FAST COMPENSATION OF GLOBAL LINEAR COUPLING IN RHIC USING AC DIPOLES

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Abstract

Global linear coupling has been extensively studied in accelerators and several methods have been developed to compensate the coupling coefficient C using skew quadrupole families scans. However, scanning techniques can become very time consuming especially during the commissioning of an energy ramp. In this paper we illustrate a new technique to measure and compensate, in a single machine cycle, global linear coupling from turn-by-turn BPM data without the need of a skew quadrupole scan. The algorithm is applied to RHIC BPM data using AC dipoles and compared with traditional methods.

INTRODUCTION

The complex linear coupling coefficient is defined as [1]

$$C = |C|e^{i\Theta} = -\frac{1}{2\pi}\oint ds\, j(s)\sqrt{\beta_x(s)\beta_y(s)}e^{-i(\phi_x(s)-\phi_y(s))+is/R\Delta}, \quad (1)$$

where R is the machine radius, $\Delta = Q_x - Q_y$ is the difference of the bare tunes (fractional part), β and ϕ are the Twiss functions and $j(s)$ is the skew quadrupolar force along the ring. The coupling strength $|C|$, also known in the American literature as ΔQ_{min}, is the tune separation on the resonance, whereas Θ denotes the phase of the coupling coefficient. $|C|$ is usually inferred from tune measurement against Δ (closest-tune approach). A repeated tune measurement is therefore needed for different working points. Other techniques such as first turn analysis and beam response after kick are described in [1]. Alternative methods using beam profile monitors are illustrated in [2].

None of the above technique provides any information on the phase of C. In [3] both amplitude and phase were measured observing the time evolution of the transverse beam profile after exciting the beam with a fast horizontal kick. In [4] the same measurement was carried out from turn-by-turn BPM data and a fit of the corresponding Poincaré map.

In machine with unsplit tunes and working point close to the difference resonance, $\phi_x(s) - \phi_y(s) \simeq 0$ along the entire ring. This results in $\Theta \simeq 0$ and a real C: one family of skew quadrupole is therefore enough for the correction. In machine with split tunes instead $\phi_x(s) - \phi_y(s)$ varies from 0 to 2π along the ring, resulting in $\Theta \neq 0$. The correction is performed by means of at least two families of skew quadrupoles, each one them represented by a complex coefficient C_{sq}, whose phase Θ_{sq} is given by

$$C_{sq} = |C_{sq}|e^{i\Theta_{sq}} = \frac{1}{2\pi}J_{sq}\sum_w\sqrt{\beta_x^w\beta_y^w}e^{-i(\phi_x^w-\phi_y^w)} \quad (2)$$

where the sum is over all the skew quadrupoles in the family, J_{sq} is the integrated strength (assuming a shared power supply), β^w and ϕ^w are the Twiss functions at the skew quadrupole locations. If Θ is unknown a scan of the two families is necessary to drive an external coupling and minimize $C + C_{sq,1} + C_{sq,2}$. The same goal is achieved without any scan, by measuring Θ, decomposing C on the directions $\Theta_{sq,1}$ and $\Theta_{sq,2}$, and making the families drive the opposite strengths as shown in Fig. 1.

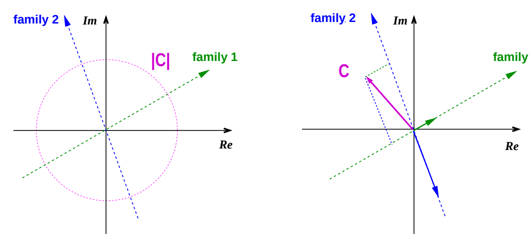

Figure 1: Left: complex plane with the measured $|C|$ and the directions of the two corrector families. Right: case with measurement of C, its decomposition along the two lines and corresponding correction setting.

