TIME DOMAIN RADIATION OF A GAUSSIAN CHARGE SHEET PASSING A SLIT IN A CONDUCTING SCREEN

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Abstract

A semi-analytical method is proposed to calculate in time-domain the radiation of a relativistic Gaussian charge sheet travelling parallel to a slotted conducting screen. The method is based on transient line current elements as basis functions which have a triangular time dependence. Making use of duality magnetic current elements are used in the slot region. The dual problem, the scattering of the fields at a conducting strip is also treated. The main purpose of the paper is to present an effective algorithm which is easy to implement for computing and visualising plane scattering and diffraction problems in time-domain.

INTRODUCTION

Two-dimensional scattering and diffraction problems like for example the diffraction of electromagnetic waves by a slit, are normally treated in the frequency domain. This leads to a system of linear equations for the coefficients of the expansion functions which has to be solved for every frequency, an inadequate approach for high frequencies. In case of transient processes a Fourier transform is required in addition.

The aim of the present paper is the development of efficient basis functions which allow for direct calculation in time-domain. These are planar electric and magnetic surface current elements with a triangular time behaviour. Electric current elements can be used e.g. for the computation of the scattering at a conducting strip, whereas, due to the duality principle, magnetic current elements are used for the diffraction by a slit. The approach is semianalytic since the basis functions are known analytical functions. A full analytical solution is only possible for a few special cases like the diffraction by a conducting wedge [1].

FIELD OF A TRANSIENT ELECTRIC LINE CURRENT

Given is a line current with a triangular time dependence parallel to the z-axis, Fig. 1.

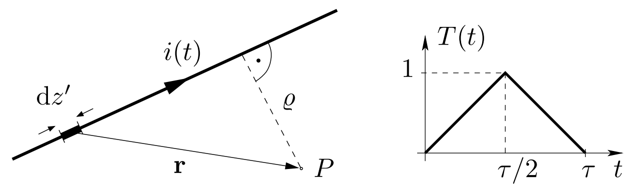

Figure 1: Geometry and time dependence of a line current $i(t) = I_0 T(t)$.

Making use of the retarded vector potential one can show that the electric field is given by

$$E_z\left(\varrho, t+\frac{\tau}{2}\right) = -\frac{Z_0 I_0}{2\pi a}\,\alpha\left(\frac{\varrho}{a}, \frac{c_0 t}{a}\right) \tag{1}$$

$$\alpha(p,q) = \begin{cases} 0 & , \quad q < p-1 \\ f\left(\frac{q+1}{p}\right) & , \quad p-1 < q < p \\ f\left(\frac{q+1}{p}\right) - 2f\left(\frac{q}{p}\right) & , \quad p < q < p+1 \\ f\left(\frac{q+1}{p}\right) - 2f\left(\frac{q}{p}\right) + f\left(\frac{q-1}{p}\right) & , \\ & \quad q > p+1 \end{cases}$$

with $f(\xi) = \operatorname{arcosh}\xi$ and $a = c_0\tau/2$. This already represents adequate basis functions in the general sense of the method of moments as shown by HARRINGTON [2]. They can be used to compute the scattering of waves at conducting surfaces. For that purpose the surface is partitioned into small segments which can be considered as line currents if the point of observation is not too close to the segment.

SCATTERING OF A PLANAR FIELD PULSE AT A CONDUCTING STRIP

A Gaussian and planar field pulse with the electric field of

$$E_z^{(i)}(\xi, t) = E_0\,\exp\left(\frac{[\xi - c_0(t-t_0)]^2}{2(c_0 T/2)^2}\right) \tag{2}$$

propagates under an angle ϑ with respect to the y-axis and is scattered by a conducting strip of width b, Fig. 2.

