ALGORITHM FOR CHROMATIC SEXTUPOLE OPTIMIZATION AND DYNAMIC APERTURE INCREASE

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Abstract

Strong chromatic sextupoles compensating natural chromaticity of a storage ring may reduce dynamic aperture drastically. In the case of several sextupole families, one can find many ways to correct a chromaticity, which provides different sizes of the dynamic aperture. Finding a solution that gives the largest dynamic aperture is an important task for the storage ring design and operation. The paper discusses several approaches to sextupole arrangement optimization in order to obtain a large dynamic aperture.

INTRODUCTION

To compensate natural chromaticity of a storage ring, two possible approaches may be mentioned. The first one uses some theoretical framework to estimate power of the nonlinear perturbation: driving terms for structural resonances, rms variation of the linear invariant (smear), tune shift with amplitude, etc. Intuitively, one may expect that linearization of such merits provides increasing of the dynamic aperture. The problems are: (a) there is no general estimation for the nonlinear perturbation valid for all cases and betatron tunes; (b) there is no direct relation between perturbation strength and the size of dynamic aperture. In simple case a solution is trivial (for instance, in the vicinity of a strong resonance where reduction of the relevant driving term opens the aperture) but for arbitrary situation the general solution is not available. An advanced example of such approach is the NSLS-II dynamic aperture optimization [1] by a least-square solving of a 52×9 nonlinear system, which includes 27 geometric modes to 3rd order, 12 tune shift coefficients to 6th order and 13 chromatic terms to 6th orders.

The second approach does not use any analytical expressions; instead of that it is based on general methods of numerical optimization. However, this method, if to enumerate all possible sextupole patterns to compensate chromaticity, requires a lot of processing time. In this paper we discuss an algorithm of the natural chromaticity compensation by “the best” pairs of sextupole magnets. The algorithm is simple and effective, does not require excessive running time, and can be applied for arbitrary lattice. Below we use this algorithm to optimize the dynamic aperture of the ALBA storage ring [2]. ALBA will be a third-generation synchrotron light source built in Spain near Barcelona. The storage ring, working at 3 GeV with a circumference of 268.8 m, has been designed for a maximum current of 400 mA. The lattice is based on an extended DBA structure and has a nominal emittance of 4 nm-rad. The machine has four-fold symmetry with 4 long straight sections (8 m), 12 medium (4.2 m) and 8 short (2.6 m). There are 14 focusing and 16 defocusing sextupole magnets in the ALBA lattice cell to adjust the natural chromaticity, and they can be arranged in different pairs to obtain the better DA.

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ALGORITHM DESCRIPTION

We propose to correct the chromaticity by N small steps along the vector $\vec{\xi} = (\xi_{x0}, \xi_{y0})$ as it is shown in Fig.1. At each step N^{-1}-th fraction of the horizontal and vertical chromaticity is compensated by a single (in some sense the best for this particular step) pair of focusing and defocusing sextupoles (SF_i, SD_j). To find the best pair of sextupoles, we try all possible (SF, SD) - combinations and the pair demonstrating the largest dynamic aperture is fixed at this step. If N_{SF} and N_{SD} are the number of focusing and defocusing sextupoles, then $N_{SF}\times N_{SD}$ combinations have to be looked through at every step.

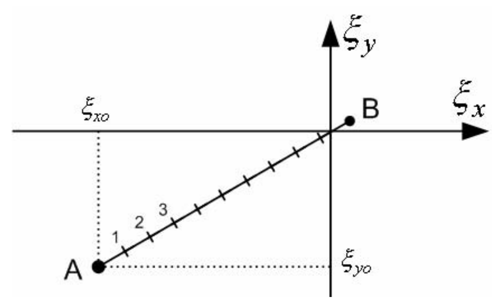

Fig.1 Step-by-step chromaticity compensation. A and B indicate initial and final points respectively.

At the next steps the procedure is repeated until the chromaticity will reach the desired value.

As the dynamic aperture represents particle stable motion area with complicated and rather ambiguously determined boundary (see Fig.2), an important problem is comparison of different apertures, provided by sextupole pairs tested at the particular step. After some study we revealed that the most simple and reliable way is to found and compare the DA area of different seeds. By area we mean all points indicating a particle survived during tracking for a definite number of turns (1000 turns in our case).

ALGORITHM APPLICATION

As an algorithm application example we use the lattice ALBA v.25 [3] with parameters summarized in Table 1. The lattice ALBA v.25 already contains set of sextupoles optimized by another approach and considered below as the reference one.