FROM F_{1001} TO $|C|$ (ΔQ_{MIN})

In [5] it is shown that the linear coupling resonance driving term (RDT) $f_{1001} = |f_{1001}|e^{iq}$ is an observable, as it is measurable from the FFT of turn-by-turn BPM data of a transversely excited beam according to

$$|f_{1001}| = \frac{1}{2}\sqrt{\frac{|V(1,0)||H(0,1)|}{|H(1,0)||V(0,1)|}} \quad (3)$$

$$q = \Phi_{H(0,1)} - \Phi_{V(0,1)} + \frac{\pi}{2}, \quad (4)$$

where $H(1,0)$ and $V(0,1)$ are the horizontal and vertical tune peaks respectively (i.e. the fundamental harmonic in the horizontal and vertical FFT space respectively), whereas $H(0,1)$ and $V(1,0)$ are the secondary harmonics excited by linear coupling close to the difference resonance. Amplitude and phase of each harmonic, in the horizontal spectrum for example, read $H(m,n) = |H(m,n)|e^{i\Phi_{H(m,n)}}$.

In [2] it is shown how to measure $|C|$ from two measurements of $|f_{1001}|$ (at any location) at two different working points. Introducing the difference of the tunes (fractional part) $\Delta = Q_x - Q_y$, it can be shown that

$$4|f_{1001}\Delta| \simeq |C| + C'_o\Delta, \qquad \text{for } |C| < \Delta << 1\,. \quad (5)$$

A linear fit against Δ is therefore enough to infer $|C|$. In the upper plot of Fig. 2 the inferred $|C|$ from two simulated measurements of $|f_{1001}|$ according to Eq. (5) is shown.

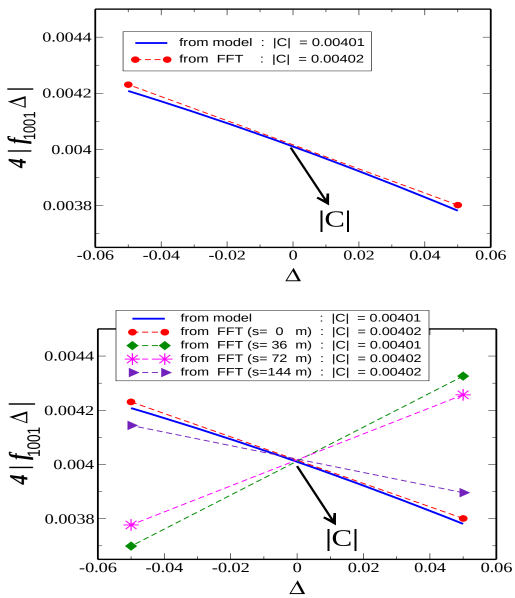

Figure 2: Examples of $|C|$ inferred from two simulated measurements of $|f_{1001}|$ according to Eq. (5) at one location (top) and at several locations (bottom).

Variation of $|f_{1001}|$ along the ring due to localized coupling sources results in different lines having nearly the same values at $\Delta = 0$, as shown in the bottom plot of Fig. 2 [5, 6]. There single particle simulations of the SIS-18 of GSI with distributed skew quadrupole kicks along the ring are run for two working points and $4|f_{1001}\Delta|$ is computed from the FFT of turn-by-turn data. The picture indicates that a single measurement of $4|f_{1001}\Delta|$ at different locations provides already a good estimation of $|C|$, being

$$|C| \simeq 4|\Delta|\frac{1}{N}\sum_{w}^{N}|f^w_{1001}|, \qquad \text{for } |C| < \Delta << 1\,, \quad (6)$$

where the normalized sum denotes the average over the BPMs (i.e. along the ring), N is the number of available monitors, and the latter condition is required for making use of Eq. (5). In simulations shown in the bottom plot of Fig. 2 the accuracy in computing $|C|$ is of about 0.5%. For an effective measurement the available BPMs should be distributed uniformly along ring: regions with large coupling uncovered by BPMs would prevent the average from describing the global amount of coupling.

