_S\right|-\pi\nu\right] - \frac{\eta_S \eta(s)}{\alpha L_0}\right\}\theta_S. \quad (8)$$

Path-Length Ripple at NewSUBARU

NewSUBARU storage ring [1] has six modified DBA bending cells. Each cell has one 8° inverted bend (IB) between two 34° normal bends. The most possible source of the path-length ripple is two power supplies for IBs at the eastern arc and the western arc.

The deflection was identified by a measurement of harmonic frequency components of the primary power line (60Hz in western Japan) in the horizontal displacement (*dX*) around the ring. The fast *dX* signal was recorded in a digital oscilloscope sweeping with the line trigger. The signals at 9 beam position monitors (BPM4-BPM12) were averaged over 100 shots. The measurement took place with normal operation parameters (α=0.0015). Fig. 1 shows one example, the distribution of 180 Hz components (3rd harmonics). The distribution was well explained by a combination of two cods expressed by Eq.(8), one produced by IBs at the western arc and another at the eastern arc. This proves that the power supplies produced the path-length fluctuation. Table 1 lists the calculated energy oscillation amplitudes of 60Hz, 120Hz, and 180Hz components. The estimated field ripple of these components was as low as 10^{-10}.

Table 1: Energy oscillation amplitudes estimated from the orbit oscillation.

harmonic frequency (Hz)		60	120	180
IB field ripple (10^{-10})	eastern arc	1.56	0.82	1.16
	western arc	0.67	1.19	0.95
energy oscillation (10^{-6})		1.11	1.17	1.34
path-length oscillation (10^{-9})		1.71	1.80	2.06

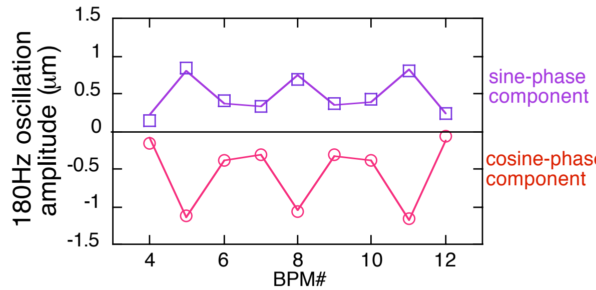

Figure 1: Horizontal oscillation amplitudes of 180Hz components at 9 BPMs. The lines are fitted function assuming 180Hz deflections at the magnets by the two power supplies, one for invert bends at the eastern arc and another for those at the western arc.

RF RIPPLE

Phase Oscillation

The phase ripple was estimated from the side-band peak height of the FFT spectrum of the beam signal. The signal was picked up by a button electrode set on the vacuum chamber. The signal was sensitive to the timing modulation but not to the horizontal displacement. Fig. 2 shows the FFT power spectrum and Table 2 shows the timing modulation (or phase oscillation) amplitude calculated from the side-band peak height.

There is no proof that the observed oscillation was true and not a monitor noise. However a small 100 Hz oscillation (not a harmonic frequency of 60 Hz) of 8 fs was known to be true. The noise source was the phase detector in the low-level control system [10].

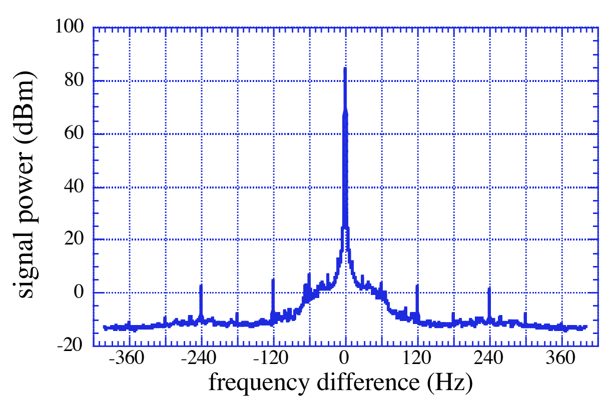

Figure 2: FFT power spectrum of the beam signal picked up from the button electrode. The center frequency is the rf frequency (500MHz).

