STATIONARY BEAM ELECTRON TRANSPORT IN AIRIX FOR THE TRAJENV CODE

O. Mouton*, CEA/DIF, Département de Physique Théorique et Appliquée,
BP12, 91680 Bruyères-le-Châtel, France

Abstract

In the framework of the AIRIX program, the electron beam propagation between the injector and the X-conversion target is routinely simulated with the 2D TRAJENV code [1]. We describe the main physical models implemented in the code for a stationary beam. Both modeling of applied electromagnetic forces in induction cells and self generated ones are presented.

LINEAR DYNAMICS

The presented expressions allow us to calculate the forces effect on the beam in an induction cell taking into account the acceleration gap and the solenoid. Typically, for a stationary beam, the equations of the beam dynamics can be reduced to linear differential equations of the second order with constant coefficients. The method, already used for magnetic fields [2] is applied here for electric ones due to the gap, and for electromagnetic fields, due to the space charge effects.

Dynamics Due to the Electric Field of a Gap

The radial equation of the relativistic dynamics due to the electric field in the gap is used in the simulation of the induction cells. We use the following approximations:

(i) paraxial approximation
(ii) beam of infinite length
(iii) cylinder symmetry of the applied electric field
(iv) neglected bound effect due to the transport tube
(v) $\left|\frac{rr'E'_z(0,z)}{2E_z(0,z)}\right| << 1$

The fundamental equation of the dynamics gives [3]:

$$\gamma\beta^2 c^2 (r'' + \frac{\gamma'}{\gamma\beta^2} r') - \frac{qE_r}{m} = 0 \,, \qquad (1.1)$$

with

$$\gamma' = \frac{q}{mc^2}\left(E_z + r'E_r\right). \qquad (1.2)$$

On the 1st order, at the z longitudinal coordinate, the E_r component of the electric field is linear with r:

$$E_r = -\frac{E'_z(0,z)}{2} r \,. \qquad (1.3)$$

So, the expr*ession (1.3) can be written as:

$$\gamma' = \frac{qE_z(0,z)}{mc^2}(1 - \frac{rr'E_z(0,z)}{2E_z(0,z)}) \,. \qquad (1.4)$$

where

$$\gamma' = \frac{qE_z(0,z)}{mc^2} \,. \qquad (1.5)$$

We deduce the transversal dynamics equations in cartesian coordinates:

$$x'' + g_1(z)x' + g_2(z)x = 0 \,, \qquad (1.6)$$

$$y'' + g_1(z)y' + g_2(z)y = 0 \,, \qquad (1.7)$$

with

$$g_1(z) = \frac{\gamma'}{\gamma\beta^2} \,, \qquad (1.8)$$

$$g_2(z) = \frac{\gamma''}{2\gamma\beta^2} \,. \qquad (1.9)$$

In a small interval $I_z = [z_0; z=z_0+\Delta z]$, the linear differential system of equations (1.9-10), with constant coefficients, can be written as:

$$x'' + 2\bar{\lambda}x' + \bar{g}_2 x = 0 \,, \qquad (1.10)$$

$$y'' + 2\bar{\lambda}y' + \bar{g}_2 y = 0 \,, \qquad (1.11)$$

with

$$\bar{g}_2 = \frac{1}{\Delta z}\int_{z^{(0)}}^{z} g_2(u)du \,, \qquad (1.12)$$

and

$$\bar{\lambda} = \frac{1}{\Delta z}\int_{z^{(0)}}^{z} \lambda(u)du \,. \qquad (1.13)$$

The expression of the $\bar{\lambda}$ coefficient is:

$$\bar{\lambda} = \frac{1}{2\Delta z} \mathrm{Log}(\frac{\gamma\beta}{\gamma^{(0)}\beta^{(0)}}) \,. \qquad (1.14)$$

To calculate the electric field in (1.5), we take the analytical expression of the electric potential on the longitudinal revolution axis of an accelerating cell. This potential is modeled by two adjacent coaxial cylinders, with a separated gap of length g at the V_1 and V_2 potentials [4, 5]. Deriving the potential, we deduce the electric field on the z axis:

$$E_z^{(e)}(0,z) = -\frac{sh(\alpha g)}{g} \times \frac{(V_2 - V_1)}{ch(\alpha g) + ch(2\alpha(z - z_s))} \,. \qquad (1.15)$$

Finally, the energy growth of a particle on path length Δz, can be integrated with the (1.5) relation:

$$\Delta\gamma(z)mc^2\Big|_{z^{(0)} \to z = z^{(0)} + \Delta z} = \int_{z^{(0)}}^{z} qE_z^{(e)}(0,z)dz \,. \qquad (1.16)$$

*olivier.mouton@cea.fr

The energy varies along the longitudinal position z of the beam according to the shape of the longitudinal applied electric field (1.15). At a z coordinate, we assume that the energy due to the gap applied electric field is a uniform function given by relation (1.16).

