MODELS TO STUDY MULTI BUNCH COUPLING THROUGH HEAD-ON AND LONG-RANGE BEAM-BEAM INTERACTIONS.

T. Pieloni*, W. Herr, CERN, Geneva, Switzerland

Abstract

In the LHC almost 6000 bunches will collide in four interaction regions where they experience head-on as well as clustered long range interactions. These lead to a coupling between all bunches and coherent beam-beam effects. For two colliding bunches this is well understood. However, for a larger number of bunches colliding with different collision patterns, it results in a complex spectrum of oscillation frequencies with consequences for beam measurements and Landau damping. To study the coherent beam-beam modes, three complementary models have been developped and will be described in this report. Two of these methods rely on self-consistent multi-bunch and multi-particle tracking while the third is an analytic model based on a complex matrix algorithms. The three methods together provide useful information about the beam-beam coupling of multi bunch beams and together provide a deeper insight into the underlying physics.

INTRODUCTION

In order to obtain higher luminosity, colliders rely on a large number of bunches per beam. The consequences are parasitic long range Beam-Beam Interactions (BBIs) and a much richer spectrum of modes of oscillation due to the coupling of many bunches [1, 2, 3]. Moreover different bunches may have different parameters (e.g emittance, intensity) and undergo different collision patterns due to multiple Interaction Points (IPs) and the non-regular bunch filling pattern of the LHC [4]. Therefore significant bunch to bunch differences in the coherent modes and Landau damping are expected. To understand this coupling we developed three complementary methods based on simulation codes and on an analytical approach. The strong-strong simulation code COMBI (COherent Multi Bunch Interactions) was written to study the coupling of multiple bunches through BBIs and allows simulating a large number of bunches for an arbitrary collision or filling scheme. It uses two different methods to simulate the beam-beam dynamics: a Rigid Bunch Model (RBM) [5] and a Multi-Particle bunch Model (MPM) [6]. Both models are fully self-consistent algorithms and allow the simulation of rigid and multi particle bunches, respectively. The analytical model is based on a One Turn Map (OTM) method to find the eigenstates of the map for the bunches colliding head-on and long range as defined by the collision scheme. Here we focus on the analytical model and how it helps understanding results from RBM and MPM simulations.

* PhD student of the École Polytechnique Fédérale de Lausanne, Institute de Physique de l'Énergie et de Particules (IPEP), Switzerland.

BEAM-BEAM MODELS

COMBI Rigid Bunch Model

In the RBM [5], bunches are treated as rigid objects where the particle distribution is assumed Gaussian with fixed transverse beam size. The transverse beam-beam kicks are calculated using the formula for the coherent beam-beam kick. Between the interactions a linear transfer is applied. The RBM is useful to study coupling of multi bunch beams and in particular the effects of collision scheme symmetries as well as of the beam filling structure. The processing time is short and allows studying different and complicated BBIs patterns. However, the model can only give qualitative results and does not take into account Landau damping effects. Many of the oscillating modes resulting from this model are damped when the MPM is used. Moreover, the beam-beam coherent tune shift for head-on collisions is underestimated due to the approximation used for the particle distribution. The non-linear terms of the beam-beam force are only partially treated, therefore the field calculation is not numerically correct. A correct quantitative approach requires multi particle bunch tracking.

COMBI Multi Particle Bunch Model

In the MPM [6], each bunch is represented by N_{tot} macro-particles. At a bunch encounter each particle receives a transverse kick calculated using the incoherent beam-beam kick. The MPM is a fully self-consistent, strong-strong code that takes into account properly the non-linear terms of the beam-beam force, reproduces Landau damping effects, gives information on dipolar and higher order coherent modes and can be used to study incoherent effects (e.g. emittance growth). Landau damping suppresses some of the coherent modes visible in the RBM spectra. Therefore RBM and MPM together help studying damping effects. A clear disadvantage of the MPM is the large processing time required. To reduce it to a reasonable order the MPM will be reconfigured for a multi processor system.

One Turn Map Model

Coherent modes from localized interactions can be analyzed with a linearized model for the beam beam force by searching the eigenmodes of the full one turn map [7]. For few equidistant bunches with multiple head-on collisions the effects are well understood [1, 2]. In this report we extend the method to more complicated beam filling schemes, such as bunch trains, and include long-range interactions. For the beam-beam force we use a linear approximation for the head-on as well as for the long-range interactions. This is justified when the separation is large enough and the amplitude of the oscillation small. The counter rotating beams are represented by a single vector containing the bunch positions and angles in both planes:

$$[\ x_{1^{b_1}}, x'_{1^{b_1}}, \ldots, x_{1^{b_2}}, x'_{1^{b_2}}, \ldots\ ] \tag{1}$$

