SIMULATION OF 3D SPACE CHARGE FIELDS OF BUNCHES IN A BEAM PIPE OF ELLIPTICAL SHAPE*

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Abstract

Recent applications in accelerator design require precise 3D calculations of space-charge fields of bunches of charged particles additionally taking into account the shape of the beam pipe. An actual problem of this kind is the simulation of transverse single bunch instabilities resulting from e-clouds in positron damping rings. In this paper a simulation tool for 3D space-charge fields is presented where a beam pipe with an arbitrary elliptical shape is assumed. The discretization of the Poisson equation by the method of finite differences on a Cartesian grid is performed having the space charge field solved only in the points inside the elliptical cross section of the beam pipe taking care of the conducting boundaries of the pipe. The new routine has been implemented in the tracking code Astra. Numerical examples demonstrate the performance of the solution strategy underlying the new routine. Further tracking results with the new method are compared to the established FFT space charge routine of Astra.

INTRODUCTION

Damping rings reduce the emittances delivered by the particle sources to values required for the linear colliders. Due to the electron cloud effects in the damping rings a single bunch emittance can be blown up. Particle tracking programs (e.g. PIC codes) which model the interactions of a single bunch and the electron clouds require calculation of the space charge fields at each discrete time step. Space charge fields along with other applied EM fields determine the Lorentz force which impacts the particle trajectory. The calculation of the space charge fields from spatially distributed charges requires the solution of the Poisson equation. The solution can be strongly influenced by the applied boundary conditions (b.c.). Conducting b.c. are being applied mostly on the rectangular boundaries of the calculation domain. However the rectangular cross section is not the best approximation of the true geometry of the beam pipe. In this paper we present an algorithm for applying conducting b.c. on an arbitrary elliptic cross section of the beam pipe. In the following section we present 3D simulation results for open (free space) and conducting b.c.. Conducting b.c. are applied on the walls of a rectangular and elliptic calculation domain. Finally we show the performance of the algorithm employed in the tracking code Astra [1].

* Work supported by DESY, Hamburg
† conference participation supported by grant of EPS

CONDUCTING B.C. ON BEAM PIPES WITH ELLIPTICAL CROSS SECTION

A 3D domain in which the Poisson equation is usually solved resembles a rectangular box Γ with $Dirichlet$ b.c. on $\partial\Gamma_1$ (transversal boundary planes) and $open$ boundary conditions on $\partial\Gamma_2$ (boundary planes of the domain in longitudinal direction). For this case the Poisson's equation

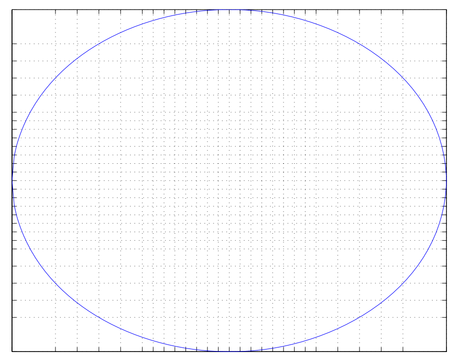

Figure 1: Cross section of the discretization domain.

reads as

$$\begin{aligned} -\Delta\varphi &= \frac{\varrho}{\varepsilon_0} && \text{in } \Gamma \subset \mathbb{R}^3, \\ \varphi &= g && \text{on } \partial\Gamma_1, \\ \frac{\partial\varphi}{\partial n} + \frac{1}{r}\varphi &= 0 && \text{on } \partial\Gamma_2, \end{aligned} \tag{1}$$

where

$\Gamma = [-a, a] \times [-b, b] \times [-c, c]$ and $\partial\Gamma = \partial\Gamma_1 \bigcup \partial\Gamma_2$. Considering beam pipes with elliptical cross section we define the Poisson equation on the cylindrical domain Ω (Figure 1) as

$$\begin{aligned} -\Delta\varphi &= \frac{\varrho}{\varepsilon_0} && \text{in } \Omega \subset \mathbb{R}^3, \\ \varphi &= 0 && \text{on } \partial\Omega_1, \\ \frac{\partial\varphi}{\partial n} + \frac{1}{r}\varphi &= 0 && \text{on } \partial\Omega_2, \end{aligned} \tag{2}$$

where $\partial\Omega_1$ is the coating of the cylinder with

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \text{ and } -c < z < c,$$

$\partial\Omega_2$ are the two elliptical bases of the cylinder satisfying

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} \leq 1$$

and being perpendicular to the z-axis.The boundary condition $\varphi = 0$ on $\partial\Omega_1$ means that the surface of the beam pipe is assumed to be an ideal electric conductor.

