CSR EFFECTS IN A BUNCH COMPRESSOR: INFLUENCE OF THE TRANSVERSE FORCE AND SHIELDING *

G. Bassi†, J. A. Ellison, K. Heinemann,
Department of Mathematics and Statistics, UNM, Albuquerque, NM 87131

INTRODUCTION

We study the influence of coherent synchrotron radiation (CSR) on particle bunches traveling on arbitrary planar orbits between parallel conducting plates which represent the vacuum chamber. Our ultimate goal is to follow the time evolution of the phase space density distribution by solving the 2D Vlasov-Maxwell (VM) system in the time domain. In contrast to macroparticle methods, the VM approach has no statistical noise and is better suited to the study of microbunching. The fields excited by the bunch are computed in the lab frame, while the nonlinear Vlasov equation, formulated in interaction picture, is integrated in the beam frame using the method of local characteristics (Perron-Frobenius (PF) method). The interaction picture allows a relatively large step using the Euler method. Details of our method can be found in [1] and [2]. The major difficulty here is the need for a detailed understanding of the support of the phase space density. Here we show recent numerical results obtained in our Liouville-Maxwell approximation (LMA) for the benchmark bunch compressor at 5 GeV studied in [3]. We analyze the role played by the transverse force and the effect of shielding and study the support of the phase space density.

THE LIOUVILLE MAXWELL APPROXIMATION

Our equations for the fields are

$$\mathcal{E}(\boldsymbol{R},u) = -\frac{1}{2\pi}\sum_{k=0}^{\infty} a_k \int_{-\infty}^{u-kh} dv \int_{-\pi}^{\pi} d\theta\, \mathcal{S}(\hat{\boldsymbol{R}},v,k) \quad (1)$$

where $\mathcal{S} = (\nabla\rho_L/\epsilon_0 + \mu_0 c\partial \boldsymbol{J}_L/\partial u, -\mu_0(\nabla\times\boldsymbol{J}_L)_Y)$, $\hat{\boldsymbol{R}} = \boldsymbol{R} + \sqrt{(u-v)^2-(kh)^2}(\cos\theta,\sin\theta)$, $\mathcal{E}(\boldsymbol{R},u) = (E_Z, E_X, B_Y)$, $a_k = (-1)^k(1-\delta_{k0}/2)$, $u = ct$ and $h = 0.01m$ is the distance between the parallel conducting plates. The lab charge and current densities $(\rho_L, \mathbf{J}_L)$ are given in terms of the beam frame spatial density by Eq. (2) in [2]. The lab and beam frames are shown in figure 1 (left frame). $\boldsymbol{R} = \boldsymbol{R}_0(s) + x\boldsymbol{n}(s)$ defines the beam to lab transformation. The beam frane phase space coordinates are (z, p_z, x, p_x). Here $z(s) = s - \beta ct(s)$, where $t(s)$ is the time of arrival at arc-lenght s, $p_z(s) = (E(s)-E_0)/E_0$ with $E_0 = \gamma mc^2$ the energy of the reference particle and $p_x(s) = v_x(s)/\beta c$ where v_x is the velocity component along $\boldsymbol{n}$. The equations of motion in the beam frame are

$$z' = -\kappa(s)x,\ p_z' = F_z,\ x' = p_x,\ p_x' = \kappa(s)p_z + F_x. \quad (2)$$

Here the collective force is $F_z = c_1\boldsymbol{V}\cdot\boldsymbol{E}$, $F_x = c_2(-X_0'(s)E_Z + Z_0'(s)E_X + v_0 B_Y)$ and $c_1 = e/(\beta cE_0)$, $c_2 = e/(\beta^2 E_0)$, $\boldsymbol{V} = v_0\big(\boldsymbol{t}(s) + p_x\boldsymbol{n}(s)\big)$, $\boldsymbol{E} = (E_Z, E_X)$ and B_Y are evaluated at $\boldsymbol{R} = \boldsymbol{R}_0(s) + x\boldsymbol{n}(s)$ and $u = (s-z)/\beta$. There is a typo in [2], the temporal argument of $\mathcal{E}$ is $(s-z)/\beta$ instead of s. However, the simulations were done with the correct argument.

In the VM approach, the fields and equations of motion are coupled. Our lab to beam transformation given in Eq. (2) of [2] relates the beam frame spatial density to the lab frame charge and current densities and will be discussed in a forthcoming paper. In the LMA, the fields are computed from (1) using the source from the unperturbed bunch. This is important as it uncouples the field calculation from the equations of motion (2).