There is a following sextupoles sequence for the half-cell (SF1 SD1 SD2 SF2 SD3 SD4 SF3 SF3 SD4 SD5 SF4 SF4 SD5 SD4 SF3). Assuming individual powering of each sextupole and keeping the symmetry of the lattice (4-fold for the whole ring and reflection for the cell), we can arrange the sextupoles in 56 pairs.

Table 1 ALBA v.25 parameters

Energy	3 GeV
Number of cells	4
Natural emittance	4 nm-rad
Circumference	268.8 m
Horizontal betatron tune (per cell)	18.18 (4.54)
Vertical betatron tune (per cell)	8.37 (2.09)
Hor. natural chromaticity (per cell)	-39.4 (-9.9)
Vert. natural chromaticity (per cell)	-28.8 (-7.2)

An example of the 20 DA seeds at the first step is shown in Fig.2. One can see quite different size and shape of the DA, and basically several of them might be used for the second step, forming a kind of optimization tree.

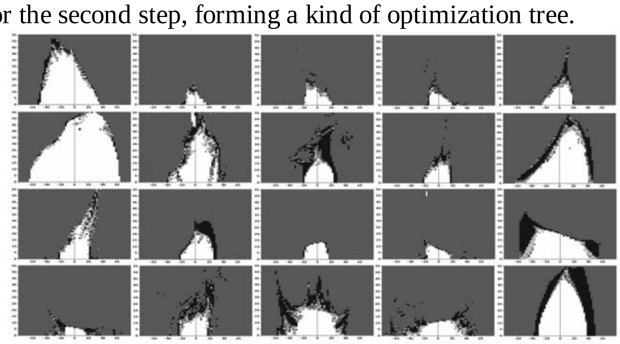

Fig.2 Twenty DA seeds at the first optimization step.

An important question is the number of optimal steps for chromaticity correction to reach the largest DA. Fig.3 shows the final area of the DA as a function of the steps number. The larger the number of steps is (the smaller chromaticity portion corrected at each step), the bigger final size of DA can be obtained. However, this process is asymptotic and for very large steps number (>100 in our case) the algorithm loses convergence and the final size of the DA starts reducing.

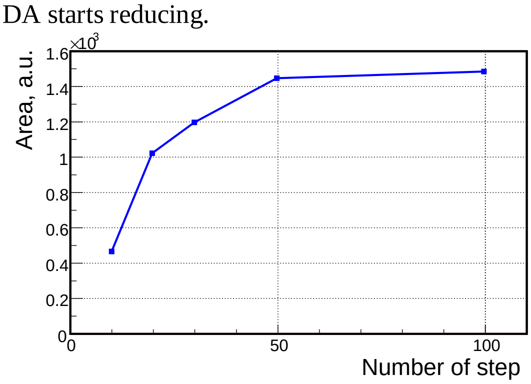

Fig. 3 Final DA size (area) as a function of the steps number required for chromaticity correction.

Possible explanation of this fact is that for very small fraction of chromaticity corrected at every step, the DA change from step to step is also small and definition of the best solution became unreliable. The optimum steps number is ~50.

Fig.4 demonstrates the change of the ALBA DA during the step-by-step chromaticity correction for 50-steps optimization.

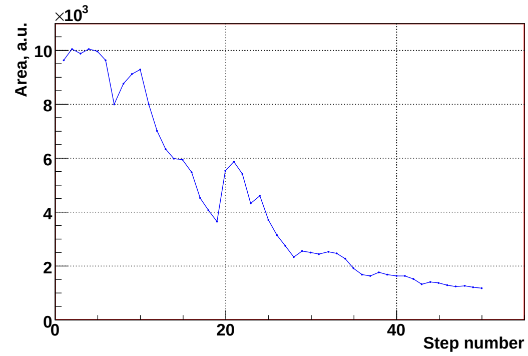

Fig.4 DA changing during step-by-step optimization.

At the first steps the DA area changes drastically because each step provides large contribution to the total perturbation of a nonlinear system. However, as the perturbation increases, the relative contribution of each next optimization step to it reduces and the change of the DA seems to converge.

The final DA in comparison with the reference one is given in Fig.5.