Dynamics Taking into Account the Space Charge

We note A_{xy} corresponds to A(X,Y), where X et Y are the rms sizes in the transverse directions (x) and (y). The space charge has a defocusing effect with a g^s_{xx} force in the (x) direction. With Kapchinskij and Vladimirskij [6] (K-V), we write that such a particle is submitted to the following dynamics equations:

$$x'' - g^s_{xy}(z)x = 0 , \quad (2.1)$$

$$y'' - g^s_{yx}(z)y = 0 , \quad (2.2)$$

with

$$g^s_{xy} = \frac{K}{X(X+Y)} , \quad (2.3)$$

where

$$X = \sqrt{\sigma_{11}} = \sqrt{< x^2 >} , \quad (2.4)$$

$$Y = \sqrt{\sigma_{33}} = \sqrt{< y^2 >} . \quad (2.5)$$

In these expressions, K is the perveance.
At the first order around the z axis, in the interval I_z, the differential equation (2.1) with variable coefficients is equivalent to the following equation with constant coefficient:

$$x'' - \bar{g}^s_{xy} x = 0 , \quad (2.6)$$

where $\bar{g}^s_{xy}$ is the mean value of g^s_{xy} on the Δz path. As we do not know the x_m and y_m variation in $[z_0 ; z]$, to calculate $\bar{g}^s_{xy}$ as a function of unknown $X^{(0)}$ and $Y^{(0)}$ at z_0, we make a limited development of g^s_{xy} around z_0. The mean value of the first order limited development of function *g* is given by:

$$\bar{g} = g^{(0)} + \frac{\Delta z}{2} g'^{(0)} . \quad (2.7)$$

We state

$$g = g^s_{xy} = \frac{K}{f_{xy}} , \quad (2.8)$$

where

$$f_{xy} = X(X+Y) . \quad (2.9)$$

The perveance can be written as:

$$K = \frac{I_b}{I_o \beta\gamma} , \quad (2.10)$$

where I_o is the characteristic courant of the beam:

$$I_o = \frac{4\pi\varepsilon_0 mc^3}{q} . \quad (2.11)$$

With the (2.10) relation and the Lorenz relation $\gamma^2(1-\beta^2) = 1$, we obtain:

$$K' = -3K\lambda , \quad (2.12)$$

with

$$\lambda = \frac{\gamma'}{2\gamma\beta^2} . \quad (2.13)$$

With the following relations

$$\sigma_{12} = < xx' > , \quad (2.14)$$

$$\sigma_{34} = < yy' > , \quad (2.15)$$

$$X' = \sigma_{12} / X , \quad (2.16)$$

$$Y' = \sigma_{34} / Y . \quad (2.17)$$

We deduce

$$\bar{g}^s_{xy} = g^s_{xy} \left(1 + \frac{\Delta z}{2} \left(\frac{K'}{K} - \frac{f_{xy}'}{f_{xy}} \right) \right) , \quad (2.18)$$

$$\frac{f_{xy}'}{f_{xy}} = \frac{\sigma_{12}(2 + Y/X) + \sigma_{34} Y/X}{X(X+Y)} . \quad (2.19)$$

Finally, we obtain the mean value of the first order defocusing due to the space charge:

$$\bar{g}^s_{xy} = \bar{g}^{s(0)}_{xy} \left(1 - \frac{\Delta z}{2} \left(\frac{\sigma^{(0)}_{12}(2 + Y^{(0)}/X^{(0)})}{X^{(0)}(X^{(0)} + Y^{(0)})} + \frac{\sigma^{(0)}_{34} Y^{(0)}/X^{(0)}}{X^{(0)}(X^{(0)} + Y^{(0)})} + 6\lambda^{(0)} \right)\right) . \quad (2.20)$$

Global Equation of the Dynamics

Taking into account the gap acceleration, the space charge and the solenoid, and assuming that the canonical angular momentum is null (constant kinetics momentum), equations (1.10) and (1.11) can be written as:

$$x'' + \bar{g}_1 x' + \bar{g}_{2xy} x = 0 , \quad (3.1)$$

$$y'' + \bar{g}_1 y' + \bar{g}_{2yx} y = 0 , \quad (3.2)$$

with

$$\bar{g}_{2xy} = \bar{g}_2 - \bar{g}^s_{xy} + \bar{k}^2_{0z} . \quad (3.3)$$

In this relation, $\bar{g}_2$ and $\bar{g}^s_{xy}$ reflect respectively the gap field effect contributions (relation 1.12) and the space charge (relation 2.20). The $\bar{k}^2_{0z}$ term corresponds to a focusing force due to the magnetic solenoid field:

$$\bar{k}^2_{0z} = \frac{1}{\Delta z} \int_{z^{(0)}}^{z} k^2_{0z}(u)du , \quad (3.4)$$

with

$$k_{0z} = \frac{qB_z(0,z)}{2\gamma m\beta c} . \quad (3.5)$$

where $B_z(0,z)$ is the axis solenoid force.

