STUDIES OF A GAS-FILLED HELICAL MUON BEAM COOLING CHANNEL*

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Abstract

A helical cooling channel (HCC) can quickly reduce the six dimensional phase space of muon beams for muon colliders, neutrino factories, and intense muon sources. The HCC is composed of solenoidal, helical dipole, and helical quadrupole magnetic fields to provide the focusing and dispersion needed for emittance exchange as the beam follows an equilibrium helical orbit through a continuous homogeneous absorber. The beam dynamics of a gas-filled helical muon beam cooling channel is studied by using Monte Carlo simulations. The results verify the cooling theory [1] of the helical magnet. The cooling performance has been improved by correcting chromatic aberration and the non-linear effects caused by the ionization energy loss process. With these improvements, a simulated cooling channel of 160 meters length has achieved a reduction of 6-dimensional (6D) phase space by a factor of 50,000.

INTRODUCTION

The fast reduction of the 6D phase space of muon beams is essential for muon colliders and beneficial for neutrino factories and intense muon sources. For instance, the desired cooling factor [2] (ratio between the initial and final 6-dimensional beam emittances) of a muon collider is ~10^6 which should be achieved within a muon lifetime (2.2 μs for a stationary muon).

In the simulations described below, muon beams are cooled by passing them through a continuous gaseous hydrogen energy absorber, where the muons are accelerated by RF cavities imbedded in a special magnetic field that provides focusing and dispersion for emittance exchange.

The HCC magnet is composed of superconducting solenoid, helical dipole, helical quadrupole, and helical sextupole current coils. Inside the magnet coils, a series of high-pressure, hydrogen gas-filled RF cavities provides the large gradient needed to compensate the ionization energy loss and to provide the needed RF bucket area to capture the large emittance of the muon beam. The high-pressure gas also serves as energy absorber for muon ionization cooling. The peak RF gradient and RF phase are chosen to compensate for energy loss in the gas, maintaining the design reference energy down the channel. Recent experiments indicate that the pressurized RF cavities used in these simulations will work inside strong magnetic fields [3].

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HELICAL COOLING CHANNEL

In ionization cooling, all momentum components of a muon are reduced in passing through an absorber, while only the longitudinal momentum is regenerated by the RF cavities. This reduces the angular spread of a beam. Appling this technique as the beam undergoes betatron motion allows the four transverse dimensions to be cooled. In the HCC, the helical dipole magnet is used to produce a continuous dispersive field needed for emittance exchange and longitudinal cooling. The beam is stabilized by adding solenoid, helical quadrupole and sextupole components. The required field calculations for each component are given in reference [1] and modified in reference [4].

SIMULATION STUDIES

Simulations have been done to verify the cooling theory in the HCC. For instance, the theory predicts the cooling decrements in the HCC, which have been studied using G4Beamline [5] and ICOOL [6] simulation programs under the condition of equal cooling decrements [7]. We found the emittance reduction rates and equilibrium emittances are well predicted by the cooling theory.

Chromatic Aberration Correction

We have added a chromatic aberration correction to the basic theory [1] with a helical sextupole component. The field strength of this sextupole on the reference orbit is given by

$$b'' = b' / aD \quad (1)$$

where b' and b'' are the field strengths of quadrupole and sextupole components on the reference orbit, a is the reference radius, and D is the dispersion function. Using this correction, the acceptance of the HCC is increased by 20% and the cooling factor is increased by about 10%.

Correction for Non-Linear Energy Loss

The analytical calculations of beam dynamics given in reference [1] assume an average energy loss rate. The assumption is valid for beam momenta down to 200 MeV/c. Below this, the betatron motion becomes non-linear due to the non-linearity of ionization energy loss as a function of momentum. This non-linear term can be corrected by tuning the dispersion function. The dispersion function is expanded by the Taylor expansion along with the beam path s and is given by

$$D \rightarrow D(s) + sD'(s) + \tfrac{1}{2!} s^2 D''(s) + \cdots, \quad (2)$$

where D is the dispersion function and prime means the deviation along the beam path. The coefficients are

$$D' = \left(\frac{\Delta a}{\Delta p}\right)\frac{\partial p}{\partial s} = \hat{D}\frac{dp}{ds}, \qquad D'' = \left(\frac{\Delta a}{\Delta p}\right)\frac{\partial}{\partial s}\frac{\partial p}{\partial s} = \hat{D}\frac{d^2 p}{ds^2}, \quad (3)$$

where $\hat{D}$ is the dispersion function determined from the linear particle motion and is given by equation (4.20) in reference [1]. The partial deviation of the momentum is transformed to the total differentiation since the momentum deviation by the energy loss process is taken along with the beam path. That is,

$$-\frac{dp}{ds} = \frac{E}{p}\frac{dE}{ds}, \quad -\frac{d^2 p}{ds^2} = \frac{dp}{ds}\frac{d\left(\frac{dp}{ds}\right)}{dp}, \quad (4)$$

where E is the total energy and dE/ds is determined from the Bethe-Bloch formula.