The initial phase space density with linear chirp is

$$F = Ke^{-\frac{p_z^2}{2\sigma^2}\frac{1}{2\sigma_u^2}(p_z-uz)^2 - \frac{1}{2\epsilon_0\beta_0}(x^2+(\alpha_0 x+\beta_0 p_x)^2)} \quad (3)$$

where $K = u/(4\pi^2\epsilon_0\sigma_u\sigma)$, and the source, in the LMA, evolves according to (2) with $F_z = F_x = 0$. Instead of calculating the fields directly, we calculate F_z and F_x. These are calculated up front and on a grid adapted to the unperturbed beam frame charge density. The equations of motion (2) are transformed to the interaction picture

$$\begin{aligned} z_0' &= -R_{56}(s)F_z - D(s)F_x,\ \ p_{z0}' = F_z, \\ x_0' &= (sD'(s) - D(s))F_z - sF_x,\ \ p_{x0}' = -D'(s)F_z + F_x \end{aligned}$$

where D, D' and R_{56} are the standard lattice functions, and integrated using the Euler method and interpolation to determine F_z and F_x at their appropriate arguments.

NUMERICAL STUDIES

In [2] we presented preliminary numerical results in the LMA for the benchmark 5GeV bunch compressor studied in [3]. Here we continue that study, giving a rather complete description of moments and reduced densities with an emphasis on the effects of shielding and the transverse force F_x. Our formula with shielding, a image charge expansion, is given by (1). In our simulations we calculated the first 10 terms of (1). The formula without shielding is given by the $k = 0$ term. We study the LMA with particles, thus the initial phase space positions of the particles are randomly generated according to (3). Our code is fast enough so that we can study reduced densities.

We first discuss the x-emittance, see figure 1 (right frame). The unperturbed value starts and ends at 10^{-6} m-rad, and on the scale shown it behaves essentially like the

* Work supported by DOE grant DE-FG02-99ER1104
† gbassi@math.unm.edu

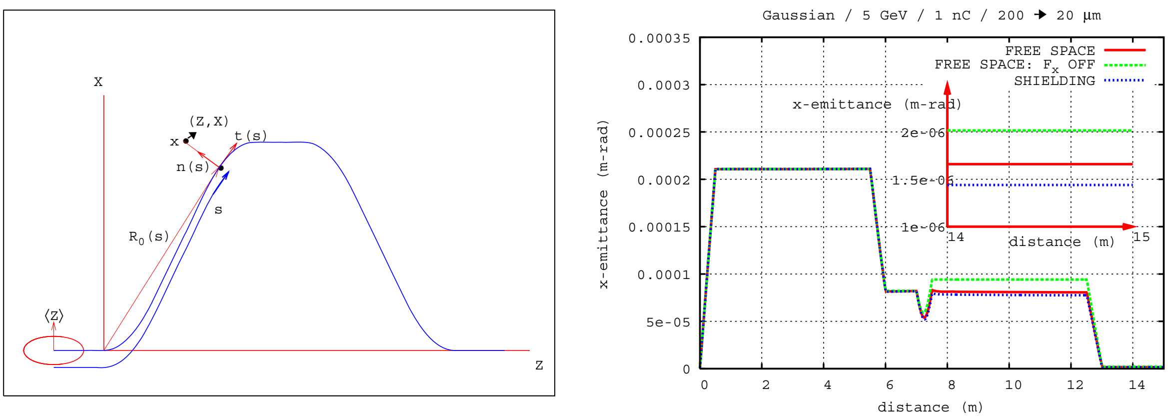

Figure 1: Left: lab frame (Z, X) and beam frame (s, x) for a chicane bunch compressor. Right: Normalized transverse emittance (x-emittance). In red free space case, in green free space case without transverse force, in blue with shielding. The values at the end of the chicane are 1.67 mm-mrad, 2.01 mm-mrad and 1.44 mm-mrad respectively.

