THE HIGH ORDER NON-LINEAR BEAM DYNAMICS IN HIGH ENERGY STORAGE RING OF FAIR

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Abstract

The High Energy Storage Ring (HESR) is a part of the international project FAIR for the antiproton physics with beam in a momentum range from 1.5 to 15 GeV/c to explore the research areas of hadron structure and quark-gluon dynamics [1]. An important feature of the project is the combination of phase space cooled beams with internal targets. Therefore there are two obvious reasons of beam heating: the target-beam interaction and the intra-beam scattering.

Another source of the beam size growth is the high order non-linear resonances. In the paper we investigate the non-linear beam dynamics together with the different schemes minimizing this affect.

HESR LATTICE

The HESR lattice consists of two arcs and two straight sections for target and cooling facilities with circumference 574 m [2]. Figure 1 shows the common view of HESR and one half super-period.

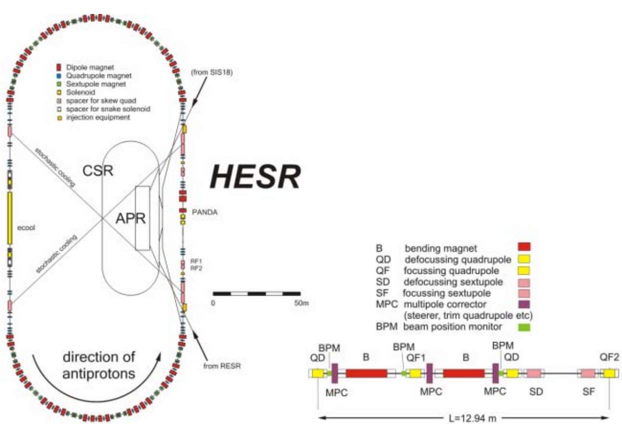

Figure1: Schematic layout of the HESR lattice and one half super-period.

HIGH ORDER NON-LINEARITY ORIGIN

Multipoles

The optics of HESR consists of the quadrupoles, the bend magnets, the sextupoles and the multipoles correctors. Besides, due to the imperfections the multipoles errors are added into the lattice. Even in ideal optics and for the monochromatic beam each n-th multipole M_n in composition with the curvature h^m creates all higher multipoles M_{n+m}.

Following the MAD presentation, Hamiltonian of system is:

$$
\begin{aligned}
&H = H_0(p_x, p_y, \delta) + V(x,y,s)\\
&H_0(p_x,p_y,x,y,z) = \frac{p_x^2+p_y^2}{2(1+\delta)} + (K_x+\Delta K_x)\frac{x^2}{2} - (K_y+\Delta K_y)\frac{y^2}{2},\\
&V(x,y,s) = \frac{S_x(z)}{6}x^3 + \frac{S_{xy}(z)}{2}xy^2 + \frac{O_x(z)}{24}x^4 + \frac{O_{xy}}{4}x^2y^2 + \frac{O_y}{24}y^4,\\
&K_x = K + h^2, \quad \Delta K_x = \delta\cdot D\cdot(2hK+S) + \frac{(\delta\cdot D)^2}{2}(3hS+O),\\
&K_y = -K, \quad \Delta K_y = -\delta\cdot D(hK+S) - \frac{(\delta\cdot D)^2}{2}(2hS+O),\\
&S_x = 2hK + S + \delta\cdot D\cdot(3hS+O), \quad S_{xy} = -hK - S - \delta\cdot D(2hS+O),\\
&O_x = 3hS + O, \quad O_{xy} = -2hS - O, \quad O_y = -h^2K + hS + O.
\end{aligned}
\tag{1}
$$

where $K(s) = \frac{\partial B_y}{\partial x}$, $S(s) = \frac{\partial^2 B_y}{\partial x^2}$, $O(s) = \frac{\partial^3 B_y}{\partial x^3}$ are quadrupole, sextupole and octupole coefficients and $\delta \equiv \Delta p/p \neq 0$ is the momentum spread.

In case of non-monochromatic beam $\delta \equiv \Delta p/p \neq 0$ each multi-pole of n-th order M_n gives all multi-poles $M_{1\div(n-1)}$ of 1÷(n-1)-th order in the place where D≠0.