FROM F_{1001} TO PHASE OF C (Θ)

In [2] it is shown how the phase of the coupling strength Θ is related to the phase of f_{1001}

$$\Theta \simeq \frac{1}{N}\sum_{w}^{N}\left[q^w - (\phi^w_x - \phi^w_y)\right] + \pi\left[1 - \text{sgn}(\Delta)\frac{1}{2}\right] \quad (7)$$

where the normalized sum denotes the same average over the BPMs of Eq. (6), q^w is the phase of f_{1001} measured at the w^{th} BPM, and ϕ^w are the BPM betatron phases (taken from the lattice model). The above approximation has a reminder proportional to Δ. It can be shown that on the resonance ($\Delta = 0$) both $|f_{1001}|$ and $q - (\phi_x - \phi_y)$ are constant along the ring. Note that also the betatron phases could be in principle inferred from the BPM spectra after choosing one BPM as reference. Nevertheless the correction relies on the model skew quadrupole Twiss parameters as shown in Eq. (2). The latters are not measurable in a straightforward and reasonably fast way. Therefore a more refined measurement of Θ would not be of help finding the best corrector setting.

In Fig. 3 the variation along the RHIC Yellow ring (injection energy) of $q^w - (\phi^w_x - \phi^w_y) + \pi\left(1 - \text{sgn}(\Delta)\frac{1}{2}\right)$ was simulated (MADX tracking) and is plotted for different working points. Five skew quadrupoles generate a coupling of $|C| = 0.01$. For $\Delta > |C|$ jumps are visible in correspondence of skew quadrupolar kicks. As you move closer to the difference resonance, $\Delta \to 0$, the jumps become less visible. From simulations at $\Delta = \pm 0.05$, Eq. (7) provides $\Theta = 2.79$ and $\Theta = 3.10$ respectively. The deviation from the correct value $\Theta = 3.0$ is therefore $\sim 5\%$ as expected.

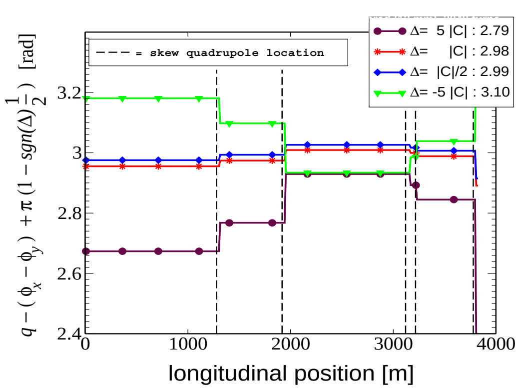

Figure 3: MADX tracking simulation of the RHIC Yellow ring at injection energy with betatron coupling $|C| = 0.01$: the quantity $q^w - (\phi^w_x - \phi^w_y) + \pi\left(1 - \text{sgn}(\Delta)\frac{1}{2}\right)$ at the BPM locations is plotted for different working points.

MEASUREMENT AND CORRECTION OF C IN RHIC DURING 2005

Eq. (6) is applied to RHIC BPM data taken during 2005 [7]. The beam was transversely excited by two AC dipoles. The inferred $|C|$ are listed in Tab. 1 and compared with the one obtained applying the N-turn map algorithm described in [8]. The standard deviation of $|f_{1001}|$ along the ring is used as error indicator. The use for several BPMs and the average make the formula be robust against failure of few BPMs (isolated large jumps in upper plot of Fig. 4). During the measurement of May 30 and June 13 2005 two

date	$<|f_{1001}|>$	Δ	$|C|$ Eq. (6)	$|C|$ [8]
May 30	20 ± 8	13	1.1 ± 0.4	1.6
May 30	50 ± 9	48	10 ± 1.7	10
June 8	25 ± 9	39	4 ± 1	3.1
June 13	30 ± 9	41	4.9 ± 1	4.4
June 13	40 ± 10	30	4.8 ± 1	4.4
June 13	25 ± 8	45	4.5 ± 1	4.4

Table 1: $|C|$ of the RHIC yellow ring at injection energy from f_{1001} measurement during 2005 using Eq. (6) compared with the ones obtained with the N-turn map algorithm [8]. All the numbers are in units of 10^{-3}.

sets of BPM data were taken turning off the three families of corrector skew quadrupoles. The corresponding $|C|$ and Θ are therefore the amplitude and the phase respectively of the natural global coupling coefficient C (see Fig. 4 for the data of May 30). Eqs. (6) and (7) yield

$$|C| = (9.6 \pm 1.7) \times 10^{-3} \quad \text{May 30} \quad (8)$$
$$|C| = (16 \pm 1) \times 10^{-3} \quad \text{June 13} \quad (9)$$

$$\Theta = (5.56 \pm 0.19)\ \text{rad} = (319 \pm 16)^{o} \quad \text{May 30} \quad (10)$$
$$\Theta = (5.58 \pm 0.17)\ \text{rad} = (320 \pm 10)^{o} \quad \text{June 13} \quad (11)$$

The standard deviation of $q - (\phi_x - \phi_y)$ along the ring is used to estimate the error.

