MAGNET SIMULATIONS FOR MEDICAL FFAG *

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Abstract

In the frame of the RACCAM project (this conference) mathematical tools have been developed for magnetic field symbolic calculations. One of the main methods to generate the non linear magnetic field $B_0(r/r_0)^k$ needed for FFAG accelerators, consists in a distribution of conductors on a flat pole. This can make the machine more compact. The main field is produced by a main coil, then conductors introduce the non linearity. This is a simple application of the Biot and Savart law that can apply to a large variety of practical cases. This calculation tools has been set up as a *Mathematica* function.

INTRODUCTION

The RACCAM project aims to study fixed field alternating gradient accelerators for medical applications. It is now focused on the design of a spiral FFAG. This kind of circular accelerator allows to obtain high energy beams and high repetition rates with a relatively compact machine that gets rid of the pulsed magnetic field in the dipoles of the machine ring. To compensate the energy variation the field gradient must increase following the radial variation $B_0(r/r_0)^k$, where B_0 is the maximum field and r the machine ring radius.

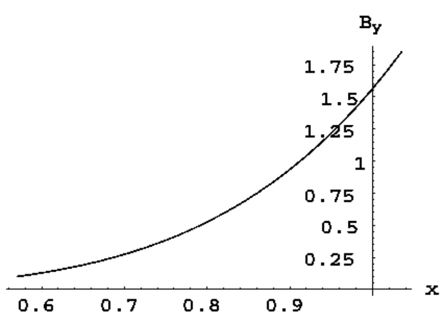

Figure 1: Vertical component of magnetic field along the machine ring radius.

There are two main methods to produce this non linear field. The most evident one is to use pole shaped yoke. But it has the disadvantage to give a non adjustable variation of the field, determined by the exponent factor k

The Japanese example showed that the best way to produce the non linear field $B_0(r/r_0)^k$ compatible with compactness of the machine, is to distribute conductors on a flat pole.

A first numerical simulation with *Poisson-Superfish* with empirically determined current intensity law $I_0(r/r_0)^k$ gave us the scale of intensities to expect, from roughly a hundred amperes to 2000 A. This way to calculate is not satisfying because the current law has to be derived directly from the magnetic field law delivered by beam dynamics.

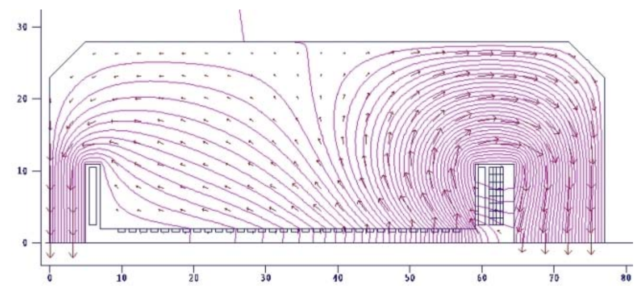

Figure 2: *Poisson-Superfish* simulation with parameters of spiral dipoles from the ADSR KURRI project in Japan [1].

PAIR OF SYMMETRICAL CONDUCTORS

To evaluate the magnetic field construction, the symbolic calculation appears to be an efficient tool of evaluation of the contribution of pairs of conductors to the total magnetic field.

One Pair of Conductors

From Biot and Savart law, a single conductor traversed by the current intensity I at location (ξ,η) generates a magnetic field in the median plane (x,0) with the following 2d components:

- $$B_x(x,\xi,\eta)=\frac{\mu_0 I}{4\pi}\frac{\eta}{(-x+\xi)^2+\eta^2}$$
- $$B_y(x,\xi,\eta)=\frac{\mu_0 I}{4\pi}\frac{-x+\xi}{(-x+\xi)^2+\eta^2}$$

A *Mathematica* function was written which evaluates the symbolic expression of the magnetic field components by a distribution of conductors. Now considering a pair of conductors located at (-a,g) and (a,g) with current intensity $4\pi/\mu_0$, the 2d components expressions are then:

- $$B_x(x,g,n)=\frac{g}{(-x-ng)^2+g^2}+\frac{g}{(-x+ng)^2+g^2}$$
- $$B_x(x,g,n)=\frac{-x-ng}{(-x-ng)^2+g^2}+\frac{-x+ng}{(-x+ng)^2+g^2}$$

where g is the gap height and ng=a the radial position, so that the conductor position is only determined with the gap height. The derivative of the vertical component then comes easily :

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$$\frac{dB_y}{dx}(x,g,n)=\frac{1}{(-x-ng)^2+g^2}-\frac{1}{(-x+ng)^2+g^2}+\frac{2(-x-ng)^2}{\left((-x-ng)^2+g^2\right)^2}+\frac{2(-x+ng)^2}{\left((-x+ng)^2+g^2\right)^2}$$

From this derives the following conditions for the magnetic fields sum:

- n<1: only two maximum because conductors are too close to distinguish each contribution. The maximum field tends to twice the maximum field of a single conductor.
- n=1: two extremum and an inflexion point at (0,0) because the distance between conductor is equal to gap height.
- n>1: four extremum because as conductors get far from each other the single conductor contribution appears.