In case of the closed orbit distortion x_{co} and y_{co} any multi-pole M_n of the n-th order is:

$$x_{co}^n y_{co}^m = x^n y^m + \sum_{i,j=1,m,n} a_{ij} x^{n-i} y^{m-j} \Delta x^i \Delta y^j .$$

Thus, due to the closed orbit distortion each n-th multi-pole M_n gives additionally all multipoles $M_{1\div(n-1)}$.

Chromatic sextupoles

Usually the strongest contribution into the non-linearity is coming from the chromatic sextupoles. In order to investigate the non-linear optics the Hamiltonian (1) is presented as:

$$
\begin{aligned}
H(I_x,\vartheta_x,I_y,\vartheta_y) &= \nu_x I_x + \nu_y I_y\\
&+ \frac{1}{2}\sum_{j,k,l,m} E_{lm}^{jk}\cdot I_x^{j/2}\cdot I_y^{k/2}\exp i(l\vartheta_x + m\vartheta_y),
\end{aligned}
\tag{2}
$$

where the coefficients $E_{lm}^{jk} = \sum_p h_{jklm}\exp ip\theta$ depend on the value and the distribution of the non-linear elements. They have the periodicity 2π with the new "time" coordinate $\theta = s\cdot 2\pi/C$. So, the non-linear part of Hamiltonian is:

$$V = \frac{1}{2} \sum_{j,k,l,m} \sum_{p=-\infty}^{\infty} h_{jklm} I_x^{j/2} I_y^{k/2} \exp i(l\vartheta_x + m\vartheta_y - p\theta) \quad (3)$$

with the Fourier coefficients $h_{jklm} = \frac{1}{2\pi}\int_0^{2\pi} E_{lm}^{jk} \exp ip\theta$. In case, when two conditions, the non-zero harmonic value $h_{jklm} \neq 0$ for some of the non-linear elements M_{j+k}, and the equality $k_x\nu_x + k_y\nu_y = p$, where $k_x = l$ and $k_y = m$, are fulfilled, we have the non-linear resonance. And on the contrary, when we wish to exclude the resonance influence, we should minimize the harmonic amplitude.

The only condition, which one cancels all coefficients E_{lm}^{jk}, is the zero value of $h_{jklm} = 0$ for all *j, k, l, m*. In particular, in case of the chromaticity correction on arcs with N super-periods the sextupoles have to be placed with the phase advances μ_x, μ_y per one super-period, when the harmonic $h_{jklm} = 0$ for all above mentioned combinations of *j, l, k, m*, and the total multipole of third order is canceled:

$$M_3^{total} = \sum_{n=0}^{N} S_{x,xy} \beta_x^{l/2} \beta_y^{m/2} \exp in(l\mu_x + m\mu_y) = 0 \quad (4)$$

In the HESR two families of sextupoles are used for the chromaticity correction: two focusing and two defocusing sextupoles. If super period number N is even and arc tunes $\nu_{x,y}$ are odd, then the phase advance between similar sextupoles of $n-th$ and $(n+N/2)-th$ super periods equals $\frac{\nu_{x,y}}{N}\cdot\frac{N}{2} = \frac{\nu_{x,y}}{2}$. It means we have an exact condition for compensating each sextuplet's non-linear action by another one [2].

THE NON-LINEAR TUNE SHIFT DUE TO MULTIPOLES

Nekhoroshev's condition

At derivation of the Hamiltonian (2) we use the first order perturbation theory, when the value h_{jklm} is taken as the small parameter. Already in the first order of the resonance theory $(\mathrm{P}=1)$ the sextupole excites four resonances $\{l,m\} = \{1,0; 3,0; 1,\pm 2\}$. The number of resonances and their order grows with the order as $2\mathrm{P}+1$. The resonance arises under the condition $p + k_x\nu_x + k_y\nu_y = \Delta\cdot(k_x^2 + k_y^2)^{1/2}$, where Δ is the detuning from resonance, and $k_x = l,\ \ k_y = m$.