blue (with shielding) curve. However, the insert shows significant differences in the drift at the end of the chicane. With shielding, which is the most realistic case, the CSR gives a significant, 44% increase. In the less realistic free space case there is a 67% increase and in the free space case without transverse force there is a 101% increase. Thus these last two cases overestimate the effect of the CSR and point out the importance of including both transverse force and shielding in understanding the x-emittance. In figure 2 (left frame) we show the p_z density at the end of the bunch compressor with (blue) and without (green) shielding and the stationary unperturbed density (black). The CSR has little effect on the tails and tilts the density to lower energy. The main point here is that it is easy to calculate reduced densities. In figure 2 (right frame) we show $\langle F_z \rangle$ as a function of distance along the chicane in the free space case. This is basically the mechanical part of the radiated power [5]. It is no surprise that the main action takes place inside the magnets, indicated by I, II, III and IV on the figure. It is interesting to note that the mechanical power becomes slightly positive at the beginning of the fourth magnet, indicating a transfer of energy from the field to the particles. The integral of $\langle F_z \rangle(s)$ is the mean energy loss and is shown in figure 3 (left frame) in three cases. The red curve clearly corresponds to the previous figure and all 3 curves are monotone decreasing except for the little blip at the entrance to the fourth magnet just discussed. In the free space case we see there is very little effect from the transverse force. However, the shielding has considerable effect. It decreases the energy loss, as is to be expected, since it cuts off the radiation at certain frequencies. The standard deviation of p_z is shown in figure 3 (right frame). Here we see very small changes. As in the case of the mean, the transverse force has very little effect, however the shielding case is quite different.

Although we can accurately calculate moments and reduced densities with particle simulations, our ultimate goal is to study the self consistent evolution of the phase space density with a PF method. The major difficulty here is caused by the strong correlation between the phase space variables which makes it hard to determine the support of the phase space density. We are learning from the LMA particle simulations how to solve this problem. Our first discovery is that the support of the spatial density can be taken to be that of the unperturbed density. We show our calculation of the spatial density with shielding at the end of the chicane in figure 4 (left frame). The density without shielding looks much the same at this scale.

In figure 4 (right frame) we show a scatter plot of the interaction picture variable $w = p_{z0} - uz_0$ vs z_0 at the beginning and the end of the chicane. The variation with s is basically to move the tip of the V slowly downward. Our original hope for the self consistent VM PF calculation was to use a fixed grid in the interaction picture, but here it is evident that a moving grid is necessary. An important aspect of our current work is to understand how to fix this grid, necessary in a VM PF method, but not in a VM code with particles, where one has only to fix the support of the spatial density. Thus we are currently also pursuing the less ambitious VM code with particles.

Acknowledgments

Thanks to Bob Warnock for helpful discussions.

REFERENCES

[1] R. Warnock, G. Bassi, and J. A. Ellison, Nucl. Instr. Meth. Phys. Res. Sect. A, Volume 558, Issue 1, 1 March 2006.

[2] G. Bassi, J. A. Ellison and R. Warnock, Proceedings of PAC2005, Knoxville, USA.

[3] ICFA Beam Dynamics Mini-Workshop on CSR, Berlin-Zeuthen, 2002, see `http//www.desy.de/csr`; L. Giannessi and M. Quattromini, Phys. Rev. ST Accel. Beams **6**, 120101 (2003).

[4] G. Bassi, T. Agoh, M. Dohlus, L. Giannessi, R. Hajima, A. Kabel, T. Limberg and M. Quattromini, Nucl. Instr. Meth. Phys. Res. Sect. A, Volume 557, Issue 1, 1 February 2006.

[5] K. Heinemann et. al, these proceedings.

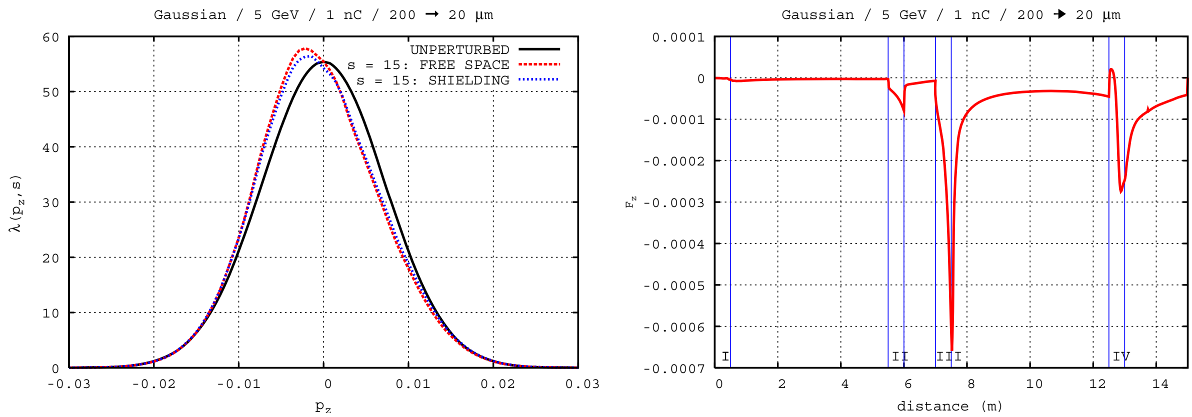

Figure 2: Left: $\lambda(p_z, s)$, the density in p_z. In black stationary unperturbed distribution, in green free space case at $s = 15$ and in blue with shielding at $s = 15$. Right: mean of F_z. The magnets are indicated by I, II, III and IV.