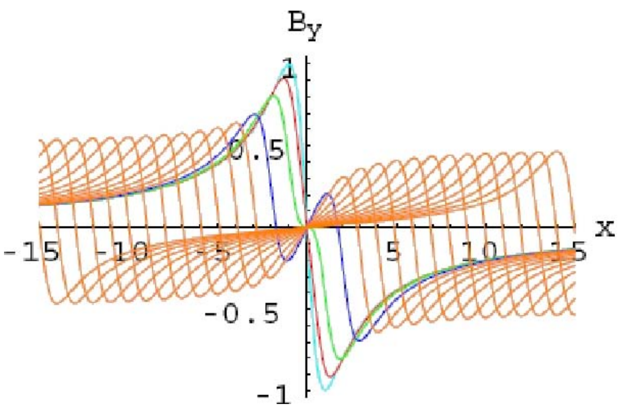

Figure 3: Pairs contributions to vertical component of the magnetic field with n<1 for light blue and red curves, n=1 for the green curve, and n>1 for the dark blue (n=2) and orange curves.

It can be noticed that all curves cross the (0,0) point. That implies a zero field gradient zone can be found immediately, just looking at the curves.

N Pairs of Conductors

Suppose a series of conductors with current intensity $4\pi/\mu_0$ and coordinates (a,±g), the field components then reads:

- $$B_x(x,g,n)=\sum_{i=1}^{N}\frac{g}{(-x-i\times ng)^2+g^2}+\frac{g}{(-x+i\times ng)^2+g^2}$$
- $$B_x(x,g,n)=\sum_{n=1}^{N}\frac{-x-i\times ng}{(-x-i\times ng)^2+g^2}+\frac{-x+i\times ng}{(-x+i\times ng)^2+g^2}$$

For a given x, B_x tends to 1/ng and B_y tends to -x/ng with increasing n. Assume given constant gap length, the vertical component is then linearized with increasing distance between conductors, whereas the maximum field diminishes (see figure 4).

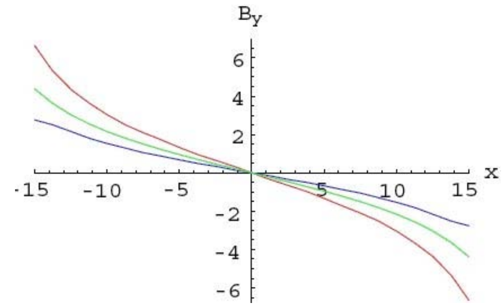

Figure 4: Sum of the fields produced by N pairs of conductors, with an interval of g (red), 1.3g (green) and 2g (dark blue). N decreases with increasing n.

From the Field Law to the Current Law

Now we know how to build the field with pairs of conductors, the current law should determined with respect to the field law calculated. Assume there are N conductors and P observation points. Then the Biot and Savart law turns to a simple matrix equation. For example, the vertical component is determined with $A\times I+B_y=0$, where:

$$B_y=\begin{pmatrix}B_{y1}\\B_{y2}\\\vdots\\B_{yP}\end{pmatrix},\quad I=\begin{pmatrix}I_1\\I_2\\\vdots\\I_N\end{pmatrix}$$

and

$$A=\begin{pmatrix}\frac{x_1-\xi_1}{(x_1-\xi_1)^2+(y_1-\eta_1)^2} & \cdots & \frac{x_1-\xi_N}{(x_1-\xi_N)^2+(y_1-\eta_N)^2}\\ \vdots & \ddots & \vdots\\ \frac{x_P-\xi_1}{(x_P-\xi_1)^2+(y_P-\eta_1)^2} & \cdots & \frac{x_P-\xi_N}{(x_P-\xi_N)^2+(y_P-\eta_N)^2}\end{pmatrix}$$

where (ξ,η) refer to conductor coordinates, (x,y) to observation point coordinates.

As N is forcedly equal to P (usually P>N), the system may not be well conditioned. To reduce the dimensions of the system to the right number of equations, each member of the matrix equation is multiplied by the transpose of A. The system now reads as follow : $A^TA\times I=A^TB_y$.

Assuming the field law $B_y=1.56\,x^{4.89}-0.533$, depending on the number of conductors, oscillations appear that could be a disaster for beam dynamics (see fig. 5).

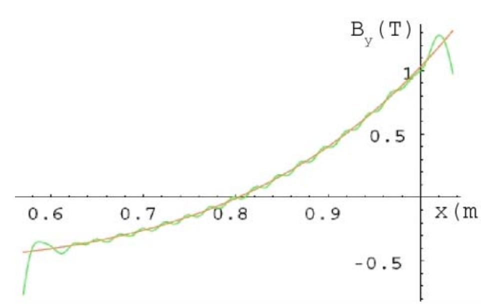

Figure 5: Comparison between theoretical field and calculated field for 20 conductors on a pole. Oscillations disappear with 80 conductors, except on extremities.