In the action-angle variables the average Hamiltonian of the motion can be written as:

$$\begin{aligned} H(I_x,\vartheta_x,I_y,\vartheta_y) &= \frac{(k_x^2+k_y^2)^{1/2}}{k_x}\Delta I_x + \frac{(k_x^2+k_y^2)^{1/2}}{k_y}\Delta I_y \\ &+ 2\langle h_{k_x,k_y,p}\rangle I_x^{k_x/2} I_y^{k_y/2} \cos(k_x\vartheta_x + k_y\vartheta_y) \\ &+ \zeta_x I_x^2 + \zeta_{xy} I_x I_y + \zeta_y I_y^2 \end{aligned} \quad (5)$$

where $\langle h_{k_x,k_y,p}\rangle = \int_0^C \beta_x^{k_x/2}\beta_y^{k_y/2} S_{x,xy}(s) \exp i(k_x\mu_x + k_y\mu_y)ds$

and the coefficients $\zeta_x, \zeta_y, \zeta_{xy}$ determine the non-linear tune shift. In the first order of the perturbation theory the non-linear tune shifts arise due to the octupoles. But already in the second order the sextupoles give contribution in the non-linear tune shift as well. The influence of the non-linearity is specified by the discriminant in the expression:

$$\hat{I}_x^{1/2} = -\frac{3h_{30p}\cos 3\hat{\vartheta}_x}{8\zeta_x} \pm \frac{1}{4\zeta_x}\sqrt{\frac{9}{4}h_{30p} - 8\zeta_x(\Delta + \zeta_{xy}I_y)} \quad (6)$$

The lattices with $\zeta_x >> h_{30p}$ have to be classified as a special lattice, since it is a case, when the value of h_{30p} is effectively suppressed, but the non-linearity remain to be under control and strong. It is obvious from (6), if the sign of the detuning Δ coincides with the sign of the tune shift ζ_x, the discriminant is negative and the system has only one centre at $I_x = 0$. Therefore this case corresponds to the maximum stable region and the lattice with these features is the most hopeful. However, we can see from the discriminant D, if ζ_{xy} has the opposite sign with the tune Δ, then under some amplitude of oscillation in the vertical plane I_y the total detuning $\Delta_{total} = \Delta + \zeta_{xy}I_y$ can make the discriminant $D \geq 0$. Following the Nekhoroshev's quasi-isochronous condition [3] the maximum stable region is when all $\zeta_x, \zeta_y, \zeta_{xy}$ have the same sign.

Correction of non-linear tune shift

Thus, as the first step, the chromatic sextupole have been compensated in the frame of the first order perturbation theory. And after the only reason of the structural resonance excitation is the working point smearing due to the non-linear tune shift.

As the second step, using the multipole correctors, we can compensate or at least minimize the total non-linear tune shift. For this purpose we investigated the individual non-linear tune shift of each type of multipoles. The calculated total tune can be represented as the function of the emittance through the radius $r_x = \sqrt{\beta_x\varepsilon_x}$:

$$\nu_x = \nu_{x0} - \Delta\nu_x + \left.\frac{\partial\nu_x}{\partial\varepsilon_x}\right|_{\varepsilon_x=0}\varepsilon_x + \frac{1}{2!}\left.\frac{\partial^2\nu_x}{\partial\varepsilon_x^2}\right|_{\varepsilon_x=0}\varepsilon_x^2 + \ldots \quad (7)$$

The non-linear tune shift is determined by the first coefficient $\left.\frac{\partial \nu_x}{\partial \varepsilon_x}\right|_{\varepsilon_x=0}$, which depends on the required corrected chromaticity $\Delta\zeta_{cor}$ (see Fig. 2). In both planes it can be approximated by the parabolic dependence $\left.\frac{\partial \nu_x}{\partial \varepsilon_x}\right|_{\varepsilon_x=0} \approx 0.075\cdot\Delta\zeta^2_{cor}$; $\left.\frac{\partial \nu_y}{\partial \varepsilon_y}\right|_{\varepsilon_y=0} \approx 0.086\cdot\Delta\zeta^2_{cor}$.