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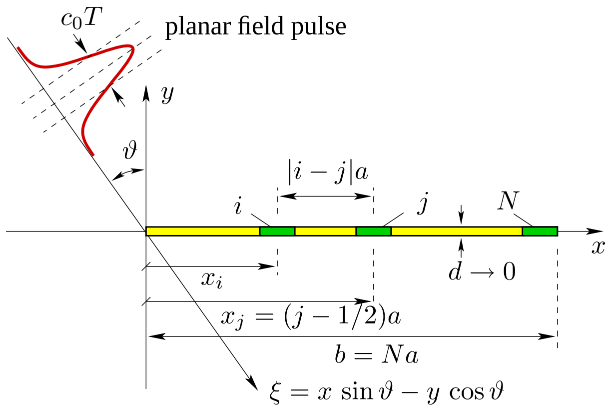

Figure 2: Planar field pulse is incident on a conducting strip.

The time shift t_0 is large enough, typically $t_0 > 2T$, such that the strip may be considered as current-free for $t = 0$. Now, the induced surface currents will be approximated by spatial pulse functions $P(x - x_i)$ of width a and by temporal triangular functions of duration $\tau = 2a/c_0$

$$J_F(x,t) = \sum_{i=1}^{N}\sum_{k=1}^{M} \frac{I_{ik}}{a}\, P(x-x_i)\, T\left(t - t_k + \frac{a}{c_0}\right) \ , \quad (3)$$

with $t_k = ka/c_0$ and $I_{i1} \approx 0$. The unknown coefficients I_{ik} for $k > 1$ follow from the boundary condition for the electric field

$$E_z^{(i)}(x = x_i, y = 0, t = t_k) + E_{ik}^{(s)} + E_{ik} = 0 \quad (4)$$

taken in the center of the current segment i and at time instants $t = t_k$. E_{ik} is the electric field due to all segments $j \neq i$ at the position of segment i and at the time instant $t = t_k$. $E_{ik}^{(s)}$ is the electric field of segment i at its center and also at $t = t_k$. Then, the normalized current coefficients $\tilde{I}_{ik} = I_{ik}Z_0/(2\pi a E_0)$ at $t = t_k$ are easily evaluated by making use of the current coefficients at earlier time instants

$$V_0 \tilde{I}_{ik} = \frac{E_z^{(i)}(\xi = x_i \sin\vartheta, t = t_k)}{E_0} - \quad (5)$$
$$- \sum_{n=1}^{k-1} \tilde{I}_{i(k-n)} V_n - \sum_{\substack{j=1\\ j\neq i}}^{N} \sum_{l=1}^{k-1} \tilde{I}_{jl}\, \alpha(|i-j|, k-l)\ .$$

Here the basis functions $\alpha(p,q)$ are given in (1) and the constants V_n follow when computing the field in the center of its proper segment

$$\begin{aligned} V_0 &= \mathrm{arcosh}\,2 + \tfrac{\pi}{3} \\ V_1 &= \mathrm{arcosh}\,4 - 2\,\mathrm{arcosh}\,2 + 4\,\arcsin\tfrac{1}{4} - \tfrac{2\pi}{3} \\ V_n &= \mathrm{arcosh}\,(2n+2) - 2\,\mathrm{arcosh}\,(2n) + \\ &\quad + \mathrm{arcosh}\,(2n-2) + (2n+2)\,\arcsin\tfrac{1}{2n+2} - \\ &\quad - 4n\,\arcsin\tfrac{1}{2n} + (2n-2)\,\arcsin\tfrac{1}{2n-2}\ . \end{aligned} \quad (6)$$

Fig. 3 shows the lines of constant electric field strength. The strip was partitioned into 25 segments. In Fig. 4 the surface current distribution is shown on the strip with 100 segments in that case. After 20 time steps the peak of the field pulse has reached the strip. As can be seen, the induced current peak propagates faster than the velocity of light $v = \sqrt{2}\,c_0$ because of the incidence tilted by $\sin\vartheta = 1/\sqrt{2}$.