Through the arcs positions and angles are transformed by a simple rotation in phase space:

$$A_{x,y} = \begin{pmatrix} \cos(\Delta\mu_{x,y}) & \sin(\Delta\mu_{x,y}) \\ -\sin(\Delta\mu_{x,y}) & \cos(\Delta\mu_{x,y}) \end{pmatrix} \tag{2}$$

where $\Delta\mu_{x,y}$ represents the phase advance in the arc. In the case of n bunches we have a band diagonal matrix with the submatrices A_x and A_y. Between the linear transfers we define a matrix for head on and long range BBIs. For both cases we use a linear approximation for the beam-beam kick and the submatrix for two bunches colliding in the horizontal plane becomes:

$$\begin{pmatrix} x_1 \\ x'_1 \\ x_2 \\ x'_2 \end{pmatrix}_{s+1} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ -k_x & 1 & k_x & 0 \\ 0 & 0 & 1 & 0 \\ k_x & 0 & -k_x & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x'_1 \\ x_2 \\ x'_2 \end{pmatrix}_{s} \tag{3}$$

where k_x represents the horizontal linear beam-beam kick computed from the local derivative of the force. In case of head-on kick, k_x is given by:

$$k_x = \frac{2\pi\xi}{\beta_x} \tag{4}$$

where ξ is the beam-beam parameter and β_x the beta function at the collision point. The long range kick depends on the beam separation [5]. The positions of the coupling terms in Eq. 3 depend on the bunch filling scheme and collision pattern. The model is very flexible and derived from the filling and collision scheme as for RBM and MPM. The one turn matrix M_{turn} is then obtained by multiplying the transfer and beam-beam matrices until the full turn is completed.

The OTM enables to compute eigenvalues and eigenvectors of the system of coupled bunches and therefore to obtain the complete set of oscillation frequencies. The calculation is fast and gives all oscillation modes. However the linear approximation of the beam-beam kick leads to an incorrect quantitative picture. The power of this method is that analyzing the eigenfrequencies and eigenvectors helps understanding bunch to bunch differences in the tune spectra produced with RBM and MPM [5, 6]. From the oscillation patterns provided by the OTM model, it is possible to predict and understand the presence of different dominant modes for different bunches.

RESULTS

Simple case: one bunch beams colliding head-on

As in the original study [7] the eigenfrequencies of the system are obtained by evaluating the eigenvalues λ_i of the M_{turn} and the frequencies by

$$Q_i = \frac{\text{arccos}\lambda_i}{2\pi} \tag{5}$$

In the simplest case of one bunch beams colliding head-on in one IP, one obtains two possible oscillating states as shown in Fig. 1. The two peaks in the tune spectrum (Fig. 1, top) corresponds to the unperturbed betatron frequency Q_σ and shifted by the coherent beam-beam tune shift ξ the perturbed one Q_π. The related eigenvectors (Fig. 1, bottom) give for each eigenfrequency the relative phase and amplitude of the bunches. Two clear modes are present: the σ-mode at which bunches oscillate in phase and the π-mode at which bunches oscillate out of phase.

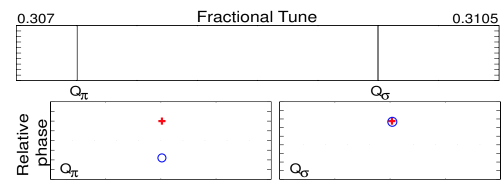

Figure 1: Eigenfrequencies and eigenvectors of a system of two bunches colliding head-on in one IP. The in and out of phase components are clearly visible for the two beams (crosses indicate beam one while circles beam two).

Beams with four equally spaced bunches

In the case of four bunches per beam colliding head-on in two non-symmetric IPs the number of possible modes increases and one expects different oscillating patterns for each eigenfrequency. We find the σ and π modes and six intermediate modes for a total of five eigenfrequencies as in Fig. 2. The difference between the two extreme modes is 2ξ due to the two head-on collisions. These modes correspond to the mode in [7]. For a given eigenfrequency one can identify the contribution of individual bunches by inspecting the corresponding eigenvectors.

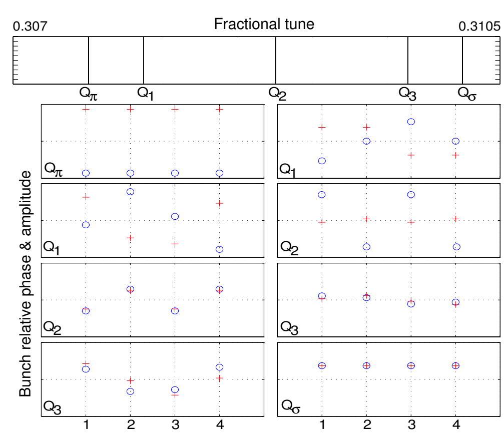

Figure 2: Eigenfrequencies and eigenvectors of a system of four bunches colliding head-on in two IPs.