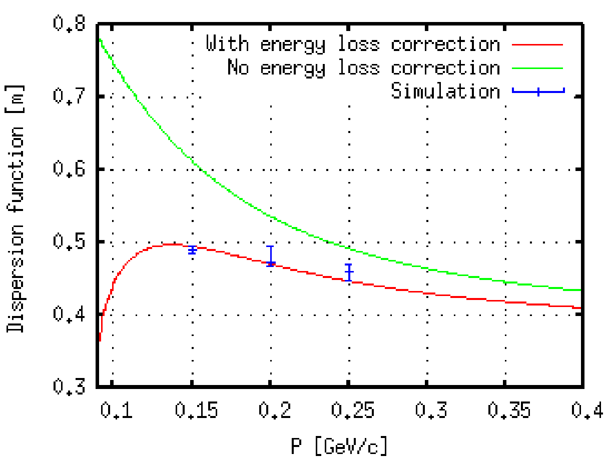

Figure 1: Tuning of the dispersion function to take non-linear energy loss effects into account. The green curve is the dispersion calculated from reference [1]. The red curve is calculated using equation (2). The blue points correspond to the best cooling factors achieved in the simulations.

Fig. 1 shows the original and corrected dispersion functions versus momentum compared to the values that give the best simulation cooling factors at different beam momenta (150, 200, and 250 MeV/c). The red line shows the expected dispersion function by using equation (2) while the green one is the calculated dispersion function with no energy loss correction.

Sequential HCC Segments

As the beam cools in 6D it can fit into smaller, stronger magnets and into smaller, higher frequency, higher-gradient RF cavities. These conditions make the cooling more effective. As an example, we designed a system of four sequential HCC segments to demonstrate this method of tailoring the channel to the beam dimensions. Each HCC has a different field configuration which is given in Table I. The first HCC has a big bore and low frequency RF for maximum acceptance. Because of the large bore, the field strength in the first one is weaker than the later ones. In this simulation, however, we kept the RF frequency at 200 MHz to avoid questions of longitudinal matching between segments.

Table 1: Series of HCCs design parameters. Helical Pitch is defined as tangent of the helical pitch angle of the reference orbit. Helical field strengths are quoted at the radius of the helical reference orbit. The pressure of gaseous hydrogen is 400 atm at room temperature

Ref. Momentum (MeV/c):	250	250	250	250
Helical Ref. Pitch:	1	1	1	1
Helix Period (m):	1	0.8	0.6	0.4
HCC length (m):	50	40	30	40
Ref. Orbit Radius (m):	0.16	0.13	0.09	0.06
Solenoid Field (T):	6.95	8.69	11.6	17.3
Hel. Dip. Strength (T):	1.62	2.03	2.71	4.06
Hel. Quad. Strength (T/m):	-0.7	-1.1	-2.0	-4.5
Hel. Sext. Strength (T/m^2):	0.20	0.32	0.56	1.26
RF Frequency (MHz):	200	200	200	200
RF Phase (deg):	140	140	140	140
RF Gradient (MV/m):	33	33	33	33

The optimum length of each cooling segment has not yet been optimized. The transition to the next segment should be made while the cooling rate is high rather than wait for the beam to come close to the equilibrium emittance for the conditions of the present HCC. Doing this should reduce the total channel length.

The HCC segments that are used in these simulations are realistic only to the extent that two new technologies come to pass. The first is the use of high-pressure RF cavities, which have yet to demonstrate that they can operate in the radiation environment of a cooling channel with large ionization energy loss. Although the operation of pressurized cavities in a magnetic field has been shown to be possible, operation in radiation fields will only be tested next year when the beam line at the Fermilab MTA is installed. The second new technology is the use of high temperature superconductor operating at low temperature to allow high field at the helical dipole coils. Although there are new solutions to overcome this requirement, the simulations that are shown here assume that the helical dipole fields are generated by coils that encompass very large RF cavities and this implies large (~20 T) fields at the conductor.