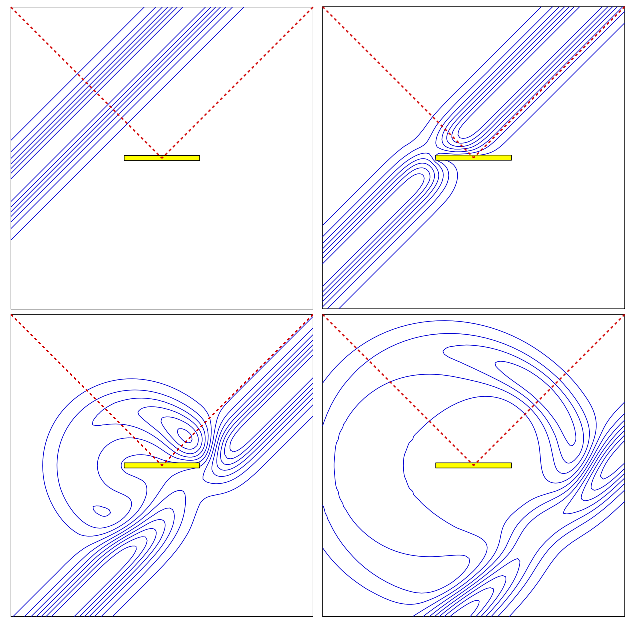

Figure 3: Lines $E_z =$ const. for a planar field pulse impinging on a conducting strip. Strip approximated by 25 segments.

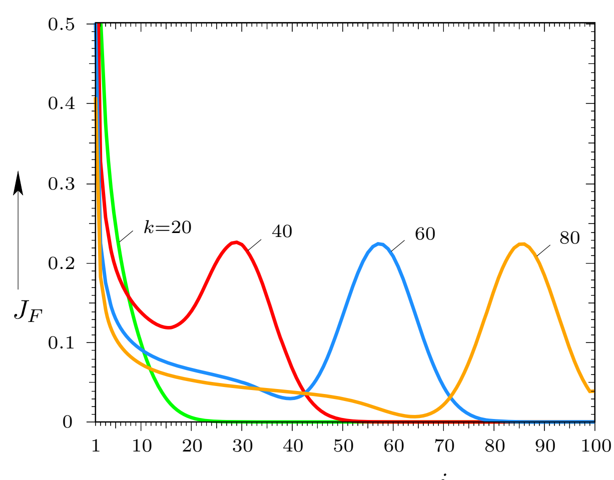

Figure 4: Surface current distribution on the strip at different time instants. At $k = 20$ the peak of the field pulse has reached the strip. Angle of incidence $\vartheta = \pi/4$, strip approximated by 100 segments.

GAUSSIAN CHARGE SHEET PASSING A SLIT IN A CONDUCTING SHEET

A Gaussian surface charge

$$q_F(x,t) = q_{F0}\ \exp\left\{-\frac{[x-v(t-t_0)^2]}{2(c_0T/2)^2}\right\}\ . \qquad (7)$$

travels parallel to a conducting sheet with a slit, Fig. 5.

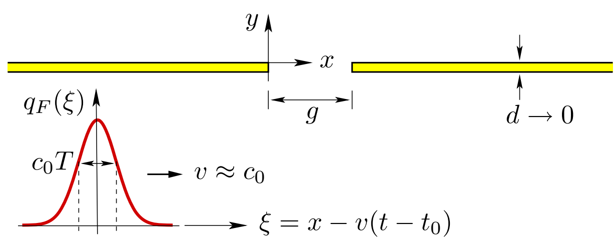

Figure 5: Gaussian surface charge travelling parallel to a conducting sheet with a slit.

In case of a closed conducting sheet the primary magnetic field of the charge is

$$H_z^{(p)} = q_{F0}c_0\ \exp\left\{-\frac{[x-v(t-t_0)^2]}{2(c_0T/2)^2}\right\}\ . \qquad (8)$$

The effect of the slit can then be described by magnetic surface currents $\pm J_F^M$ above and below the sheet, Fig. 6.