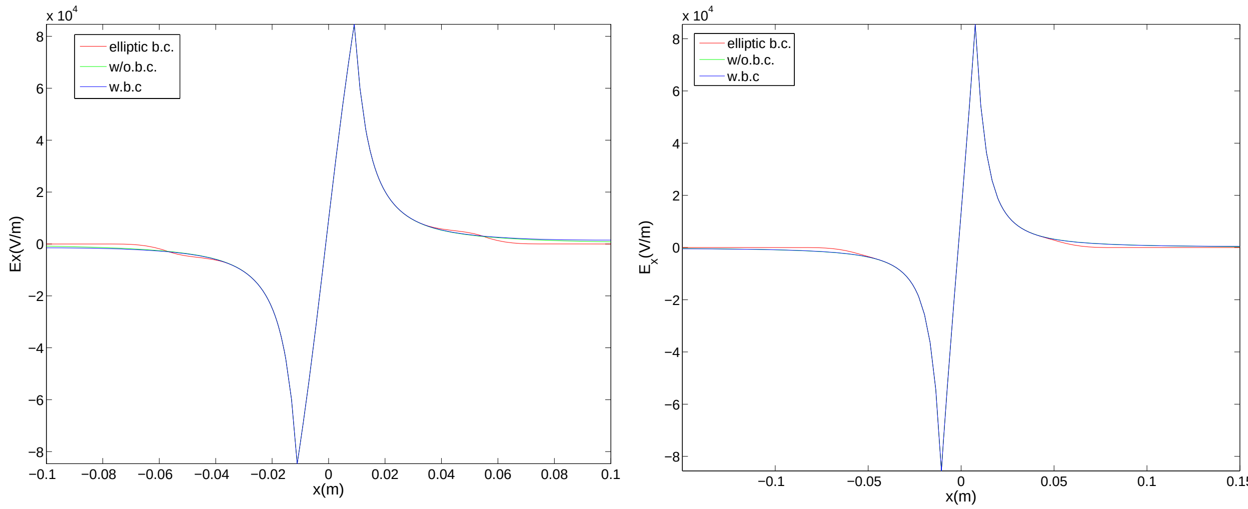

Figure 2: Electric field E_x along x-axis of a square $a = b$ (left) and a rectangular box $a = 1.5b$ (right) computed with open and conducting b.c. on a rectangular and elliptic pipe.

