PARAMETERS FOR ABSORBER-BASED REVERSE EMITTANCE EXCHANGE OF MUON BEAMS*

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Abstract
The normalized longitudinal emittance of a muon beam after six-dimensional ionization cooling appears very small compared to the value that could be utilized or maintained after acceleration to muon collider energy. This circumstance offers the possibility for further reduction of the transverse emittance by introducing absorber-based reverse emittance exchange (REMEX) between longitudinal and transverse degrees of freedom before acceleration to high energy. REMEX follows Parametric-resonance or Phase Ionization Cooling (PIC) [1] and is accomplished in two stages. In the first stage the beam is stretched to fill the RF bucket at the initial cooling energy. In the second stage the beam is accelerated to about 2.5 GeV, where energy straggling begins to limit the absorber technique, and stretched again. The potential transverse emittance reduction and the intrinsic limitations of the REMEX technique have been analyzed earlier [2]. In this report, we discuss and develop the requirements to achieve the maximum REMEX effect.

INTRODUCTION

After 6D cooling [3], the longitudinal emittance is small enough to allow high frequency RF for acceleration. However, the longitudinal emittance after the beam has been accelerated to collider energy is thousands of times smaller than necessary to match the beta function at the collider interaction point. We plan to repartition the emittances to lengthen the muon bunch and shrink the transverse bunch dimensions using linear cooling channel segments and wedge absorbers. In addition, a new concept developed to coalesce muon bunches will be included in the optimization of the longitudinal phase space to enhance collider luminosity.

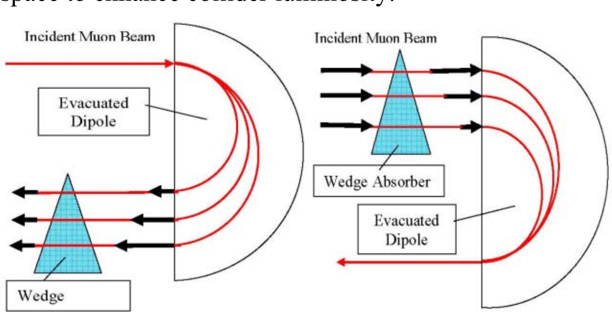

Figure 1: (a): Emittance Exchange. (b): REMEX.

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Figure 1a is the conceptual diagram of the usual mechanism for reducing the energy spread in a muon beam by emittance exchange, where an incident beam with small transverse emittance but large momentum spread (indicated by black arrows) enters a dipole magnetic field. The dispersion of the beam generated by the dipole magnet creates a momentum-position correlation at a wedge-shaped absorber. Higher momentum particles pass through the thicker part of the wedge and suffer greater ionization energy loss. Thus the beam becomes more monoenergetic. The transverse emittance has increased while the longitudinal emittance has diminished.

Figure 1b shows the conceptual diagram of the new mechanism for reducing the transverse emittance of a muon beam by reverse emittance exchange where an incident beam with large transverse emittance but small momentum spread passes through a wedge absorber creating a momentum-position correlation at the entrance to a dipole field. The trajectories of the particles through the field can then be brought to a parallel focus at the exit of the magnet. Thus the transverse emittance has decreased while the longitudinal emittance has increased.

CONSERVATION CONSIDERATIONS

Both the conventional and reverse emittance exchange processes are governed by the *dissipation theorem* [4]:

$$\Lambda_1 + \Lambda_2 + \Lambda_E = \Lambda_6 \equiv 2E_l'/E \qquad (1)$$

where Λ_1 and Λ_2 are the two transverse decrements and Λ_E is the energy spread decrement, $E = \gamma mc^2$ is the muon energy, E_l' is the energy loss rate in the absorber, and Λ_6 is the cooling decrement of the six-dimensional beam emittance. Each of the three individual decrements may be positive or negative with no limitation in principle on its magnitude. But the sum of the decrements, as the theorem states, is not influenced by coupling or emittance exchange at all, and is simply determined by the energy loss rate averaged along the beam path. When applied to the 6D or basic cooling of the initial muon beam (where each decrement should be positive), the theorem shows that none of the three decrements exceeds the Λ_6 value. In an optimum linear or ring cooler design, the decrements are equalized to $\Lambda_6/3$, as in the case of the 6D helical cooling channel in the 6D column of Table I.