FUTURE IMPROVEMENTS

Although we achieved a cooling factor 50,000 in the four sequential HCC segments, a muon collider may require something 20 times better. This may not be so difficult to achieve, in part because the present simulation codes use a scattering model for hydrogen that overestimates multiple scattering from hydrogen [8]. First tests indicate that the new model will give almost a factor of two improvement in each of the transverse planes and something less in the longitudinal direction.

Another improvement may be the development of techniques to cool at lower momentum where the effective beta function and equilibrium emittance will be proportionally better. Operating at 100 MeV/c rather than 250 MeV/c corresponds to two and a half times better equilibrium emittance in each transverse plane.

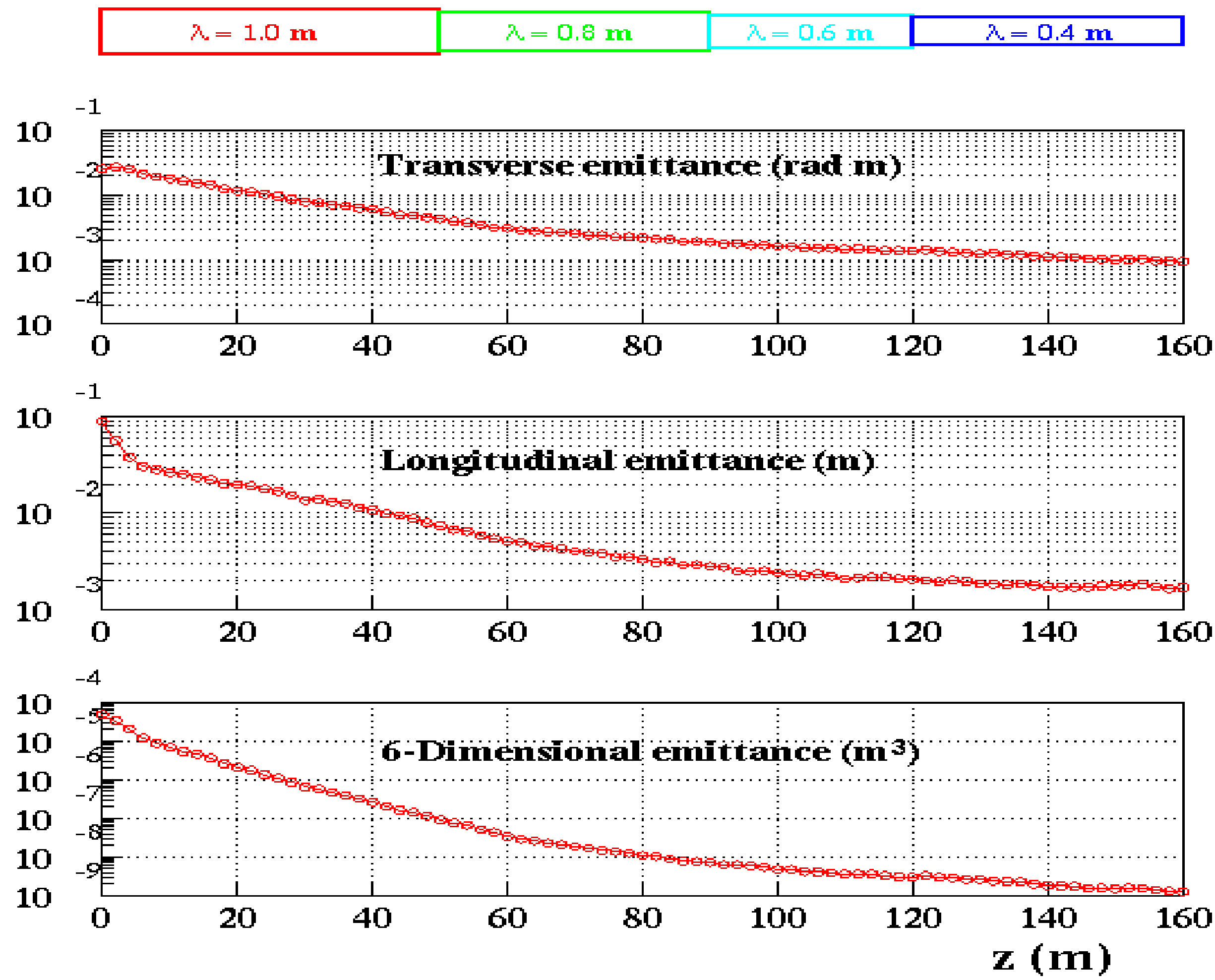

Figure 2: Emittance evolution of a muon beam in 4 sequential HCC segments. The cooling factor is 50,000 in a total channel length of 160 meters. The final emittances will be improved with a better simulation model.

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