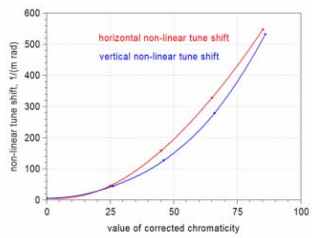

Figure 2: The horizontal and vertical non-linear tune shift versus the required corrected chromaticity $\Delta\zeta^2_{cor}$.

Besides, the errors in the bend magnets affect on the non-linear tune as well. Figure 3 shows the non-linear tune shift versus the sextupole and octupole component errors in the bend magnet measured in the units $10^4\times\Delta b_{sext\_mag}/B_0$ and $10^4\times\Delta b_{oct\_mag}/B_0$ correspondingly.

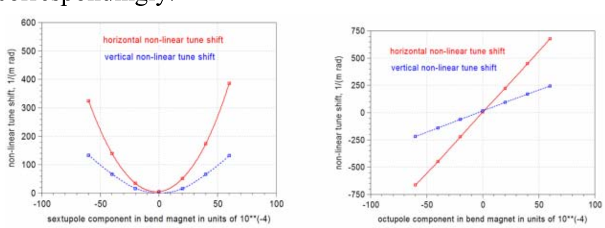

Figure 3: The non-linear tune shift vs the sextupole $\Delta b_{sext\_mag}/B_0$ and octupole $\Delta b_{oct\_mag}/B_0$ components in bend magnet.

From the numerical simulation we found out that the horizontal tune shift is more sensitive to the errors in the bend magnet.

In order to compensate the non-linear tune shifts we use the multipole correctors located near each quadrupoles (see fig. 1 and 4).

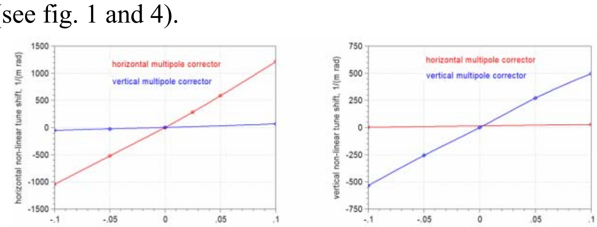

Figure 4: The non-linear tune shift versus the octupole components in multipole horizontal b^{hcor}_{oct} and vertical b^{vcor}_{oct} corrector.

The Table 1 shows the coefficients between the horizontal, vertical tune shifts and the corresponding parameters. Due to the different value of coefficients for the correctors they are decoupled each from other and allows compensating the tune shifts in both planes.

Table 1: Coefficients.

	$\Delta\zeta^2_{cor}$	$\left(\frac{\Delta b_{sext\_mag}}{B_0}\right)^2$ x10^8	$\frac{\Delta b_{oct\_mag}}{B_0}$ x10^4	b^{hcor}_{oct}, mT	b^{vcor}_{oct}, mT
$\left.\frac{\partial \nu_x}{\partial \varepsilon_x}\right\|_{\varepsilon_x=0}$	0.08	0.1	11.2	11.2	0.6
$\left.\frac{\partial \nu_y}{\partial \varepsilon_y}\right\|_{\varepsilon_y=0}$	0.09	0.04	4.0	0.13	5.2

THE SPACE CHARGE TUNE SHIFT

Due to the space charge two effects are observed: the mismatching and the structural resonances crossing. In our case, since the mismatching is determined by ratio $(\nu_{x,y}-\Delta\nu_{x,y})/\nu_{x,y}$ it has not significant effect. It is another situation with a structural resonance. By SIMBAD for each energy 1, 4 and 6 GeV we investigated the behavior of beam crossed the structural resonance.

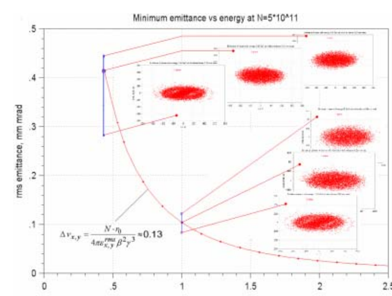

Figure 5: The structural resonance crossing.

Figure 5 shows the resonance crossing for the different energy value. The curve indicates how due to the resonance crossing in the vertical plane with initial tune $\nu_y = 12.13$ the particles in bunch are redistributed. Thus, at W=1 GeV the minimum achieved emittance is about 0.1 mm mrad.

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