Trains with five bunches

The main goal of these studies is a better understanding of the bunch to bunch differences observed in the RBM and MPM when simulating bunch trains. Depending on the coupling, i.e. collisions, bunches of a train show different spectra. The Figs. 3 and 4 show the tune spectra obtained with the MPM "observing" only the first and third bunches of a train of five. Due to the long-range interactions all bunches are coupled and as a result the σ and π modes split into sidebands. This coupling leads also in a breaking of symmetries therefore no mode degeneracies are visible (Fig. 5 for each eigenfrequency one eigenvector). The system has less degrees of freedom for the coherent motion. The spectrum of the first bunch in the train (Fig. 3) shows ten frequencies as obtained with the OTM (Fig. 5 top). Bunch number three shows only seven (Fig. 4). Bunches contribute differently to the different modes.

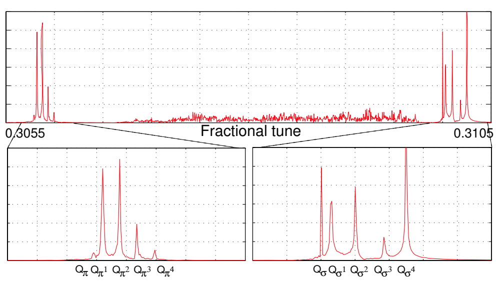

Figure 3: Top: MPM tune spectrum of the first bunch of a train of five undergoing head-on and long-range collision. Bottom: zoom of the sidebands around the π and σ-modes.

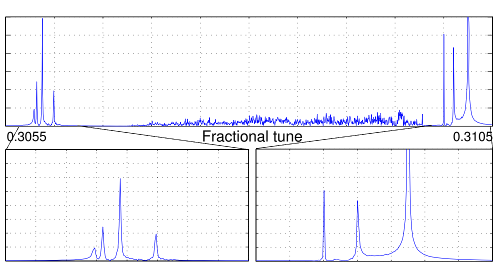

Figure 4: Top: MPM tune spectrum of the third bunch of a train of five undergoing head-on and long-range collision. Bottom: zoom of the sidebands around the π and σ-modes.

Looking at the eigenvectors in Fig. 5, the ones corresponding to the missing and/or smaller amplitude frequencies of Fig. 4, (Q_{π^1}, Q_{π^3},Q_{σ^1} and Q_{σ^3}), show that bunch three remains at zero level and is not contributing to the related coherent mode. The oscillating pattern of mode Q_{π^4} shows that bunch one of the train is contributing less with respect to bunch three which explains the differences in amplitude in Figs. 3 and 4.

Consequences for measurements

The analysis of the eigenfrequencies and especially of the eigenmodes obtained from this model allows understanding the oscillation pattern of multi bunch modes for different eigenfrequencies. Moreover, using this model it is possible to predict the different responses of individual bunches to measurements. This enables us to correctly interpret the observations from single bunch measurements.

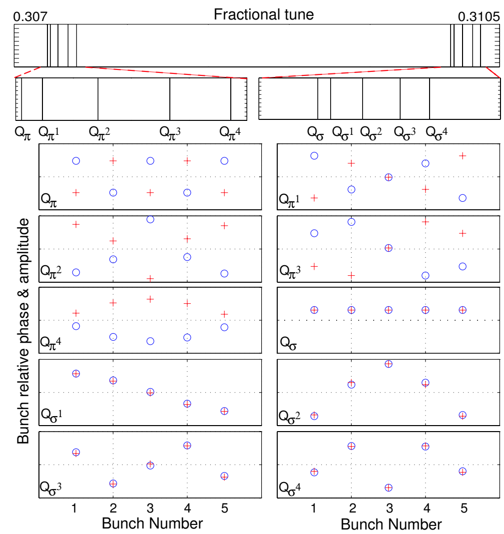

Figure 5: Eigenfrequencies and eigenvectors of bunch trains undergoing head-on and parasitic collision.

CONCLUSIONS

For any beam filling scheme and collision pattern it is possible to identify all the eigenfrequencies associated with the coherent beam-beam modes. The corresponding eigenvectors allow to associate oscillating pattern to eigenfrequency. The OTM is consistent and complements results of RBM and MPM. The eigenvectors analysis is a further step in explaining bunch to bunch differences due to coherent BBIs and can be used to correctly interpret measurements from single bunch diagnostics.

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