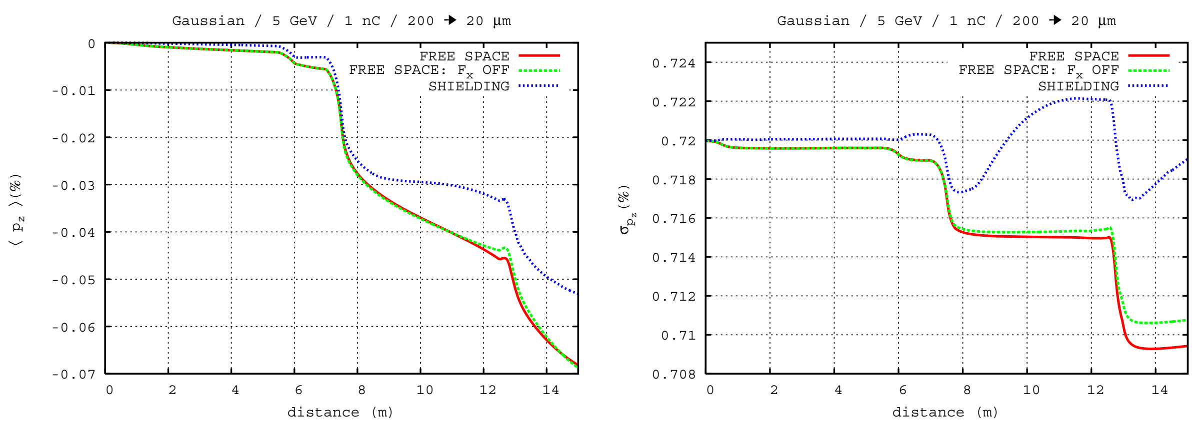

Figure 3: Left: mean of the relative energy loss. Right: standard deviation of the relative energy deviation. In red free space case, in green free space case without transverse force, in blue with shielding.

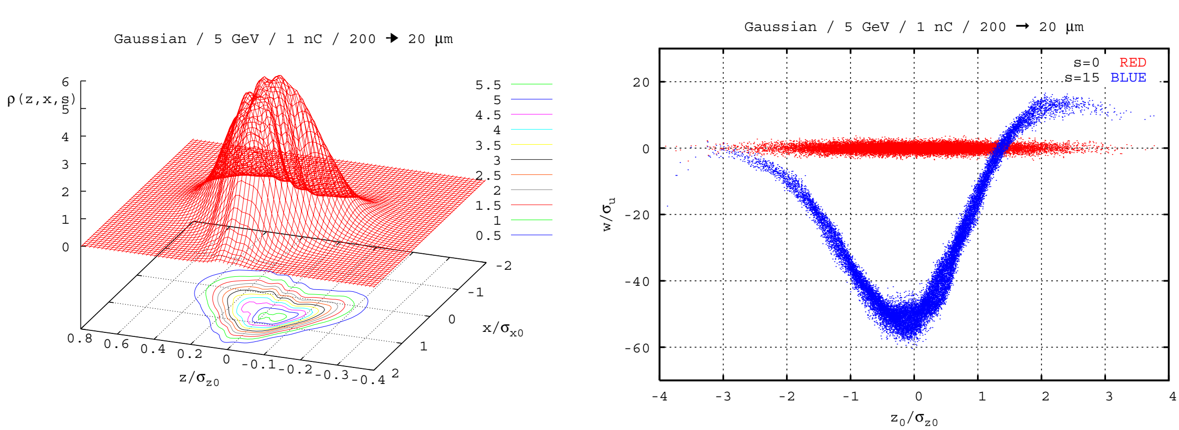

Figure 4: Left: perturbed charge density with shielding at s = 15m (end of chicane). Here $\sigma_{z0} = 2mm$ and $\sigma_{x0} = 0.64mm$. Right: Scatter plots of the interaction picture variable $w = p_{z0} - uz_0$ vs z_0 at $s = 15$, where u is the slope in the initial p_z, z correlation. Here $u = -36m^{-1}$ and $\sigma_u = 2 \times 10^{-6}$.