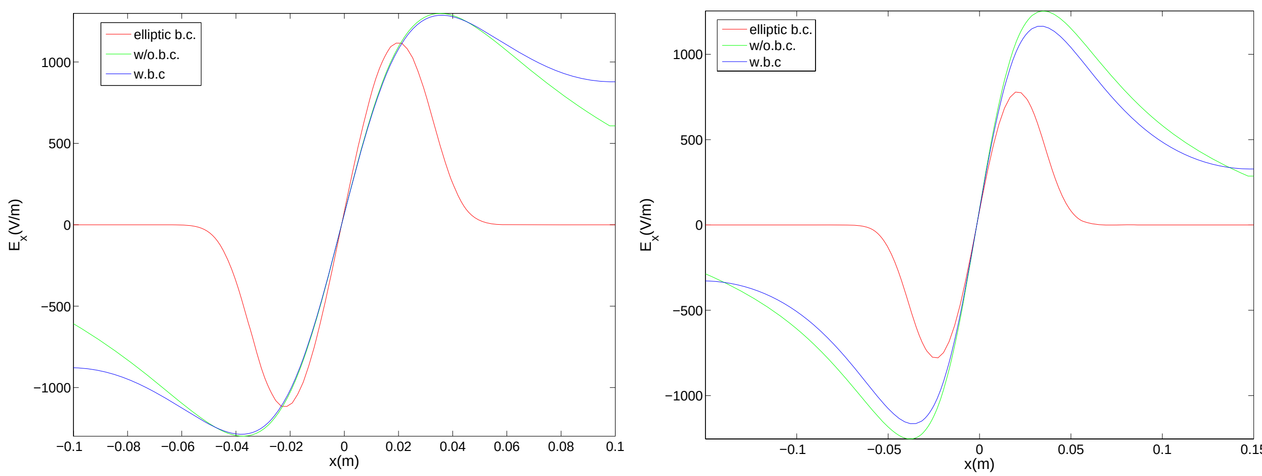

Figure 3: Electric field E_x along $y = \pm b/2$ of a square $a = b$ (left) and a rectangular box $a = 1.5b$ (right) computed with open and conducting b.c. on a rectangular and elliptic pipe.

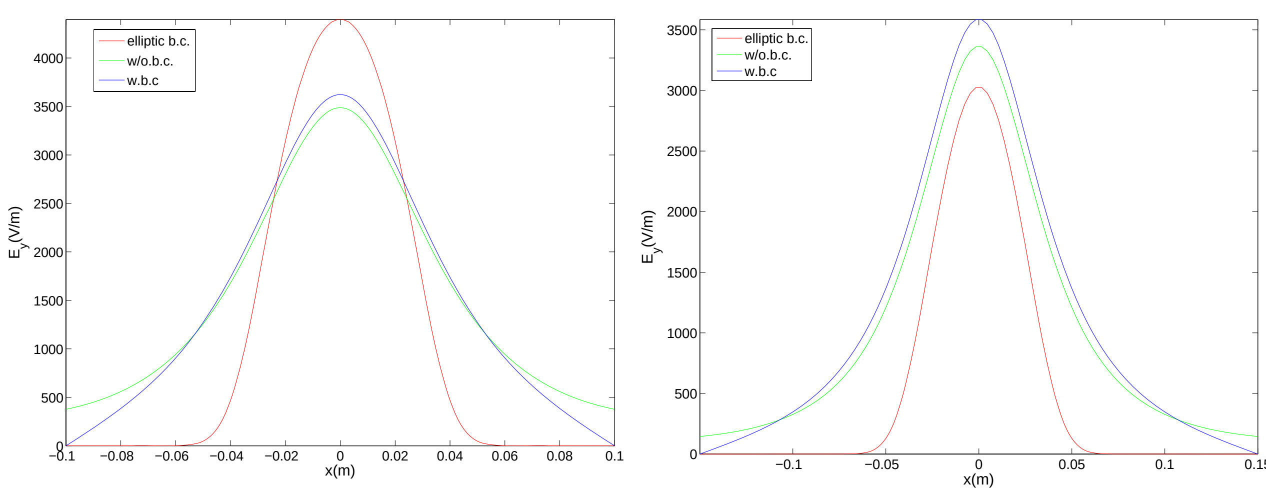

Figure 4: Electric field E_y along $y = \pm b/2$ of a square $a = b$ (left) and a rectangular box $a = 1.5b$ (right) computed with open and conducting b.c. on a rectangular and elliptic pipe.

The discretization volume in which the cylindrical computational domain Ω is embedded (Figure. 1) is the same rectangular box Γ as in (1). The domain Γ is discretized along the x-, y- and z-axis in N_x, N_y and N_z (in general case non equidistant) steps, respectively. So, the discretization of (2) with second order finite differences leads to a linear system of equations $\boldsymbol{Au} = \boldsymbol{b}$ where u is the vector with the potentials on the grid points.

The above system for the domain Ω contains only the equations for the points which are inside of Ω. The number of unknowns is considerably smaller because in each (x, y)-plane all grid points which are outside the ellipse are skipped. The system matrix $\boldsymbol{A}$ is a block structured, however the blocks will have different dimensions and the symmetry of $\boldsymbol{A}$ is lost in contrast to the system we get for the rectangular domain Γ. It turns out that the system matrix $\boldsymbol{A}$ is positive definite (more details in [2]) therefore we apply the BiCGSTAB algorithm to solve the system.