BEAM TRANSPORT MATRICES

Solving the differential equations (3.1) and (3.2), we obtain the transport matrix:

$$\mathbf{U} = \mathbf{U}_F \times \mathbf{U}_R = \mathbf{U}_R \times \mathbf{U}_X . \qquad (4.1)$$

In this expression, $\mathbf{U}_F$ is the focusing or defocusing matrix and $\mathbf{U}_R$ is the rotating one:

$$\mathbf{U}_F = \begin{bmatrix} \mathbf{U}_{xy} & 0 \\ 0 & \mathbf{U}_{yx} \end{bmatrix}, \qquad (4.2)$$

and

$$\mathbf{U}_R = \begin{bmatrix} I\cos\Delta\theta & I\sin\Delta\theta \\ -I\sin\Delta\theta & I\cos\Delta\theta \end{bmatrix}, \qquad (4.3)$$

where

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}. \qquad (4.4)$$

For the rotating matrix, $\Delta\theta$ is the angular variation due to the solenoid for the longitudinal step Δz:

$$\Delta\theta = -\int_{z^{(0)}}^{z} k_{0z}\, dz . \qquad (4.5)$$

For a focusing force ($\bar{g}_{2xy} > \bar{\lambda}^2$), we have:

$$\mathbf{U}_{xy}(z) = \sqrt{\frac{\gamma^{(0)}\beta^{(0)}}{\gamma\beta}} \times \begin{bmatrix} \cos\kappa_{xy}\Delta z + \bar{\lambda}\dfrac{\sin\kappa_{xy}\Delta z}{\kappa_{xy}} & \dfrac{\sin\kappa_{xy}\Delta z}{\kappa_{xy}} \\ -(\kappa_{xy}^2 + \bar{\lambda}^2)\dfrac{\sin\kappa_{xy}\Delta z}{\kappa_{xy}} & \cos\kappa\Delta z - \bar{\lambda}\dfrac{\sin\kappa_{xy}\Delta z}{\kappa_{xy}} \end{bmatrix} . \qquad (4.6)$$

For a defocusing force ($\bar{g}_{2xy} \le \bar{\lambda}^2$) we have:

$$\mathbf{U}_{xy}(z) = \sqrt{\frac{\gamma^{(0)}\beta^{(0)}}{\gamma\beta}} \times \begin{bmatrix} \mathrm{cha}_{xy}\Delta z + \bar{\lambda}\dfrac{\mathrm{sha}_{xy}\Delta z}{a_{xy}} & \dfrac{\mathrm{sha}_{xy}\Delta z}{a_{xy}} \\ (a_{xy}^2 - \lambda^2)\dfrac{\mathrm{sha}_{xy}\Delta z}{a_{xy}} & \mathrm{cha}_{xy}\Delta z - \bar{\lambda}\dfrac{\mathrm{sha}_{xy}\Delta z}{a_{xy}} \end{bmatrix} . \qquad (4.7)$$

A particle vector **X** in the phase space and the momentum beam matrix **σ** vary according to [6]:

$$\mathbf{X}^{(1)} = \mathbf{U} \times \mathbf{X}^{(0)}, \qquad (4.8)$$

$$\boldsymbol{\sigma}^{(1)} = \mathbf{U} \times \boldsymbol{\sigma}^{(0)} \times \mathbf{U}^{t} . \qquad (4.9)$$

The figure 1 shows an example of electron transport in AIRIX using the TRAJENV code. The upper curve is the K-V electron beam envelope radius, and the lower ones show the centroid trajectory.

CONCLUSION

We described the matrix model integrated in the beam transport code TRAJENV which is routinely used on AIRIX. Currently, it can also simulate successfully the transport of the electron beam through thin foils [7, 8]. Now, new improvements are under development for the end-user of the radiographic facility.

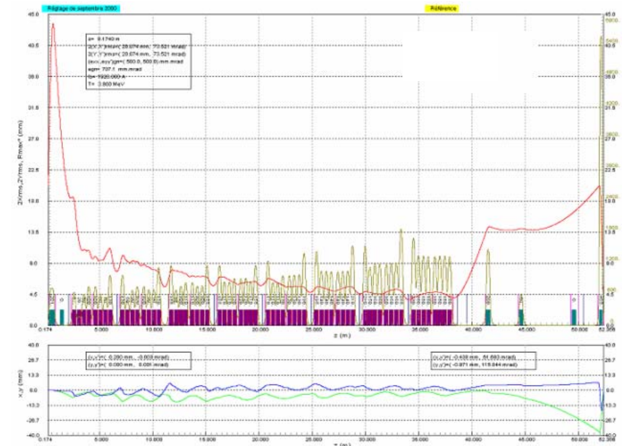

Figure 1: Electron beam envelope radius and centroid trajectory in AIRIX with the TRAJENV code.

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