Table 2: Side-band peak height to the main peak in the FFT power spectrum.

harmonic frequency (Hz)	60	120	180	240
Sideband peak height (dB)	-80	-81	-95	-82
Oscillation amplitude (fs)	.32	30	6	24

QI OPERATION

The energy and phase oscillation amplitudes in case that $\omega/2\pi = 180\text{Hz}$, $\Delta_L = 2\times10^{-9}$ and $\Delta_{RF} = 3\times10^{-14}\text{sec}$ were calculated. Fig. 3 shows an α dependence of the oscillation amplitudes induced by the path-length ripple and the rf ripple. It shows that the path-length ripple is serious in QI operation at $\alpha < 10^{-5}$.

The α dependence of dX at the dispersion section was measured in the range of $10^{-5} < \alpha < 10^{-3}$ [10]. The harmonic oscillation amplitudes were roughly proportional to the inverse of momentum compaction factor as was expected.

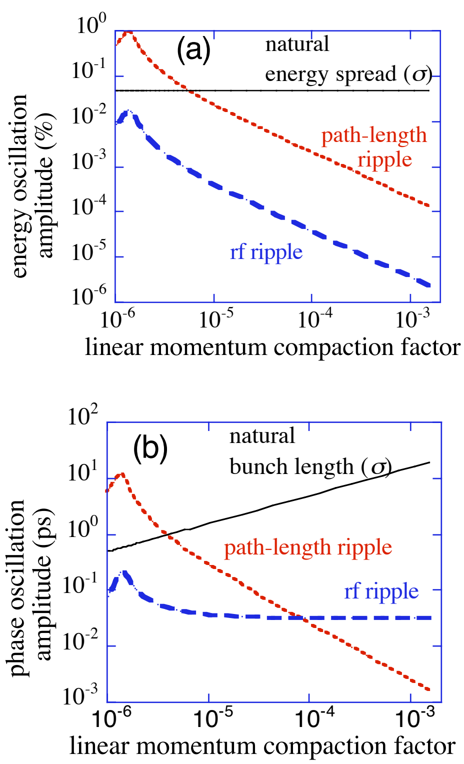

Figure 3: Estimation of (a) energy oscillation and (b) phase oscillation amplitude as a function of momentum compaction factor.

SYNCHROTRON OSCILLATION

White-Noise Induced Oscillation

In most cases the coherent oscillation at the resonant frequency, induced by a white (or broad band) rf noise, is larger than the harmonic oscillations explained above. However this oscillation is less harmful in QI operation. When $\omega = \omega_S$ and $\Delta_L = 0$, Eq. (5b) is reduced to

$$\tau = \frac{\omega_S}{2j\alpha_E}\Delta_{RF}e^{j\omega t}. \quad (9)$$

When α approaches to zero, the phase oscillation amplitude (τ) decreases with the natural bunch length if Δ_{RF} is a constant of the frequency ω. However in normal cases Δ_{RF} is smaller for lower frequency [10], therefore the resonant coherent oscillation is less harmful at low α.

CSR Induced Oscillation

One problem of the coherent synchrotron oscillation in QI mode is a large oscillation, which cannot be explained by the white-noise [2]. The oscillation amplitude depends on the rf frequency shift (Δf_{RF}), in other wards the shift of the stored energy from the reference energy. The amplitude also depended on the stored beam current as shown in Fig. 4. One possible source of the oscillation is a burst of coherent synchrotron oscillation by a longitudinal instability. It could produce a sudden energy loss and excite a coherent synchrotron oscillation. However we have no explanation for the observed Δf_{RF} dependence.

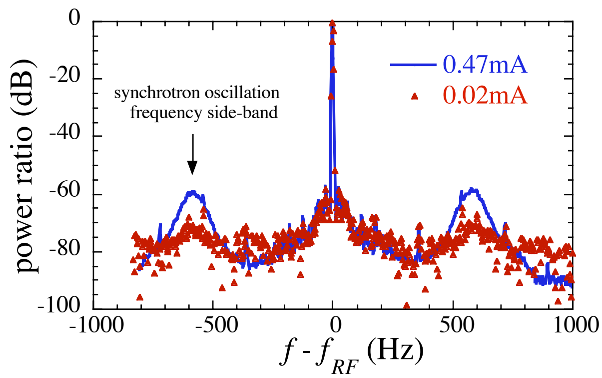

Figure 4: Beam current dependence of the synchrotron oscillation side-band. The main peak (rf frequency f_{RF}) is normalized to 0dB. White noise was subtracted from the data at 0.02mA.

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