In order to prepare the beam for a high energy collider after 6D and PIC cooling, we would like the transverse emittance to damp ($\Lambda_1 = \Lambda_2 = \Lambda_{tr} > 0$) and the longitudinal emittance to grow. In this case, the dissipation theorem does not prohibit a transport line design with a large emittance exchange rate, e.g. $-\Lambda_e >> \Lambda_6$ and $\Lambda_{tr} >> \Lambda_6$. Let us introduce a ratio parameter $\chi = -\Lambda_E / \Lambda_6$. Then the transverse decrement is found as

$$\Lambda_{tr} = \frac{\chi+1}{2}\Lambda_6 = (\chi+1)\frac{E_l'}{E}.$$

A large χ value, i.e. a large REMEX rate, is desirable to get the maximum reduction of the equilibrium transverse emittances. A large χ ratio can be obtained by introducing absorber wedges with large angles at positions with the appropriate energy-orbit dispersion.

The product of the three normalized emittances is approximately conserved in REMEX, so that a simultaneous reduction of each transverse emittance by a factor k causes an increase of the energy spread by a factor k^2. There are two obvious ways to reduce the initial energy spread to accommodate this rapid increase of energy spread from REMEX. First, it can be reduced by *longitudinal decompression* of the cooled muon bunches at the same time as or before the REMEX process. By decompression we mean that we can perform a bunch rotation to exchange short bunch structure and large momentum spread for a longer bunch with smaller momentum spread. Second, the energy spread can be reduced by *accelerating the beam*, so that $\Delta E / E$ decreases with increasing energy.

BALANCE EQUATIONS

In this section we derive some useful relationships in order to estimate the potential emittance and corresponding luminosity improvements that might be expected from REMEX. The reader may wish to concentrate on the final balance equations (8) and (9), for example to see the explicit dependence on the beam energy and the straggling terms.

Because of beam acceleration and decompression, we consider the balance in terms of the Courant-Snyder dynamic invariants. First, we assume that the beam is bent in the horizontal plane with no coupling between horizontal and vertical motion in the focusing magnetic field. The Courant-Snyder or adiabatic invariant of particle motion in a horizontal plane can be defined in its simplest form as

$$I(x,x',\Delta p) = \frac{\gamma\beta}{2f}[(x - D\frac{\Delta p}{p})^2 + f^2 x'^2] \qquad (2)$$

Here x, x$'$ and Δp are the deviations of particle coordinate, angle, and momentum from the reference orbit, $\gamma = 1/\sqrt{1-\beta^2}$ is the Lorentz factor, f is the local focusing parameter (beta-function), and D is the momentum-orbit dispersion function. Both f and D may vary along the beam path with periods of the focusing structure, although we neglect here the terms in the expression for Courant-Snyder invariants associated with f' and D', assuming them insignificant in the absorber plates. The x coordinate, generally, is represented as a superposition of the dispersion part and so-called betatron oscillation part x_b:

$$x = D\frac{\Delta p}{p} + x_b; \quad x_b \equiv \sqrt{2If/\gamma\beta}\cos\psi$$
$$x' \approx -\sqrt{2I/f\gamma\beta}\sin\psi; \quad \psi' = 1/f \ . \qquad (3)$$

Here ψ is the betatron oscillation phase. The transverse normalized emittance is defined as $\varepsilon_{tr} = \overline{I}$, and the longitudinal normalized emittance as $\varepsilon_z = \left(\overline{s^2}\,\overline{(\Delta p)^2}\right)^{1/2} / mc$. Here s is the particle longitudinal position relative to the reference particle averaged over the fast beating associated with betatron oscillations of particle transverse position along a curved orbit.

For the balance equations we must account for the dissipation terms associated with particle energy and angle decrease due to loss of vector momentum in the absorber and also for the diffusion terms due to angular multiple scattering and energy straggling:

$$I' = \frac{\gamma\beta}{f}\left[-D\frac{p'}{p}x_b + f^2 p' x_b'^2 + \frac{1}{2}D^2\frac{(\Delta p)^{2\,\prime}_{str}}{p^2} + \frac{1}{2}f^2\left(x^2\right)'_{scat}\right] \qquad (4)$$

$$\varepsilon_z^{2\prime} = \overline{s^2}\left[2\overline{p'\Delta p} + \overline{(\Delta p)^{2\prime}}\right] + \overline{(\Delta p)^2}\left(\overline{s^2}\right)'_{scat} \qquad (5)$$

In an absorber with transverse energy loss gradient, we have to expand p' as

$$p' = p_0' + \frac{\partial p'}{\partial x}x \equiv p_0'\left[1 + \frac{1}{h}\left(D\frac{\Delta p}{p} + x_b\right)\right]. \qquad (6)$$

Here we have introduced the notation $h = p_0' \big/ \partial p'/\partial x$ as an effective height of the absorber wedge.

Let us also introduce a parameter ξ as the ratio of the beam transverse size and h:

$$D^2\frac{1}{q^2} \equiv D^2\frac{\overline{(\Delta p)^2}}{p^2} = (\xi h)^2, \quad \overline{x_b^2} = (\xi h)^2 \qquad (7)$$

Here the value q is introduced as the inverse of the relative energy spread. We assume $\xi = 1/3$.