In May 30 a traditional scan using two independent skew quadrupoles (`SQ11C2Y` and `SQ01C2Y`) was performed to minimize $|C|$, providing the following best setting

$$J_{1,\,\mathrm{SQ01C2Y}} \simeq 6 \times 10^{-4} \quad \mathrm{m}^{-1} \quad \text{(scan)}$$
$$J_{1,\,\mathrm{SQ11C2Y}} \simeq 7 \times 10^{-4} \quad \mathrm{m}^{-1} \quad \text{(scan)}\,.$$

A similar result is obtained without any scan by just decomposing C on the axes defined by the skew quadrupoles. Their directions in the complex plane, defined by $e^{-i(\phi_x - \phi_y)}$ according to Eq. (2), are plotted in Fig. 5 together with C and its decomposition on the two axes. The gradient J_{sq} are obtained by C_{sq} using the model beta functions and inverting Eq. (2). The best corrector setting eventually reads

$$J_{1,\,\mathrm{SQ01C2Y}} \simeq (5.2 \pm 1.9) \times 10^{-4} \quad \mathrm{m}^{-1} \quad \text{(RDT)}$$
$$J_{1,\,\mathrm{SQ11C2Y}} \simeq (6.3 \pm 1.9) \times 10^{-4} \quad \mathrm{m}^{-1} \quad \text{(RDT)}\,.$$

The error is inferred from the decompositions of the upper and lower values of both $|C|$ and Θ defined by their error bars.

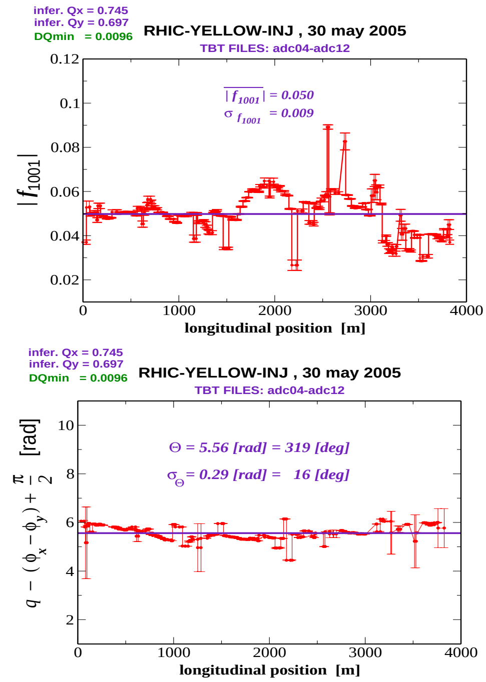

Figure 4: Variation along the Yellow ring at injection of $|f_{1001}|$ (top) and $q^w - (\phi_x^w - \phi_y^w) - \mathrm{sgn}(\Delta)\frac{\pi}{2}$ (bottom). The corrector skew. Data points (i.e. BPMs) with fluctuation larger than 30% were rejected.

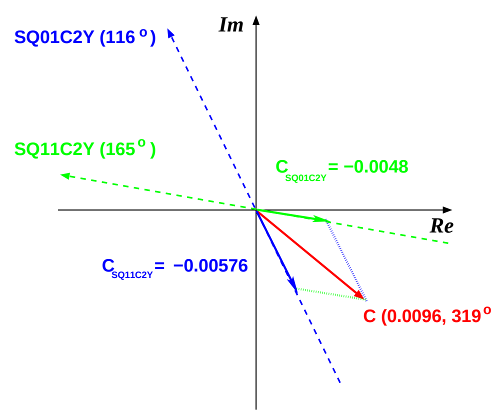

Figure 5: Scketch of the measured C decomposed on the axis defined by the two skew quadrupoles used for correction.

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