Another problem is the wide oscillation of the current intensity values near the extremities of the distribution (see fig. 6), which can reach unrealistic values for water-cooled conductors of 10mm×10mm section. This is due to missing parameters in the treatment of the problem as the finite dimensions of the system and the iron yoke influence. A solution to correct this quickly is to enlarge the conductors distribution length. If the density of conductor remains unchanged and the length is sufficiently enlarged, the oscillations disappear from the initial distribution position, and current intensity increases because of decreasing influence of the conductors from extremities (see fig. 6).

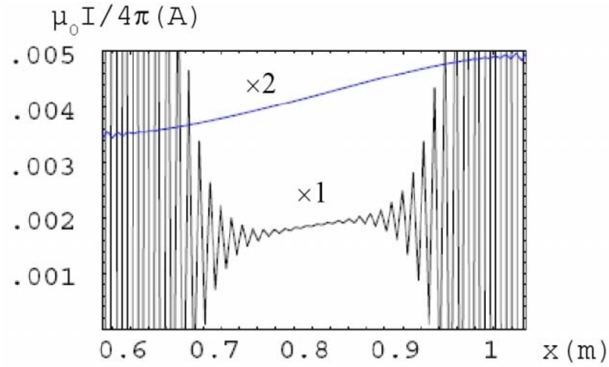

Figure 6: Current values for once (black) and twice (blue) the conductors distribution length.

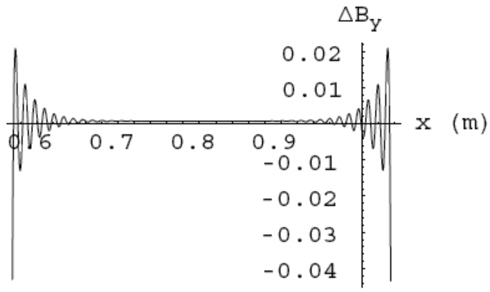

Figure 7: Difference of field values calculated from different lengths of conductors distributions (see fig. 6).

Whereas current intensities are different, one would expect that fields are equal. But there are differences, that can reach 1% of the maximum field.

PAIR OF THICK SHEETS

Uniform Thickness

To get rid of the discrete distribution of conductors, N may tends to infinity so that the result is a conducting sheet of uniform thickness. The field components in the median plane are then the integral over ξ. If the limit of the sheet along the ξ axis are p and q, and the current density $4\pi/\mu_0$, the 2d field components the reads:

$$B_x(x,\xi,\eta)=\arctan\left(\frac{-p+x}{\eta}\right)-\arctan\left(\frac{-q+x}{\eta}\right)$$

$$B_y(x,\xi,\eta)=\frac{1}{2}\log\left(\frac{(-q+x)^2+\eta^2}{(-p+x)^2+\eta^2}\right)$$

The field produced by this sheet can then be compared to the field produced by 21 conductors with the same current intensity ($4\pi/\mu_0$, see fig.8). The result show that the maximum of the field is closer to the centre of the distribution, and the absolute value of the field is greater in the region delimited by p and q.

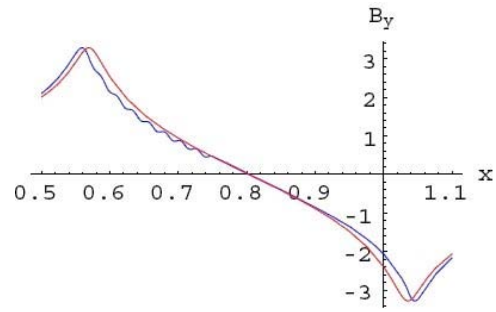

Figure 8: Fields created with a conductors distribution (blue) and a uniform thickness sheet (red).

The most remarkable result is the smoothness of the field produced by the sheet. Oscillations have totally disappeared.

Variable Thickness

To produce the nonlinear field that FFAGs need with a sheet, the physical parameter that has to vary is the sheet thickness. If h(ξ) is the thickness variation along the ring radius, then 2d field components are written as:

$$B_x(x,\xi,\eta)=\int_p^q\int_{h(\xi)}^{g}\frac{\eta}{(-x+\xi)^2+\eta^2}\,d\xi\,d\eta$$

$$B_y(x,\xi,\eta)=\int_p^q\int_{h(\xi)}^{g}\frac{-x+\xi}{(-x+\xi)^2+\eta^2}\,d\xi\,d\eta$$

These integrals lead to symbolic solutions in a few cases. They have to be integrated numerically most of the time.

TO DO...

These calculations are the first attempt to set up a tool for 2d magnetic field calculations that would allow to converge quickly towards the best magnet design. These tools are written as *Mathematica* functions which need improvements:

- include the iron yoke influence,
- calculate fields with respect to the finite dimensions of the system,
- improve equations writing.

ACKNOWLEDGMENTS

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REFERENCE

[1] Y. Ishi, "Accelerator design and construction for FFAG-KUCA ADSR", presentation, Oct. 15th 2004.