SIMULATION EXAMPLES

As example, to compare the calculated fields with open and conducting b.c. on a rectangular and elliptic pipe we choose a spherical bunch with uniformly distributed charge of -1nC, situated in the center of the beam pipe. The bunch radius r is 10 mm ($r << a, b$). The electric field is significantly different especially in the proximity of the boundary. Figure 2 shows the E_x field on the x-axis in a square box ($a = b$) and in a rectangular box with $a = 1.5b$.

The difference between the electric field in open space, in rectangular box and in elliptical cylinder becomes more evident as we move closer to the boundaries in both directions. Figure 3 and 4 show both transversal components, E_x and E_y on the line $y = \pm b/2$.

TRACKING EXAMPLE

Recently the 3D space charge algorithm including a beam pipe with elliptical shape has been implemented into the tracking code Astra [1]. The tracking example presented here was performed with a bunch of 10,000 macro particles tracked along a distance of 0.3 m without any additional external fields. The macro particles within the bunch were assumed to have Gaussian distribution with $\sigma_x = \sigma_y = 0.75$ mm and $\sigma_z = 1.0$ mm. Further, the bunch has a total charge of -1 nC and an average energy of 2 MeV. The beam pipe has a diameter of 24mm.

Figure 5 shows the transverse electric field after a drift of $z = 0.3$ m in a beam pipe with circular shape. For comparison the same drift was simulated with the 3D FFT space charge routine of Astra (see Figure 6) which realizes free space boundary conditions. The typical Astra output shows that there is good agreement of the results close to the bunch. It has to be mentioned preconditioned conjugate gradients were applied to solve the system of equations, because it turned out that the BiCGSTAB algorithm does not yet perform stable in the tracking context. Here further investigations are necessary.

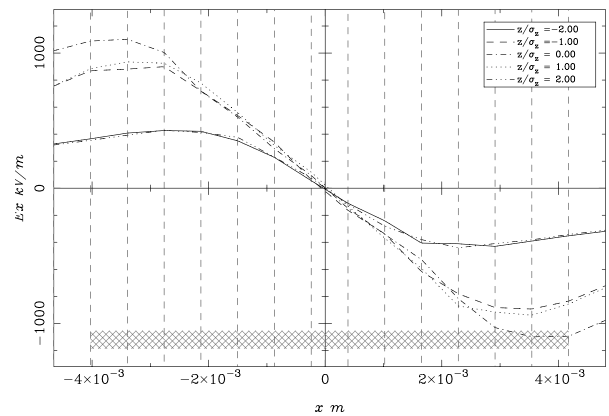

Figure 5: Iterative solver: Transverse electric field component at the position $z = 0.3$ m.

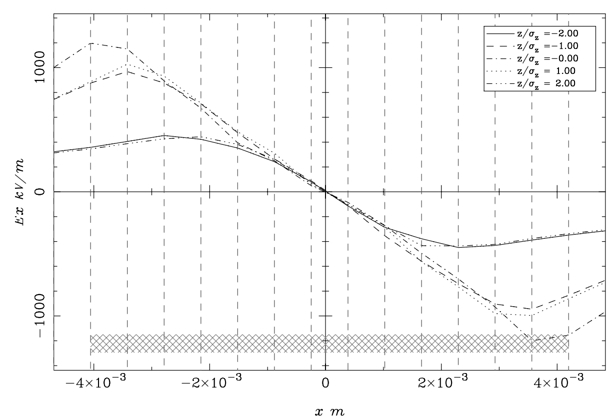

Figure 6: FFT Poisson solver: Transverse electric field component at the position $z = 0.3$ m.

CONCLUSION

General observation from the simulations with elliptical beam pipe is that the field differences get significant as we move towards the boundary of the pipe. The future work will be to benchmark the routine against experimental data.

REFERENCES

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[2] A. Markoviḱ, A numerical computation of space-charge fields of electron bunches in a beam pipe of elliptical shape. Hamburg DESY - DESY-TESLA-05-21 (05/09) 43 p.