Next, we average over the betatron oscillation phase ψ in (4) and (5). To make the horizontal and vertical transverse emittance reduction equal on average, we can assume that the beam bend and dispersion interchange frequently between the horizontal and vertical planes. Taking also into account the relationships between the diffusion rates and fundamental 6D cooling decrement,

$$\left(x'^2\right)'_{scat} = \frac{Z+1}{2\gamma\beta^2}\frac{m_e}{m_\mu}\Lambda, \quad \frac{((\Delta p)^2_{str})'}{p^2} = \frac{\gamma^2+1}{4\log\gamma\beta^2}\frac{m_e}{m_\mu}\Lambda,$$

we obtain the final balance equations for normalized transverse and longitudinal emittance:

$$\varepsilon_{tr}' = \frac{\Lambda}{4\beta^2}\{-[q(\xi - \frac{m_e}{m_\mu}\frac{\gamma^2+1}{4\gamma\log}q)+2]\varepsilon_{tr} + (Z+1)f\beta\frac{m_e}{m_\mu}\} \quad (8)$$

$$\varepsilon_z' = \frac{\Lambda}{2\beta^2}\{[q(\xi + \frac{m_e}{m_\mu}\frac{\gamma^2+1}{4\gamma\log}q)+\frac{2}{\gamma^2}]\varepsilon_z + (Z+1)f\beta\frac{m_e}{m_\mu}\frac{\varepsilon_{tr}}{2\varepsilon_z}\} \quad (9)$$

To enhance efficiency, the optical concepts developed for PIC [5] can be applied to REMEX so that f can be as small as the plate thickness, w. The asymptotic solution of equation for $\varepsilon_\perp$ can be found as follows:

$$\frac{\varepsilon_\perp(z)}{\varepsilon_{\perp 0}} \Rightarrow \approx \sqrt{\frac{\varepsilon_{z0}}{\varepsilon_z}} + \frac{4\beta\beta_0}{3\xi(1-\eta)}\frac{w}{w_0}\frac{\Delta p}{p} \quad (10)$$

where the parameter η is associated with straggling impact: $\eta \equiv \frac{\gamma}{4\xi}\frac{p}{\Delta p}\frac{m_e}{m_\mu \log} \quad (<1)$

The initial value of the transverse emittance $\varepsilon_{\perp 0}$ that can be obtained after PIC is

$$\varepsilon_{\perp 0} = \frac{3}{4\beta}(Z+1)\frac{m_e}{m_\mu}w_0 \quad (11)$$

In equation (10), the first term describes the reverse emittance exchange and the second term takes into account the transverse scattering and energy straggling. Angle scattering and energy straggling will limit the achievable transverse emittance. The optimum maximum energy of REMEX is determined by energy straggling, which grows with energy. The following shows the expected REMEX change in the minimum $\varepsilon_\perp$ that can be found using equation (10) with appropriate choices of p, Δp, and ε_z. Note that the integrated energy loss in such a fast process as REMEX is relatively small, as is the beam loss.

Table 1: Main REMEX Parameters

Parameter	*Unit*	*Initial*	*After 1st stage*	*After 2nd stage*
Momentum	MeV/c	100	100	2500
Bunch length	cm	.5	10	10
Momentum spread	%	3	3	3
Longitudinal norm. emittance	cm	1.5×10^{-2}	.15	7.5
Transverse norm. emittance	µm	25	8	2

Equation (10) can be used to estimate the potential reduction of the transverse emittance in REMEX. As discussed earlier, the maximum effect can be achieved with two steps of REMEX. The first step can be performed at constant energy immediately after the basic 6D cooling. Here, the REMEX can be accompanied or preceded by bunch lengthening in the RF bucket. Maintaining the momentum spread achieved by the basic 6D cooling, the transverse emittance will be decreased by a factor $\sqrt{l/l_0}$ for a bunch length change from l_0 to l. For the second step, REMEX follows or is accompanied by beam acceleration and is effective up to the point that the growing energy straggling stops the transverse emittance decrease, determined by the nullification of equation (8). If REMEX ends at an energy where straggling is still not crucial (i.e. for $\gamma >> 1$, $\frac{m_e}{m_\mu}\frac{\gamma}{4\log}q << \xi$), then we can estimate ε_{tr} and $\Delta p/p$ as functions of energy.

There is a new idea to create intense bunches for muon colliders by coalescing several less-intense bunches in a high-energy storage ring [6]. Since coalescing and REMEX both use the momentum space generated by the acceleration to the high energy of a muon collider, the compatibility and optimum usage of a combination of these new techniques will have to be examined.

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