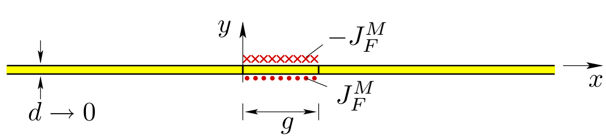

Figure 6: Equivalent magnetic surface currents for the conducting sheet with slit.

These magnetic surface currents are easily found in a way analogous to the approach in the preceding chapter. Since the diffraction by the slit is the dual problem to the scattering by a strip, one can use the basis functions (1) and replace E_z/E_0 by Z_0H_z. After fulfilling the boundary conditions a result similar to (5) is obtained, where instead of electric magnetic current segments are used. The exact procedure can not be outlined here because of the space restrictions.

In Fig. 7 the electric field lines are shown at time intervals with 20 time steps.[1] In addition, the induced wall current on the sheet is given in Fig. 8. The reflection of the current pulse at the sheet end can clearly be observed.

[1] Animated field plots under `http://www-tet.ee.tu-berlin.de/EPAC06/`

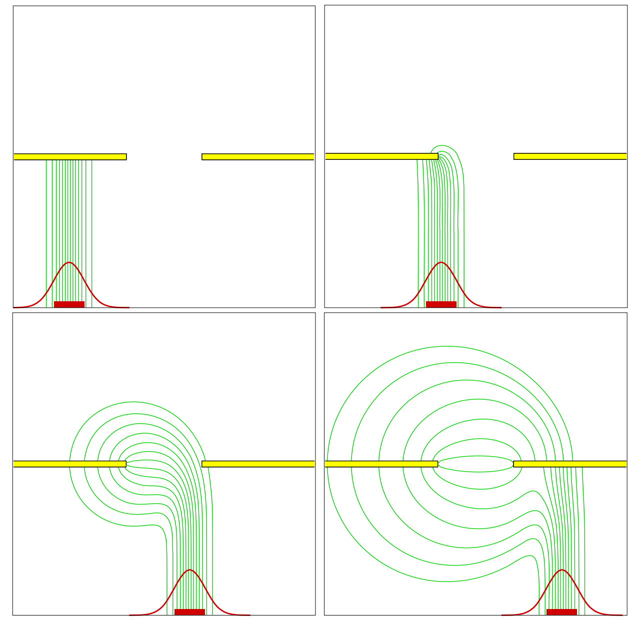

Figure 7: Electric field lines for a Gaussian charge travelling parallel to a conducting sheet with slit. Slit approximated by 25 segments, time duration of the basis function $\tau = 2a/c_0$ and mean width of the charge $c_0T = 10a$.

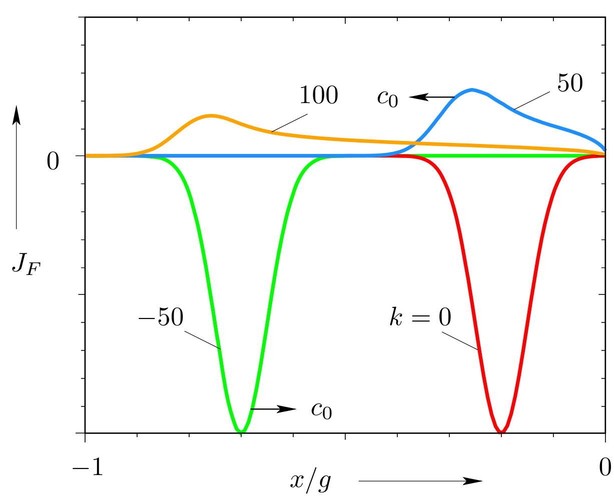

Figure 8: Wall current in the sheet of the problem in Fig. 7. At time step $k = 20$ the charge peak has reached the slit. Slit approximated by 100 segments.

REFERENCES

[1] H. Henke, Proc. IEEE Part. Accel. Conf., San Francisco, 1991, p. 380.

[2] R. F. Harrington, "Field computation by Moment Methods", Wiley, 1993.