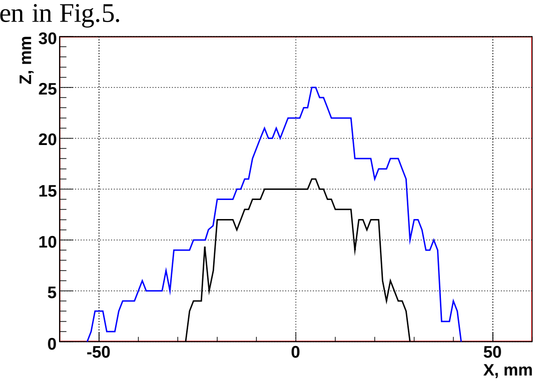

Fig.5 Final DA reached by the best sextupole pair (blue) compared to the reference one (black).

A frequency of repetition of the best pairs turning during the chromaticity compensation and the DA optimization is depicted in Fig.6.

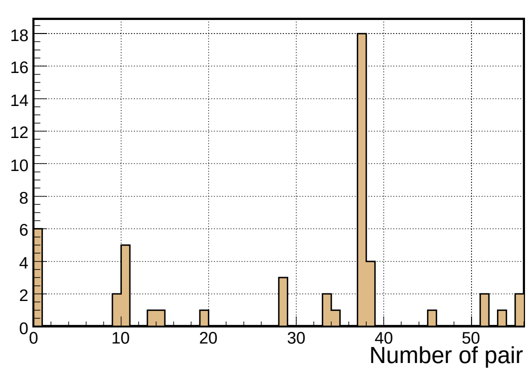

Fig.6 Repetition frequency of the best sextupole pairs occurrence during chromaticity compensation.

One can see that some pairs have never been chosen as the best ones while others occurred rather frequently (especially pair 37, which seems very effective for the DA optimization).

DA AND PHASE TRAJECTORIES

As usual, measurement of DA in a real storage ring is a difficult task requiring special equipment. It is easier to measure of phase space trajectories by means of turn-by-turn BPM system. So it seems worth finding a correlation between the DA size and some measurable characteristics of phase space curves.

Phase trajectories can be approximated analytically with the help of different perturbation approaches (see for instance [4]), which give the solution in the form of invariant of motion as a function of original phase space variables (for simplicity horizontal motion (J_x, ϕ_x) only):

$$\overline{J_x} = const = J_x + J_x^{3/2}\left(A_{1,3}(\theta)\cos 3\phi_x + A_{1,1}(\theta)\cos\phi_x\right) + \\ + J_x^2\left(A_{2,4}(\theta)\cos 4\phi_x + A_{2,2}(\theta)\cos 2\phi_x + A_{2,0}(\theta)\right)$$

The above equation was used to fit the phase space trajectories, started with the same initial values, for three DA (the best, the worst and the intermediate one) at some particular step. Fig.7 shows these trajectories in (x, p_x) variables.

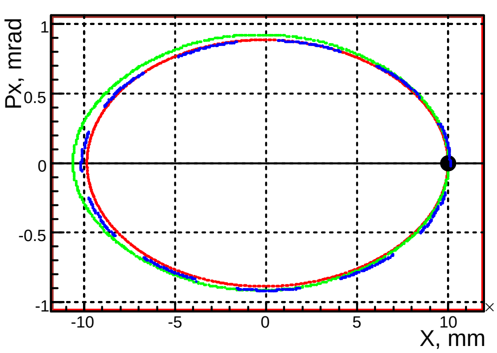

Fig.7 Three phase space trajectories for the best (red), the worst (blue) and intermediate (green) side of the DA.

The fitting coefficients and the maximum value of the phase space distortion $\Delta J_x = \bar{J}_x - J_x$ are summarized in Table 2.

Table 2 Phase space distortion coefficients

+DA	Best	Intermed.	Worst
$A_{1,3}$	4.95	-0.62	0.05
$A_{1,1}$	2.55	-9.02	-28.3
$A_{2,4}$	870	941	2730
$A_{2,2}$	-410	2100	950
$A_{2,0}$	-2600	1470	7100
ΔJ_x (μm)	0.14	0.24	0.64

The maximum value of the action variable distortion provides indication of the DA size: the larger ΔJ_x definitely corresponds to the smaller DA and vice versa.

Fig.7 demonstrates ~1-mm difference at the level of 10 mm in betatron oscillation amplitudes for the three cases considered. Such value can be easily observed by modern turn-by turn BPM diagnostics and used for DA optimization at operating machines.

CONCLUSIONS AND PLANS

The present work describes and illustrates the algorithm for the DA optimization by selecting of the best sextupole pairs as the natural chromaticity is corrected step-by-step. The results of the algorithm application seem promising and we plan to develop it further, in particular, to take into account energy DA optimization by including a second order chromaticity $(\partial\xi_{x,z}/\partial\delta)$ as the weight factor for the best sextupole pair definition.

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